RE-APPRAISAL OF TERZAGHI'S SOIL MECHANICS.

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1. Two different theories of the strength of soil.

In his recent biography of Terzaghi, Goodman (1999, p 213) has described Terzaghi and Peck's difficulties in writing 'Soil Mechanics in Engineering Practice' which, published in (1948), quickly established itself as a main pillar of geotechnical education. He quotes a judgement they formed on Donald Taylor's (1948) book 'Fundamentals of Soil Mechanics' as a 'theoretical and esoteric portrayal of soils'. But when Taylor's book was published it won praise from academics. Students read it and used his interlocking equations (1 and 2 below) to develop an alternative to the normal soil mechanics theory taught by Coulomb, Rankine, Mohr and Terzaghi. Both theories fit data of peak strengths of soil. The alternative theory also fits data of other phenomena. This Lecture will re-appraise the normal soil mechanics theory by contrasting it with the alternative theory.

Goodman (1999, p 82) describes how Terzaghi developed the concept that pressure acting in the solid phase of soil, today known as the 'effective' stress, controls soil strength and deformation. The normal theory of soil strength (it pre-dates Coulomb) is that effective stress on potential failure surfaces attains limiting values, and bodies of dense soil crack, or slip with a shear strength that is a sum of cohesion and friction. In that normal theory peak strength includes cementing or bonding of soil grains. Adhesion, friction, and cohesion are the strengths with which soil resists cracks, or resists slip plane failure. The alternative theory treats soil as a 'paste continuum'. It explains how the strength of an un-bonded aggregate of strong, stiff, soil grains is affected by compaction and by effective pressure. It questions both Coulomb's slip plane concept and Terzaghi's concept of 'true' cohesion and friction c' and ϕ' resisting failure on slip planes.

Taylor reported strain controlled tests of dense sand in a shear-box with constant effective normal stress σ' and steady increase of shear displacement x, Fig.1a. The stress ratio τ/σ' was related to dilation y normal to the plane of shear displacement, Fig 1b. In Figs.1c and d at peak stress work is done by τ to overcome interlocking, in addition to the dissipation in friction, as in Eqns. 1,

$$\tau\delta x = \sigma'\delta y + \mu\,\sigma'\,\delta x \qquad \text{hence} \qquad \tau\delta x - \sigma'\delta y = \mu\,\sigma'\,\delta x \qquad (1),$$

Peak shear stress ratio τ/σ' involves strength components of interlocking $(\delta y/\delta x)$ and of friction μ

$$\tau/\sigma' = (\delta y/\delta x) + \mu \qquad (2).$$

Taylor's interlocking equations apply to aggregates both of fine and coarse soil grains and also to peak strength of over consolidated clay where dilation and suction increase the water content on a slick slip plane. Data of a test of clay in a shear box can mislead if the rate of dilation $(\delta y/\delta x)$ at peak strength in Fig 1c is unknown.

Terzaghi's student M. J. Hvorslev (1937) interpreted peak strengths as shown in Fig. 2. He had no data of values of interlocking $(\delta y/\delta x)$ at peak strength. He did not know if strength was due to interlocking, or to his components of 'true' friction ϕ', (the slope of his lines of peak strengths) and 'true' cohesion c', (his c' depended on water content w). Schofield and Wroth (1968, Fig. 8.8) note that Hvorslev's peak strengths lie between bb′ and cc′ in Fig. 3a (sector II). To the right (sector I) are data of ductile plastic yielding and left (sector III) tensile cracking. In Fig 3b three similar sectors are shown on a graph of v against ln p′, in which $p' = (\sigma_1' + \sigma_2' + \sigma_3')/3$, and $v = (1+e)$. The line dd′ indicates isotropic plastic compression under spherical effective stress $q=0$.

Schofield (1982) linked cracked clastic debris at low mean effective pressure (sector III) with liquefaction at high hydraulic gradient. In Fig. 3 lines bb′ indicate tensile cracking, and cc′ ultimate Critical States (CS). Aggregates dilate in shear in sector II (positive interlocking). Aggregates contract in shear in sector I (negative interlocking). Truly plastic soil behaviour is predicted by application of Taylor's equations to the yielding of soil paste in sector I.

Coulomb was educated in the École du Corps Royale du Génie at Mézières. Teaching there was based on Belidor's (1737) book that taught that motion of interlocked slip surfaces causes friction. He represented surface-roughness by hemispheres that must climb over each other. The angle of dilation gave a coefficient of friction. He did not consider 'negative interlocking'. Navier's (1819) new edition of Belidor reproduced this theory without change. Leslie (1804) (and no doubt others) noted that dilation could not apply to friction in steady continuous sliding, only to initial motion. But Belidor, Coulomb, and Navier accepted the premise that slip planes exist and that the observed coefficient of friction is due to interlocking. Either friction or interlocking came into their design calculation, not both. To correct their error we must reverse the conventional way of thinking.

Taking account of all possible circumstances and establishing connections between phenomena that have been thought unrelated we arrive at a new premise and a more profound explanation as follows. Internal friction relates to the angle of repose of a loose aggregate of grains, not to slip of solid sliding surfaces. The sum of both interlocking $\delta y/\delta x$, and critical state internal friction angle shown in Fig 1d, gives strength both to normally and over-consolidated soil. Test data of soft clay yielding in sector I, Fig. 3, fit plastic soil behaviour predicted by Roscoe and Schofield (1963) quite closely. This proves that there is no 'true cohesion' at all in re-consolidated disturbed soil.

