Scaling of Seepage Flow Velocity in Centrifuge Models

CUED/D-SOILS/TR326 (March 2003)

N.I.Thusyanthan[1] & S.P.G.Madabhushi[2]

Research Student[1], Senior Lecturer[2], Cambridge University Engineering Department

Abstract: It is a well-established fact that the scaling factor for seepage velocity in a centrifuge model is N where Ng is the gravity experienced by the model. However the derivation of this scaling law has lead to some controversy because the scaling law can be satisfied either by scaling the permeability or by scaling the hydraulic gradient. This technical note explores this issue in depth by considering the original Darcy's formulation. It will be shown that by separating the driving force (energy gradient) on the flow and the intrinsic permeability of the soil, the scaling law of seepage flow velocity can be better understood.

Key words: Darcy's permeability (hydraulic conductivity), intrinsic permeability, scaling law, centrifuge modelling

List of Symbols

A	cross section of soil sample
c	kozeny constant
d_m	effective particle size
g	acceleration due to gravity
$\Delta\bar{h}$	total head difference
h_1, h_2	Pressure heads
i	hydraulic conductivity
i_m, i_p	hydraulic conductivity in model, in prototype
K	intrinsic permeability
k	Darcy's permeability
k_{1g}	Darcy's permeability at 1g
ΔL	soil sample length
N	centrifuge acceleration level
n	porosity
ΔP	difference in fluid pressure
Q	total volume of fluid
s	specific surface
t	time in seconds
v	seepage velocity
v_m, v_p	seepage velocity in model, in prototype
$\Delta(v^2)$	difference in square of fluid velocity
Δz	height difference between two points
γ	unit weight of fluid
γ_{1g}	unit weight of the fluid at 1g
μ	dynamic viscosity of fluid
ρ	density of the fluid

1. Introduction

Centrifuge modelling technique is being used to study many different aspects of soil behaviour. Flow velocity in a centrifuge model at Ng will be N times faster compared to the prototype it represents. Consequently the scaling law for seepage velocity has been established as $v_m = N\,v_p$ (Schofield, 1980) and has been confirmed experimentally (Arulanandan et al.,1988). This scaling law for seepage velocity has been accepted and commonly used, but the question of whether it is the Darcy's permeability (hydraulic conductivity) or the hydraulic gradient that is a function of gravity has not been addressed properly. This issue was highlighted by Goodings (1979), who points out to the multiplicity of the concepts in scaling flow velocity. Butterfield (2000) and Dean (2001) also discussed this issue.

Pokrovsky and Fyodorov (1968), Cargill and Ko (1983) , Tan and Scott (1985) and more recently Singh and Gupta (2000) are among many others who have considered permeability (k) to be directly proportional to gravity and hydraulic gradient (i) to be independent of gravity. While this explains why seepage velocity has a scaling law of N ($v_m = N\,v_p$), there is an alternative explanation for the increase of seepage velocity in a centrifuge. Schofield (1980), Hussaini et al. (1981), Goodings (1984), and Taylor (1987) have all suggested that permeability to be independent of gravity and it is the hydraulic gradient which has got a scaling factor of N. Since both sides of the explanation result in the same final answer $v_m = N\,v_p$ and it is the final seepage velocity that is considered important in many cases, the controversy has often been overlooked.

In this technical note we attempt to resolve this controversy by using the energy gradient as the driving force on the pore fluid.

2. Darcy's law

In 1856, French hydraulic engineer Henry Darcy (1803-1858) published a report on the water supply of the city of Dijon, France. In that report, Darcy described the results of an experiment designed to study the flow of water through a porous medium. Darcy's experiment resulted in the formulation of a mathematical law that describes fluid motion in porous media. Darcy's Law states that the rate of fluid flow through a porous medium is proportional to the potential energy gradient within that fluid. The constant of proportionality is the Darcy's permeability (or hydraulic conductivity). Darcy's permeability is a property of both the porous medium and the fluid moving through the porous medium. In fact, Darcy's law is the empirical equivalent of the Navier-Stokes equations. Darcy's flow velocity for lamina flow is defined as the quantity of fluid flow along the hydraulic gradient per unit cross sectional area.

$$v = k\,i \qquad [1]$$

where k = Darcy's permeability or hydraulic conductivity (m/s)

i = hydraulic gradient

Hydraulic gradient i is defined as the rate of drop of total head along the flow path.

$$i = -\frac{\Delta \bar{h}}{\Delta L} \qquad [2]$$

i is the space rate of energy dissipation per unit weight of fluid.