2. Coulomb's planes of limiting stress and Rankine's conjugate planes.

The slip plane concept derives from Amontons (1699). He described resistance to sliding of solid surfaces in machines by a constant coefficient of friction denoted by μ. In his theory ϕ is a limiting angle at which vectors of force can be inclined to solid surfaces, where $\mu = \tan\phi$. Drained conical heaps of loose soil stand with a constant 'angle of repose' ϕ_d to any height. Both Coulomb and Rankine used ϕ_d to define internal friction in geo-material. When Coulomb learned equation (3)

$$\tau = c + \sigma \tan \phi_d \quad (3)$$

he learned as a fact that tensile adhesion in geo-materials is the same as shear cohesion c. But when Coulomb (1773) tested rock (Bordeaux limestone) he found that this is not a true fact; it had less strength in tension (adhesion) than in direct shear (cohesion). The difference was small. He felt it was safe in his own design work to take the value of adhesion measured in a tension test for use as c, the cohesion in shear. Disturbance breaks cemented bonds and destroys adhesion. He notes several times that newly disturbed soil must be assumed to have zero cohesion. Coulomb and Rankine both rely only on internal friction ϕ_d of soil in design, not cohesion, so for them c = 0 in Coulomb's equation. The theory of strength adopted by Terzaghi follows Mohr's interpretation of equation (3) (see below). Terzaghi differed from Coulomb and Rankine when he interpreted peak strengths of newly disturbed, re-consolidated, clay samples as containing a 'true cohesion' component, and also when he used ϕ' a 'true friction' angle less than their internal friction ϕ_d.

The teaching in Schofield and Wroth (1968) did not make a radical review of Coulomb's concept of slip surfaces. What follows below here introduces soil in a loose heap at an angle of repose. The Critical State (CS) is seen as the ultimate states in a triaxial test. It also exists in loose heaps at the angle of repose. Let two closed cylindrical glass jars (jam jars) half full of sand, one with voids filled with air and one water, be lain on their sides on a table and slowly rolled along, and slowly tilted to stand on their ends. The sand is seen to form slopes at the same angle of repose ϕ_d in both cases. Water is not a lubricant. Below such loose slopes the states in soil at all depths must fit a line like cc′ in Fig 3a. If increase of stress made internal friction ϕ_d decrease, then higher heaps would have flatter slopes. Loose slopes form repeatedly in an hourglass without significant grain fracture; the sand grains do not turn to dust. Elastic buckling of lines of stressed stiff grains may perhaps explain ϕ_d, but we do not need to link it with limiting stress vectors on slip surfaces or explain ϕ_d by micro mechanics. Our soil continuum mechanics can be developed with the least number of assumptions if, in the manner outlined below, we associate the property of CS internal friction with aggregates of soil grains that form loose slopes at an angle of repose ϕ_d.

Rankine had studied thermodynamics before becoming Professor at Glasgow University in 1855. To prepare for teaching he read Lamé's (1852) lectures on stress and strain in elastic material. He developed a graphical construction of the (Lamé) ellipse of plane stress for his textbook 'A Manual of Applied Mechanics' (1858). He found two directions of conjugate stress in all stress fields. He found a limiting ratio of major and minor stress, and the lateral pressure in bodies of heavy loose earth in limiting states. He noted that, unlike Coulomb, he did not need to assume a failure plane to calculate lateral earth pressure. But his limiting stress field equations were found to be wrong by Boussinesq (1874) who noted that Rankine (1857) had written 'heat' equations rather than the correct 'wave' equations. Soon after Rankine died in 1872, aged only 52, Mohr's stress circle was taught, for example in instruction at MIT (Swain 1882), and not Rankine's stress ellipse. A stress circle, Fig. 4, contains the same information as a stress ellipse but is more easily understood. Near O where the major and minor stresses are σ_1 on a horizontal plane and σ_2 on a vertical plane, Fig 4a shows a plane aa′ inclined at an angle α to the horizontal. On this inclined plane the normal and shear stress effective in the aggregate of grains are

$$\sigma' = (\sigma'_1 \cos \alpha) \cos \alpha + (\sigma'_2 \sin \alpha) \sin \alpha = (1/2)(\sigma'_1 + \sigma'_2) + (1/2)(\sigma'_1 - \sigma_2) \cos 2 \alpha \quad (4),$$

$$\tau = (\sigma'_1 \cos \alpha) \sin \alpha - (\sigma'_2 \sin \alpha) \cos \alpha = (1/2)(\sigma'_1 - \sigma'_2) \sin 2 \alpha \quad (5).$$

All points with co-ordinates (τ,σ') lie on a circle with centre at $(\sigma'_1 + \sigma'_2)/2$ and radius $(\sigma'_1 - \sigma'_2)/2$. Fig 4b shows this stress circle. The point A represents components of stress on the inclined plane at angle α. Through the point A the line parallel to this plane intersects the circle in the point P called the 'Pole of planes'. A line drawn through P at any other angle β intersects the circle at another point B; the co-ordinates of B give the stress components on the plane that is inclined at that other angle β. Rankine's limiting-stress concept and Mohr's stress circle construction extend soil strength theory beyond the point reached by Coulomb. Behind his retaining wall Coulomb showed one slip plane embedded in a family of parallel planes which let him calculate a triangular distribution of pressure on the wall. Rankine's ellipse let him discover another 'conjugate' family of surfaces of limiting stress at every point in the soil body, as explained in the following paragraph.

Fig. 5 shows a drained soil slope at an angle of repose ϕ_d. A slab of soil of thickness z below a slope rests on a 'Coulomb slip plane' ss′. In the stress field of Fig. 5 a block of soil of weight γz applies a vector of stress of magnitude $\gamma z \cos\phi_d$ on the $(1/\cos\phi_d)$ length of plane ss′, inclined at a limiting angle ϕ_d with (anticlockwise) positive shear stress. This vector is a tangent to the stress circle at the pole P. The maximum principal stress is $\sigma'_1 = \gamma z\ (1+\sin \phi_d)$. The minor principal stress is $\sigma'_3 =\gamma z\ (1-\sin \phi_d)$. The circle centre is at $\sigma' = \gamma z$. P is vertically above a point p′ on the circle. At this 'conjugate' stress point p′ shear stress is clockwise; this stress acts on a 'conjugate' vertical section pp′ through the slope. Fig. 5 also shows an inclined soil cylinder on which the major and minor principal stress in the limiting slope will act. In a cone of loose soil at an angle of repose the intermediate stress σ'_2 acting on the cylinder in the slope falls to the minor principal stress value $\sigma'_2 = \sigma'_3$. The physical phenomenon of a loose aggregate of grains at repose in a long slope remains the basis for both soil mechanics theories.

Coulomb had a family of slip planes such as ss′ in Fig. 5 parallel to the slope at repose. Rankine extended that concept with a second family of vertical conjugate planes such as pp′ in Fig. 5. The CS view of such a slope is of many cylinders such as might be tested in a triaxial cell, as shown in Fig. 5. Close observation of sand in an hourglass will show that a steep little cone of more dense sand with a higher stress ratio builds up at the tip of a heap. This steep tip grows to a limited height and slumps in one direction or another down slope. As a loose heap is built up, successive small avalanches of loose grains run down the slopes and increase effective axial and radial pressures on every buried volume of soil below the slope. Each cylinder length shortens. Each radius swells. Stress circles increase in size and are tangential to the pair of lines OP and Op′ in Fig. 5 at angles +/- ϕ_d. Rankine did not observe, and CS theory does not postulate, slip of soil grains on Coulomb's plane ss′ or on the conjugate plane pp′. All planes have equal status. In dense soil, whether it is originally disturbed or undisturbed, an unstable process forms slip planes. No planes exist before the failures. In a loose random aggregate of irregular solid grains of diverse sizes, motion seen at close range is a process of random movement; at a distance it looks like continuous flow. The word 'paste' applies to a mixture of water with this aggregate of fine solid grains. Coulomb's description of an 'angle' of friction on a slip plane implies that intermediate stress plays no part in failure. In CS theory intermediate stress is taken into account in the generalised friction coefficient M (the Greek capital and small letters are M and μ). Each cylinder of soil yields and flows on a stress path defined by values of mean normal effective stress p' and generalised deviator stress q

$$p' = (\sigma_1' + \sigma_2' + \sigma_3')/3 \qquad (6),$$

$$q = [(½)\{(\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma_1)^2 + (\sigma'_1 - \sigma'_2)^2)\}]^{1/2} \qquad (7).$$

When applied to the cases of axial symmetry with $\sigma'_2 = \sigma'_3$, in a drained conical heap and in a triaxial compression test, these two stress invariants become

$$p' = (\sigma_1' + 2\sigma_2')/3 \quad \text{and} \quad q = (\sigma'_1 - \sigma'_2) \qquad (8).$$

CS limiting internal friction defines the ratio of the deviator and the normal stress invariant

$$q / p' = \mathrm{M} \qquad (9).$$

If $\sigma'_1 / \sigma'_3 = (1+\sin\phi_d)/(1-\sin\phi_d)$ then $q / p' = 3(\sigma'_1 - \sigma'_2) / (\sigma_1' + 2\sigma_2') =$

$$= 3((1+\sin\phi_d) - (1-\sin\phi_d)) / ((1+\sin\phi_d) + 2(1-\sin\phi_d)) = \mathrm{M} = 6 \sin \phi_d / (3 - \sin \phi_d) \qquad (10).$$

So for example if the drained angle of repose for an aggregate is $\phi_d = 30°$, then $\sin\phi_d = 1/2$ and $\mathrm{M} = 6 \sin \phi_d / (3 - \sin \phi_d) = 1.2$. The friction coefficient M can be calculated from ϕ_d in this manner for loose soil slumping (shearing) into limiting equilibrium, for any aggregate for which all drained slopes exhibit the same angle of repose. Mohr's circle still represents plane stress. Eqns. 9 and 10 replace the Mohr-Coulomb failure criterion. The angle of repose ϕ_d gives M.

Equation 9 is one of two CS equations. The other equation relates the water content or packing density of an aggregate of grains to the effective pressure in the aggregate. This is introduced in Fig 3a where one axis represents the normal and the other the shear stress-state in an aggregate of grains, and Fig 3b where the axes are the specific volume or water content of the aggregate of soil grains and ln p′. Casagrande (1936) originally defined a critical voids ratio that did not vary with pressure p′. Later Taylor (1948), and also Schofield & Togrol (1966), discuss the variation of the critical voids ratio with pressure. Section 3 below relates classification tests of disturbed soil to fundamental CS properties; Section 4 considers effective stress changes and pore pressures in tests of soil paste and predictions of yielding that fit drained and undrained triaxial test data.

3. Soil classification tests.

Most soil selected for construction has stiff, hard grains that are broken rock fragments with a size distribution that is not degraded in handling. The grains are easily separated. Loose layers can be formed easily with a grader blade. Compaction into a dense state strengthens a layer. When a road pavement is formed with compacted aggregate weakly bound by a paste of very fine soil grains the interlocked rough irregular grains make a sort of ‘concrete’. In CS theory compaction increases strength not because grains adhere more to each other when they approach more closely, but because they become interlocked. To find if a soil aggregate will perform well when compacted, a materials-engineer must cause no more damage to the grains than will occur in construction, washing and separating the coarse and fine grains by hand on a nest of sieves. Dry soil must not be crushed with a pestle and mortar, for if the coarse grains are reduced to dust the elusive interlocking quality of the compact aggregate is altered. A cone is dropped on a paste of the fine grains to find how the strength varies with water content. ‘Index’ tests give fundamental soil properties useful in theoretical disturbed soil plasticity. The simple ideas set out in Schofield and Wroth (1968) are summarised in the paragraphs that follow below.