Total head difference $\Delta \bar{h}$ is the sum of differences in velocity head, pressure head and elevation head.

$$\Delta \bar{h} = \left[\frac{\Delta P}{\rho g}\right] + \left[\frac{\Delta\left(v^2\right)}{2g}\right] + \Delta z \qquad [3]$$

$\Delta \bar{h}$ is the rate of energy loss per unit weight of the fluid. In many practical seepage flows the velocity heads are so small that they can be neglected.

Muskat(1937) produced the relationship between Darcy's permeability and the unit weight (γ) of the fluid.

$$k = K\frac{\gamma}{\mu} = K\frac{\rho g}{\mu} \qquad [4]$$

where K is known as intrinsic permeability which is a function of particle shape, diameter and packing , μ is the dynamic viscosity of the fluid, ρ is the density of the fluid and g is the acceleration due to gravity. While this relationship is broadly correct, this indicates that the permeability k is directly proportional to gravity. Is that right? According to the definition of i in

Eq. [2] the answer will be yes. However this poses other problems. For example let us consider a soil mass in space. If permeability is a function of gravity then soil will be impermeable at zero gravity as if it is a solid such as a rock with no pore space. However, at zero gravity, soil will still have pore spaces and if a fluid with pressure gradient were present across its boundaries then there would be flow. This contradicts the zero permeability concept. The same argument was first presented by Taylor (1987).Note that this flow is not driven by gravity but by exerted pressure difference. The reason for this apparent conflict is in the definition of Darcy's permeability k and hydraulic conductivity. To understand this better let us go back to the Kozeny-Carman equation described below.

Kozeny (1927) derived a relationship between intrinsic permeability (K) and porosity (n) by considering the porous medium as a group of channels of various cross sections of the same length. He solved the Navier-Stokes equations for all channels passing a cross section normal to the flow to obtain Kozeny's equation

$$K = \frac{c\,n^3}{s^2} \quad [5]$$

where c is the Kozeny constant whose value depend on the shape of the capillary(c=0.5 for a circular capillary). S is the specific surface of the channels (surface area per unit volume of the channel).

Carmen (1937,1956) improved Kozeny's equation to obtain

$$K = \frac{d_m^2}{180}\frac{n^3}{(1-n)^2} \quad [6]$$

which is commonly known as Kozeny-Carmen equation. The intrinsic permeability (K) of the porous media is linked to a characteristic particle diameter (d_m) and the porosity (n). The intrinsic permeability, unlike Darcy's permeability, is independent of gravity. This definition offers clarity to the centrifuge modellers studying seepage flows.

3. Fluid flow in prototype, model at 1g and model in a centrifuge

Using Darcy's permeability in a centrifuge could lead to some confusion. Lets consider an example in order to understand the conflicts faced in the usage of Darcy's permeability in centrifuge modelling. The model in Fig.1 could represent a prototype soil of height N×ΔL if placed in a centrifuge at Ng gravity. The static pressure difference between points B and C in the centrifuge model is ΔLρ(Ng), while that between corresponding points in the prototype is (NΔL)ρg. Similarly potential energy difference between these two points in the model and the prototype is the same. Hence it should be understood that the total driving force on the flow between B and C in the centrifuge model is the same as that between corresponding points in the prototype. Since the flow in model travels N times shorter distance, driving force on the flow per unit length is N times higher in the model. This suggests that $i_m = N i_p$. However the formal definition of hydraulic conductivity contradicts this. The hydraulic conductivity is the ratio of drop in total head to the length over which that drop occurs. Since both the total head difference and the flow path are N times smaller in the model, hydraulic conductivity can be argued to be the same in model and prototype(i.e $i_m = i_p$). This conflict could have been avoided if the hydraulic gradient was defined in terms of the energy gradient (pressure gradient + potential energy gradient) that actually drives the flow.