In Fig 6a the specific volume v (space occupied by unit volume of solid grains) increases as water content increases. The volume of pores is e and v = (1+e). The solids have specific gravity G_s. If water unit weight is γ_w and the pores are full of water then the water content of the paste is w = (e /G_s). Disturbed soil paste with water content w is placed in a small tub. A 30° cone is allowed to fall under gravity into the smooth upper surface of the paste. The measured depth of penetration is d. The area on which the cone comes to rest increases with the square of d. The strength q of the paste in that area supports the cone weight. Soft soil paste distorting in shear below a cone comes into critical states, like those below a slope of loose grains at an angle of repose. The plot of w versus ln d is a straight line, Fig 6b. A standard cone has 80 grams mass. The standard Liquid Limit w_L is the water content such that the 80 gram cone penetrates 20 mm into the paste. Undergraduate teaching in Cambridge in 1974 introduced a second non-standard test with a 240-gram cone to get two parallel lines, Fig. 6b. Both are plastic compression lines for the paste. Water evaporates from the paste into the laboratory air. The meniscus on each free surface pore maintains pore suction and effective stress in the paste, like a constant load in an Oedometer Test. Pore suction can be measured if the saturated paste is placed on a suction plate.

Classification of soil in site investigation gives data of the general form evident in the lines of Fig. 6b. Equations relating decrease of w to ln d fit data for a wide range of fine grain undrained soil. Site profiles show the proportions of fine and coarse grains, the water contents w against plastic and liquid limits w_P and w_L at all depths. Liquid Limit w_L and Plastic Limit w_P give the Plasticity Index ($w_L - w_P$) = I_P. The Liquidity Index is ($w - w_P$)/($w_L - w_P$) = I_L. In CS theory their fundamental significance is this. From Equation 9, based on a slope at repose, it follows that q and p′ increase together. Comparison of conditions at points L and L′ in Fig. 6b links loss of water content Δw to an increase of strength q of the paste by a factor (240/80) = 3. From Eqn. 9 it follows that during plastic compression of the paste of fine grains from L to L′ with water content loss Δw, there must be a factor of 3 increase in p′. Skempton and Northey (1953) found a loss of water content equal to I_P increased soil paste strength by a factor of about 100. The increase of 100 is about 33 times the increase caused by Δw. Fig 7 suggests that site investigation could introduce tests as shown in Fig. 6b, could measure values of Δw, and plot a profile of values of (w_L - 33.3 Δw) instead of w_P. The position of the in-situ water content w line between the Liquid and Plastic limits indicates the Liquidity and undrained strength of the ground in-situ after full disturbance. The lines in Fig 6b relate strength (ln. q or p) to density (w or v); they lead to the CS equation for plastic compression

$$v + \lambda \ln p' = v_\lambda = \Gamma \quad \text{(a constant)} \qquad (11).$$

Experience with fall cone tests should lead geotechnical engineers to expect a relationship between soil behaviour and the state of stress and compaction such as is set out in Fig 3a. Data of slopes at repose and of Index Tests lead to the two CS equations 9 and 11. However, development of CSSM did not begin in this way but from a need to interpret shear test data.

4. CS interpretation of triaxial test data, and ‘Wet clay’ (Original Cam-clay).

It was data of both coarse and fine grain disturbed soil tested in triaxial compression cells and shear boxes that led Roscoe, Schofield, and Wroth (1958) and Parry (1959) to CS lines with equations (9) and (11). The specimens were fully saturated reconstituted soil with a history only of compression and swelling under changes of effective stress, such as Hvorslev, Terzaghi's student, also tested. I regard undisturbed soil sampled from a natural soil deposit with bonds between grains more as soft rock than as soil. For me, CS theory is about aggregates of separate stiff elastic grains held together only by effective stress. The general pattern is shown in Fig 3b.

Loose aggregates of fine soil grains on the isotropic compression line dd′ lose water during shear deformation and approach a line of ultimate critical states cc′ that lies parallel to it. These lines are evident in Hvorslev (1937) who credits their discovery to Casagrande and Albert. They are also clear in Haefeli (1950) in the Proceedings of the London Conference held at the ICE in June 1950, 'Measurement of shear strength of soils in relation to practice' as reported in Geotechnique Vol.2. The opening paper (Skempton and Bishop 1950) considers Coulomb's equation to be fundamental to practice, and does not recognise the CS line. Bishop (1950, on page 114) gives a calculation equivalent to Taylor's Eqn 2, expressing peak strength τ/σ' as an angle of internal friction μ plus $(\delta y/\delta x)$, a dilatant strength in dense soil. Gibson (1953) later extended this work by deducting an 'energy correction' $(\delta y/\delta x)$ from τ/σ' to get a value of 'corrected' true friction. Roscoe (1953) thought progressive failure on the shear plane meant that the shear box of Bishop and Gibson at Imperial College could not give a reliable 'correction'. The energy correction is zero in critical states with shear deformation at constant volume. Roscoe devised his own simple shear box apparatus (SSA) Fig 9 with rotating end flaps to cause uniform strain in his soil specimen. When Roscoe, Schofield, and Wroth (1958) later interpreted test paths as approaching critical states, Fig 10, they commented that

"in an undrained test a sample is distorted in shear so that the relative positions of the particles are shifted without the distance between the centres of particles changing. In a drained test the relative positions of the particles are again shifted, and this certainly involves the absorption of work internally; but in addition the average distance between the centres of particles changes and it is possible that additional internal work is absorbed if there is a marked inelastic hysteresis in the compressibility of the grain structure. If this is the case, then values of τ obtained from the two tests will not be comparable even after applying the above boundary energy correction. But if, as proves to be the case, the two surfaces obtained from the two sets of results coincide, then within the limits of experimental accuracy the work internally absorbed will be independent of the rate of dilatation and, in addition τ_i will be purely a function of σ' and e."

To find if drained and undrained test path data, with or without Bishop's correction, gave a unique surface, Roscoe set his research students the task of testing soil in triaxial cells and in his SSA. His students looked for what they could call a 'Roscoe surface'. Unfortunately he put lubricated rubber sheets on the SSA end flaps to allow more dilation. Lubrication eliminated complimentary shear stress. The SSA stress-state was not uniform. A different approach was needed.

In Taylor's Eqn 1 the work $(\tau\delta x - \sigma'\delta y)$ done by forces on boundaries is assumed to be $(\mu\,\sigma'\,\delta x)$. It is dissipated in internal friction. His dissipation function is σ' the normal effective pressure times shear distortion δx times μ the friction coefficient. He assumes that no additional internal work is dissipated as average distance between the centres of particles changes. In Cambridge a value

$$v_\kappa = (v + \kappa \ln p') = (\text{a constant for a given aggregate of grains}) \qquad (12),$$

defined the state of an aggregate of stiff grains held together by effective pressure. Our v_κ involved both packing density and pressure. Roscoe's student Thurairajah (1961) analysed tests with additional 'elastic energy corrections' to allow for the elastic energy stored or released as effective stress increased or decreased in the stressed aggregate of fine elastic soil grains. His data fitted a dissipation function $Mp'\delta\varepsilon^p$ that simply generalised Taylor's function $\mu\,\sigma'\,\delta x$ in Eqn. 2,

$$p'\delta v^p + q\delta\varepsilon^p = Mp'\delta\varepsilon^p \qquad (13).$$

Thurairajah's dissipation function for a general deformation in a drained or undrained test path in Eqn. 13 can be expressed in words as a simple premise. Instead of correlated test data defining an experimental 'Roscoe surface', this generalisation of Taylor's dissipation function became the premise from which Roscoe and Schofield (1963) deduced a purely theoretical yield surface.

Stress paths to the CS line are shown in Fig. 10. ABC is a drained path of a test controlled so that there is no change of initial effective pressure p'_C as specific volume increases from v_A to v_B and then to v_C. The inclined line dd′ through D in Fig 10b indicates states of plastic compression under spherical stress. DC is an undrained test path, ending at the point C with effective pressure p'_C. Peak strengths lines from Figs. 2 and 3 are copied in Figs. 10a. Fine soil grains at A are relatively densely packed. They dilate in shear and suck in water. An engineer who remoulds a piece of such soil with a wet hand will feel the hand become dry, so soil in states A and B is said to be on the 'dry side'. Fine grain soil initially at D develops positive pore pressure and exudes water in shear, so it is on the 'wet side'. Wet and Dry states in Fig 10b could be called lightly and heavily over-consolidated respectively. On the path ABC in Fig 10a the strength rises to a peak value q_B and falls to a CS value q_c. There is small strain from A to B, and larger strain from B to C. The dashed lines from B to C and the fall from q_B to q_c in Fig. 10 indicate a part of a test path with bifurcation. In fine grain soil shear strain is localised in thin slip surfaces, and water sucked in to wet paste on those surfaces gives slickensides. They appear slick and greasy. Such surfaces are a result of instability. They do not indicate that a stress vector mattered before peak stress.

On the undrained path DC the specific volume v_c is unchanged but the effective stress falls from initial pressure p_D to p_C. Ductile soil behaviour is named 'plastic' from the Greek verb 'to mould' which describes the way that a potter forms clay into pots or makes clay models. In ductile metal the atoms share electrons and are held close by a metallic bond while slipping from one contact to the next during plastic deformation. Early studies of extrusion mechanisms in metal plasticity used illite clay modified with petroleum jelly and not metal in model experiments. Plastic theory with constant volume applies equally to metal and to fine-grain water-saturated soil in states on the wet side of CS. However fine grain soil particles are held together by pore suction slip from one contact to the next in plastic deformation, not bonded. The frictional work is proportional to the effective stress and the distortion. Undrained shear strength is found from Eqns 9 and 11

$c_u = c(v) = q(v)/2 = Mp'/2 = (M/2) \exp(\Gamma - v)/\lambda$.

About the time that Thurairajah was finding his dissipation function, our Cambridge colleague Calladine (1963) proposed that the intersection of a surface of elastic swelling states v_κ = const. with a 'Roscoe surface' would be a yield surface to which the associated flow rule of the theory of plasticity should apply. The associated flow rule of plasticity to be applied was

$$\delta p'\delta v^p + \delta q \delta \varepsilon^p = 0 \qquad (14).$$

Calladine's contribution was crucial. Elimination of $(\delta v^p/\delta\varepsilon^p)$ from equations 13 and 14 gave a differential equation involving $(\delta p' / \delta q)$ and (p'/q) which was integrated, Roscoe & Schofield (1963), to give the ideal yield surface for what we at first called 'Wet-Clay'

$$q/Mp' = 1 - \ln(p'/p'_c) \qquad (15).$$

At the CS point C in Fig.10b the effective stress is p'_c. Equation (15) predicts the yielding of soil in Fig 10 between isotropic compression and critical state lines. Instead of a 'Roscoe surface' based on correlation of drained and undrained test data, our theory deduced the mechanical behaviour of our Wet-Clay. Both drained and undrained test paths were predicted to lie on a line DC in Fig 11. From Eqn (15) the equation for line DC in Fig. 11 is

$$v + \lambda \ln p' = v_\lambda = \Gamma + (\lambda-\kappa)(1 - q/Mp') \qquad (16).$$

In Kyoto, Murayama's research student Shibata had performed drained triaxial tests in which the mean-normal effective-pressure p′ was held constant by continuous adjustment of the triaxial cell pressure. He found that specific volume reduced linearly with increase in q/p′, as in Fig 11. Their colleagues, Ohta & Hata used a 'Roscoe surface' derived from Shibata's line to predict yielding of soil on any test path. In undergraduate lectures I called the Wet Clay soil model 'Cam-clay' after our river Cam. Independently Ohta & Hata (1971) could have called their identical Kyoto model 'Kamo-clay' after their Kamo (or duck) river in Kyoto. If the equation giving this straight line DC in Fig 11 is plotted on the axes of Fig 10a it gives a slightly curved test path, close to the straight line in Fig 10a, which is the path predicted by Skempton's pore pressure parameters. Roscoe, Schofield & Thurairajah (1963) and Schofield & Wroth (1968) show other good data close to this curve. Close predictions of test data on the whole of the wet side and at critical states, using a model with Taylor's frictional dissipation as in Eqn. 2 and 13 and no cohesive strength, strongly support Coulomb's repeated statements that newly disturbed soil has zero cohesion.