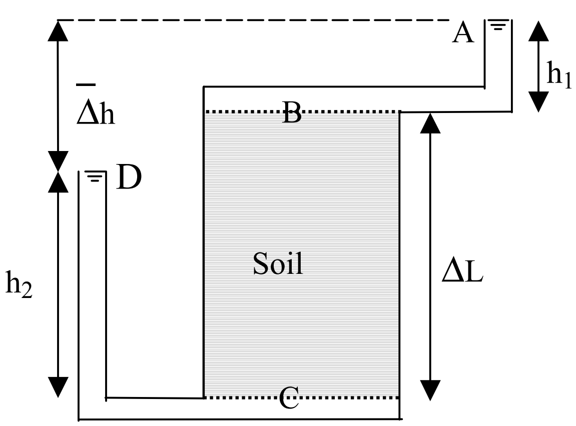

Figure 1. Centrifuge Model

The model in Fig. 1 can also be used to measure the Darcy's permeability of a soil sample. A constant head test can be done if the water level is maintained at levels A and D while flow through the sample occurs from B to C. Let the Darcy's permeability of soil sample at 1g be k_{1g} . k_{1g} can be measured using the following relationship.

$$k_{1g} = \frac{Q}{A\ i\ t} \qquad [7]$$

where Q = total volume of the fluid collected in time t (overflow collected from D)

$$i = \frac{\Delta \bar{h}}{\Delta L}$$

A = cross sectional area of soil column

Same test can be carried out in a centrifuge at Ng. Water levels at A and D need to be maintained as in the 1g test. If the water levels are maintained at A and D, the hydraulic gradient will not have changed because $\Delta \bar{h}$ and Δs are the same at Ng and 1g. The test result would show that the permeability at Ng to be N times the permeability at 1g. Singh and Gupta (2000) confirm this by similar tests (falling head rather than constant head). It should be noted that the gravitational force experienced by the model varies linearly with the radius. In case of small centrifuges, the rate of change of gravitational force within the model is high. Hence using N as a scaling factor for forces in the model is not valid. This fact was overlooked in water pressure distribution shown in Singh and Gupta (2000). Fig. 2 shows the variation of gravitation force experienced by the model in Singh and Gupta (2000).

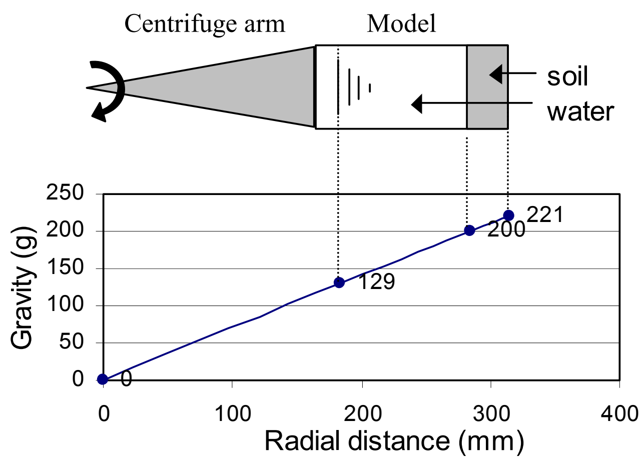

Figure 2. Centrifuge model used in Singh and Gupta (2000) and variation of gravitational field experienced by the model at 200g.

Use of Darcy's law in the form Eq. [8] gives the concept that Darcy's permeability is a function of gravity while hydraulic gradient is not.

$$v = K\frac{\rho g}{\mu} \times \frac{\Delta\left[\frac{P}{\rho g} + z\right]}{\Delta L} \qquad [8]$$

If definition given by Eq. [8] is used in centrifuge modelling of a prototype, it can be seen that the Darcy's permeability in the model at Ng is N times greater than that of prototype while the hydraulic gradient in the model at Ng is same as in the prototype. This is because both the total head difference $\Delta\bar{h}$ and flow path distance ΔL are N times smaller in the model. It is important to realise that the pressure gradients in the prototype and in the model at Ng are the same but the pressure head is N times smaller in the model placed in the centrifuge.

A better way to view the seepage flow is to consider energy gradient per unit volume of fluid as the driving force for flow. It is the energy difference that drives the flow and not the head difference.

$$v = \frac{K}{\mu} \times \frac{\Delta[P + z\rho g]}{\Delta L} \qquad [9]$$

The intrinsic permeability has the units of m^2, the energy gradient driving the flow has the units of N/m^3 and the dynamic viscosity has the units of Ns/m^2. We can establish the dimensions of velocity as m/s by substituting the dimensions for the quantities on the RHS of Eq. [9] as shown below:

$$v = \frac{m^4}{Ns} \times \frac{N}{m^3} = \frac{m}{s} \qquad [10]$$

While Eq. [9] offers a clear definition for seepage flow through soil, it is possible to link Eq. [9] to the Darcy's permeability as shown in Eq. [11] below.

$$v = \frac{k_{1g}}{\gamma_{1g}} \times \frac{\Delta[P + z\,\rho\,g]}{\Delta L} \qquad [11]$$

where k_{1g} and γ_{1g} are Darcy's permeability at 1g and unit weight of fluid at 1g respectively. Use of Eq.[11] enables us to use Darcy's permeability measured at 1g in centrifuge models at higher gravities.