Roscoe, Schofield & Wroth (1958) showed curved paths of early data of undrained triaxial tests performed quickly that achieved pore pressure equilibrium only at failure and not along the whole undrained test path. They approximated more closely to a parabola than a straight line. The Wet-clay model always has shear strain when work is dissipated. There is so much shear deformation that a lateral earth pressure coefficient of $K_0 = 1$ is predicted for one-dimensional consolidation. Roscoe and his next research student Burland (1967) modified Eqns 13, 14 and 15 to get an elliptical yield locus and a lateral earth pressure coefficient of $K_0 = 0.7$. Burland incorporated these modifications in a widely used early FE analysis. Wood (1990) proposed that Roscoe & Burland's 'Modified Cam-clay' could be called 'Cam-clay' and Roscoe & Schofield's model could be called 'Original Cam-clay'. An alternative is to call them Burland's and Schofield's models respectively, and to present the Cambridge soil mechanics research by Roscoe's group as his work with successive students before his early death, rather than as one body of finished work.

5. Terzaghi's statement that 'Rankine's earth pressure theory' has a 'fundamental fallacy'.

Sokolovsky (1960) and others have shown that Rankine's plane (2D) earth pressure theory, when rightly expressed, considers three equations in three unknowns, (σ_x σ_y τ_{xy}). The two partial differential equations of plane equilibrium are

$$\partial \sigma_x / \partial x + \partial \tau_{xy} / \partial y = 0, \text{ and } \partial \tau_{xy} / \partial x + \partial \sigma_y / \partial y = 0 \qquad (17).$$

The third equation (18) expresses Mohr's hypothesis that limiting stress circle in Fig. 4 and 5 touch the so-called Mohr-Coulomb envelope (two lines with equations +/- $\tau = c + \sigma \tan\phi$)

$$\{(\sigma_x - \sigma_x)^2 + 4\,\tau_{xy}^2\}^{1/2} = \{c \cot \phi + (\sigma_y + \sigma_x)/2)\sin\phi\} \qquad (18).$$

Sokolovsky showed these equations to be hyperbolic, so everywhere in a limiting Mohr-Coulomb stress field there are two conjugate directions of characteristic curves along which information about the magnitude and direction of the vectors of stress at points on boundaries are transmitted through the field. Information sent from two boundary points by these 'wave equations' to every point in Rankine's limiting stress field determines the stress there. Terzaghi (1936) noted that 'the factor, strain' does not enter the theory but strains really do affect earth pressures. He concluded

"The fundamental assumptions of Rankine's earth pressure theory are incompatible with the known relation between stress and strain in soils, including sand. Therefore the use of this theory should be discontinued."

The title of his paper stated that 'earth pressure theory' contains a 'fundamental fallacy' but he did not see that this arose from the soil properties used in the Mohr Coulomb equation. There is no error in equation (17). If equation (18) is true then stress boundary conditions determine the stress throughout the field regardless of variations of strain boundary conditions. Since he knew the importance of ‘the factor strain’ from his own experience, he should have deduced that there must be an error in equation (18). The explanation follows from Figure 14.2 in Taylor’s book that shows data of a shear box test of dense sand. Interlocking causes dense sand to dilate in initial strain increments. If Terzaghi wants the factor strain to enter earth pressure theory his Mohr Coulomb strength parameters (c',ϕ') can not be constants; they must depend on strain. If Eqn 18 is replaced by CS equations Sokolovsky’s limiting stress field calculations no longer apply.

Plasticity applies to soil mechanics. Disturbed saturated fine-grained clay-silt is archetypal plastic material. Soil classification detects the plastic properties of the paste of solid grains in disturbed ground after construction. The effect of interlocking is clearly seen in softening of gouge material in stiff undisturbed clay. The plot of peak strength data gives Terzaghi’s and Hvorslev’s straight line, but design based on the “true” friction and cohesion interpretation of the values of the slope and intercept of this line is less safe than plastic design based on disturbed soil properties. Terzaghi spoke in Harvard of the need for geotechnical engineers to be ‘well grounded in mechanics’. The theory of plasticity is an essential part of their education. Schofield and Wroth (1968) explain the use of undrained cohesion and drained friction parameters c_u and ϕ_d in plastic calculations of limiting equilibrium. Plastic design is based on disturbed soil strength rather than on peak strengths of undisturbed soil samples. The introduction of kinematics by Bolton (1993) extended the possibilities of plastic design. Plastic design emphasises the benefits of ductility in any structure. CS theory explains that when soil structures such as earth dams are compacted so much that parts of the soil body operate at a value $v_\lambda < \Gamma$, cracks in those parts may not self-heal if the dam is filled quickly. Soil states in which hydraulic fractures occur, plot in sector III of Fig 3b.

6. Conclusion.

Goodman tells us that after investigation of ground conditions for design of construction works Terzaghi was constantly vigilant to detect nuances in new information from a site about ground conditions. New theories of soil behaviour, and new mathematical formulation and experimental validation of solutions to problems, need equal vigilance. If En. 18 is replaced by Eqns. 9 and 16 strain compatibility equations must also be introduced. If 'bearing capacity factors' that Terzaghi based on fragments from theory of plasticity are replaced by yield locus concepts, such studies of structure-foundation interaction must be validated by experiments, (see Dean, James, Schofield, Tan and Tsukamoto 1993). To Terzaghi the possibility of successful mathematical treatment of problems involving soils appeared very limited and small-scale model-tests ‘utterly futile’. But the time when Terzaghi recommended a shift of centre of gravity of research from the study and the laboratory into the construction camp has passed. Laboratory geotechnical centrifuge analogue modelling, and digital modelling by numerical methods, together with field observational methods, are now available as research methods that can correct soil mechanics errors (Schofield, 1998).