4. Conclusion

Darcy's permeability and its relationship with intrinsic permeability can lead to some confusion in centrifuge modelling. It has been shown that the cause of misconception is in the conventional definitions of hydraulic gradient and Darcy's permeability. Conventional definition of hydraulic gradient represents the driving force for flow, but without the gravitational part of the driving force. Hence the gravitational part has been included into the Darcy's permeability as the unit weight of the fluid. This fact makes Darcy's permeability directly proportional to gravity. Hence some centrifuge modellers are using a scaling law for permeability as N. It is preferable to separate the permeability of the soil and the energy gradient driving the flow. This would lead to a clear definition of permeability being a function of the pore space (called intrinsic permeability) that does not change with the g level, and the energy gradient that drives the flow which changes with the g level in a centrifuge model.

Thus in modelling seepage flow in the centrifuge, it is advisable to use Eq. [9] or Eq. [11]. This would solve the conflicting views expressed in deriving the scaling law for the seepage velocity in centrifuge modelling. It could therefore be concluded that the seepage velocity in a centrifuge model is N times faster than in a prototype because the energy gradient that sets up the flow is N times greater in the centrifuge model.

References

Al-Hussaini, M. M., Goodings, D. J., Schofield, A. N., and Townsend, F.C. 1981. "Centrifuge modelling of coal waste embankments", Journal of the Geotechnical Engineering Division, ASCE, 107(GT4): 481-499

Arulanandan, K., Thompson, P.Y., Kutter, B.L., Meegoda, N.J., Muraleetharan, K.K.,and Yogachandran, C. 1988, "Centrifuge modelling of transport processes for pollutants in soils. Journal of Geotechnical Engineering", ASCE, 114(2): 185-205.

Butterfield, R., 2000, "Scale-modelling of fluid flow in geotechnical centrifuges", Soils and Foundations, Vol. 40 (No.6): 39-45.

Cargill, K.W., and Ko, H.Y. 1983. "Centrifuge modelling of transient water flow". Journal of Geotechnical Engineering, ASCE, 120(2): 366-387.

Carmen, P.C. 1956. Flow of Gases through Porous Media. Butterworths, London

Carmen, P.C. 1937. Fluid flow through a granular bed. Transactions of Institute of Chemical Engineers, London, Vol.15, 150-157.

Darcy, H. 1856. Les Fontaines publiques de la ville de Dijon, Paris.

Dean, E.T.R., 2001, Discussion on "Scale-modelling of fluid flow in geotechnical centrifuges", Soils and Foundations, Vol. 41(No.4): 108-110.

Goodings, D.J. 1984. "Relationships for modelling water effects in geotechnical models". Proceedings. Application of Centrifuge Modelling to Geotechnical Design. University of Manchester, 1-23

Goodings, D.J. 1979. "Centrifuge modelling of slope failures". Thesis presented to University of Cambridge, at Cambridge, England, in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Kozeny, J. 1927. Ueber kapillare Leitung des Wassers im Boden, Wien. Akad. Wiss.

Muskat, M. 1937. "The Flow of Homogeneous Fluids through Porous Media". McGraw- Hill Book Company, New York.

Pokrovsky, G.I. and Fyodorov, I.S., 1968. "Centrifuge Modelling in the Construction Industry", Vol. 1, in translation Building Research Establishment, U.K.

Schofield, A.N. 1980. "Cambridge Geotechnical Centrifuge Operations", Geotechnique, Vol. 30, No. 3: 227-268

Singh, D.N., and Gupta. K.A. 2000. “Modelling hydraulic conductivity in a small centrifuge”, Canadian Geotechnical Journal, 37(1): 1150-1155

Tan, C.S. and Scott, R.F. 1985. “Centrifuge scaling considerations for fluid-particle systems”, Geotechnique, Vol. 35, No. 4, 461-470

Taylor, R.N. 1987. Discussion on Tan & Scoot(1985), Geotechnique, Vol. 37, No. 1, 131-133.