ISSMGE International Conference orators regularly honour Karl Terzaghi. Exceptionally I asked, and the Organisers of the Istanbul Conference have allowed me, to re-appraise Terzaghi’s soil mechanics in this Special Lecture. Taylor lived long enough to be ISSMFE Secretary General in the Zurich conference, but did not live long enough to question the Mohr-Coulomb equation or apply the result he found for sand to the clay tested by Hvorslev. However this lecture has shown his interlocking analysis at MIT to be as significant as Terzaghi’s primary consolidation analysis in Istanbul. Today we should honour the author of 'Fundamentals of Soil Mechanics' as much as we honour the authors of 'Soil Mechanics in Engineering Practice'.

This Technical Report uses information that was available fifty years ago. I will work in the NGI Terzaghi Library in June 2001 to finalise this lecture, and give the lecture in Istanbul in August 2001. It will help me better to honour the memory of Terzaghi and Taylor and re-appraise their work, if those who knew them at the time will review the ideas I put forward above and write to me at ans@eng.cam.ac.uk. My home page at http://www-civ.eng.cam.ac.uk/geotech.htm has copies of other papers. Contributions on all these papers will be welcome. While I take sole responsibility for this paper, I thank Malcolm Bolton for comments on an earlier first draft.

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τ

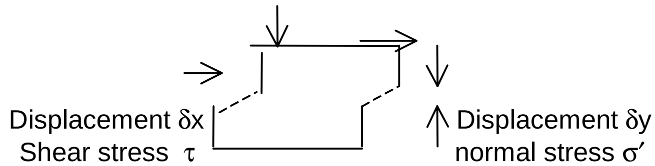
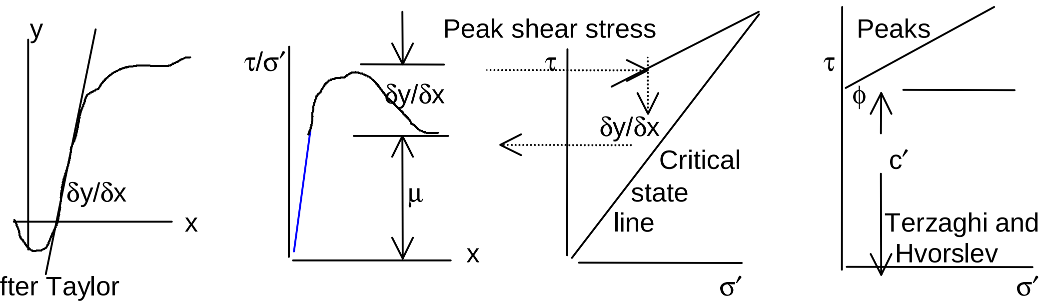


Fig. 1a Soil in shear box



after Taylor

Fig. 1b Fig.1c Fig.1d Fig1e

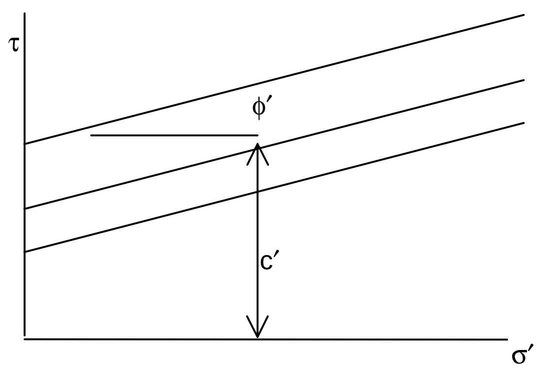


Fig. 2 Terzaghi and Hvorslev's interpretation of peak strength data

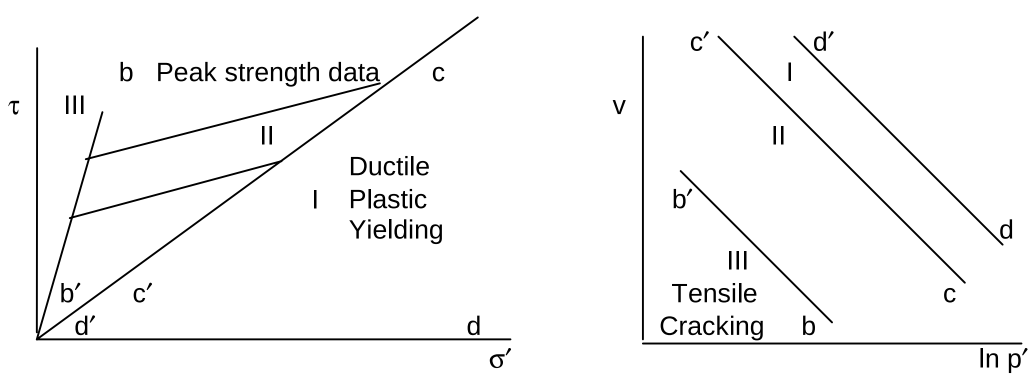


Fig.3a (compare with Fig 1c, d and e) Fig. 3b

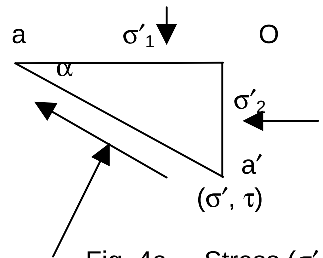


Fig. 4a Stress (σ′, τ) at a point O

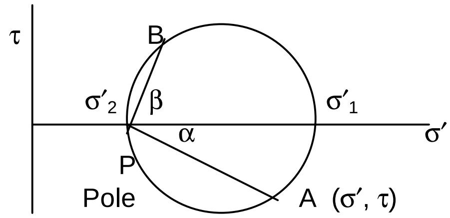


Fig. 4b Stress circle

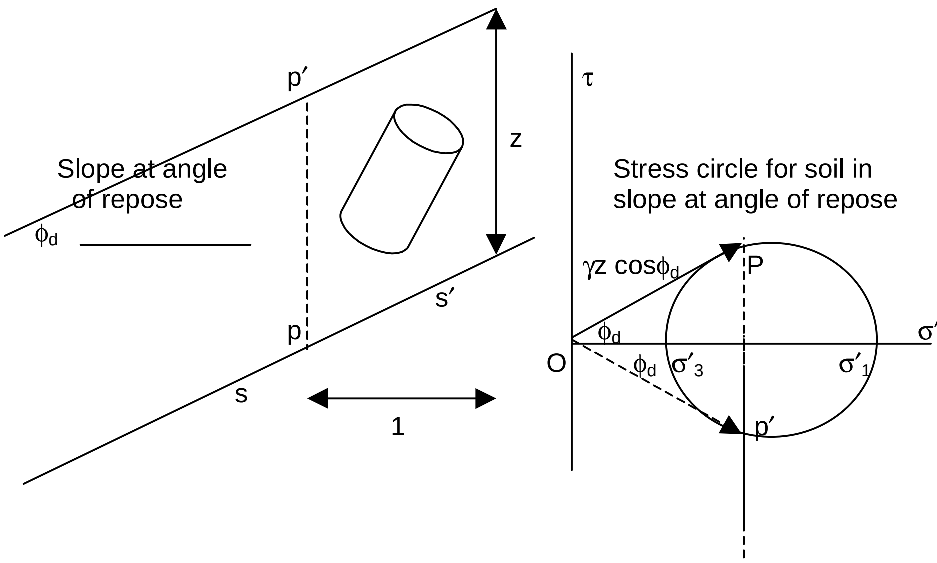


Fig. 5 Slope at angle of repose

Stress circle for soil in slope at angle of repose

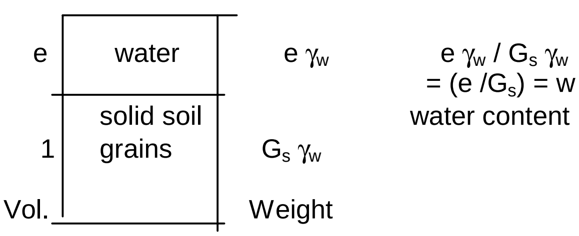


Fig. 6a

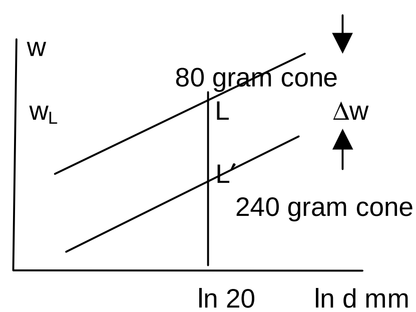


Fig 6b

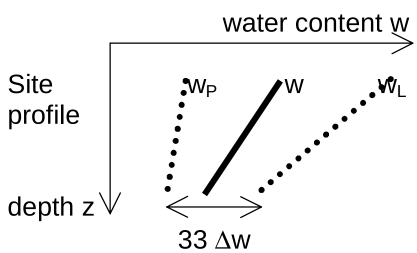


Fig. 7

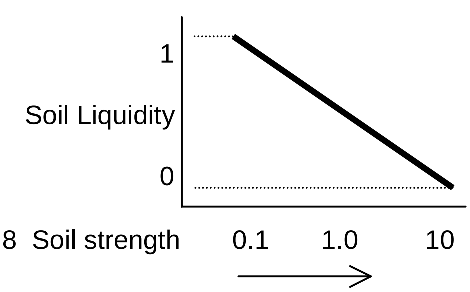
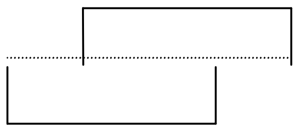
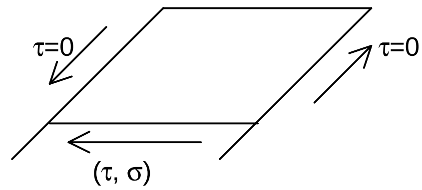


Fig. 8 Soil strength

Fig. 9





In Bishop's shear box the shear strain was not uniform

Roscoe's SSA had end flaps with no complimentary shear stress

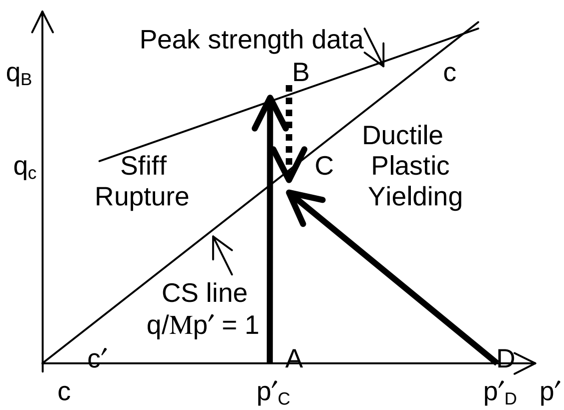


Fig.10a

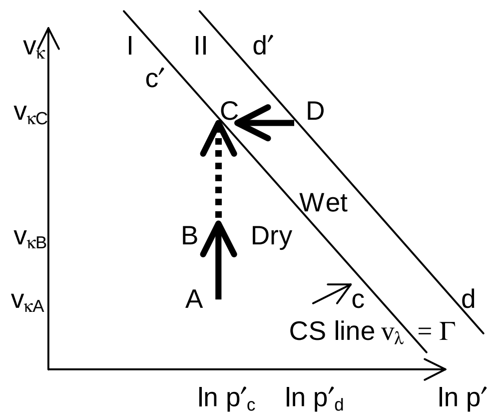


Fig. 10b

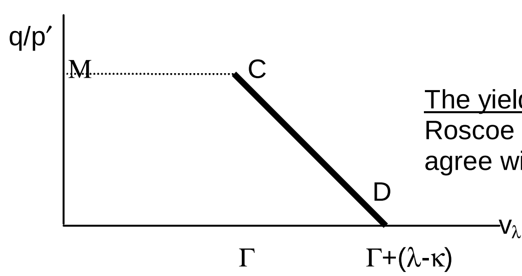


Fig 11

<u>The yielding of 'Kamu-clay'</u>
Roscoe & Schofield (1963)
agree with Shibata (1963)