Particle Shape Characterisation using Fourier Analysis

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CUED/D-Soils/TR315 (2000)

Abstract

A novel technique for the objective assessment of particle shape is presented. The technique uses complex Fourier shape descriptors and image analysis of SEM photographs of sand grains to provide an accurate quantification of particle morphology and texture. Three lower order Fourier descriptors, denoted "Signature Descriptors" provide measures of Elongation, Triangularity and Squareness, whilst an additional descriptor, denoted "Asymmetry" provides a measure of particle irregularity. They describe the overall shape of soil particles (defined as "morphology"). A summary of higher order descriptors provides textural information which is related to local roughness features (defined as "texture"). The results of studies on three silica sands (two standard, laboratory-use and one natural, unprocessed) and one carbonate sand are presented. Breakage of particles by crushing is shown to affect the morphological signature differently depending on the type of sand, though it does not significantly alter texture. The study highlights the importance of microscopy in revealing sand grain shape and texture and shows that simple statistical tools may be used to translate the information provided by relatively few grains to that of a larger body of soil.

Introduction

The mechanical behaviour of a granular material is governed by its structure and the effective stresses applied to it. Structure is considered to be a function of the packing of particles - giving overall density *(or* relative density) and degree of anisotropic behaviour. Factors that affect this are the particle size, size distribution and shape, and the arrangement of grain contacts (fabric).

It has been suggested that more angular particles produce materials more liable to creep (Human 1992; Leung et al. 1996) under constant effective stress, although Oda (1972) thought that rearrangement of relatively spherical particles would occur more easily than elongated ones under triaxial shear. The effect of "surface roughness" has also been investigated by Santamarina and Cascante (1998), and surface characteristics have been shown to influence small strain deformation behaviour. In many discussions regarding particle shape, however, the concepts of "angularity", "roundness" and "roughness" are vague, and it is evident that there is some difference in use of these terms on a day-to-day basis. This has generated the search for a quantifiable method of measurement of particle shape.

A quantifiable approach to particle characterisation is presented here, whereby shape description may be undertaken of individual grains using electron microscopy and computer aided techniques to generate Shape Descriptors via complex Fourier analysis. The fineness of detail to which a grain or collection of grains may be assessed is controlled by the operator, however it is shown that three lower order descriptors, applicable to magnitudes of "Elongation" (or aspect ratio), "Triangularity" and "Squareness", respectively, together may be used to form a signature of the particle morphology. It is also shown that the "fine detail" or textural roughness may be simply estimated via a summary of the higher-order descriptors of groups of grains.

Results of comparisons between several sand types (three silica and one carbonate) are given to illustrate the use of this method and in so doing, illustrate the value of microscopy in soil mechanics. Microscopy shows that sand grains of an apparent similar nature may be very different in shape than appears to the naked eye, whilst breakage of sand grains under load can produce very different changes in characteristic shape, dependant on the type of sand under load.

Finally, as engineers are interested in the behaviour of populations of particles (i.e. soil bodies), statistical analysis should be a central tool in the search for a link between micro and macro behaviour. In this study, simple statistical analysis enables generalisations to be made about the shape characteristics of populations of soil grains from discrete measurements.

Review of Alternative Methods of Shave Characterisation

Various attempts have been made to characterise particle shape. Some methods measure the overall shape or form, whilst others concentrate more on features such as angularity versus roundness, and others on the still finer textural differences between shapes (Barrett 1980). Barrett's definitions are given in Fig. 1. In this paper, the term "morphology" is effectively used to combine Barrett's "form" with "roundness", while "texture" defines the very local roughness features.

Measurement of morphology in soil mechanics has historically required the use of standard charts against which individual grains may be compared. "Sphericity" was defined by Krumbein (1941) as the ratio of particle volume to that of the smallest circumscribing sphere. A two-dimensional sphericity chart was then developed by Rittenhouse (1943) to facilitate the application of this method by visual means. "Roundness" was defined as the ratio of the curvature of the corners and edges of the particles to that of the overall particle (Wade11 1932), and again a two dimensional chart was developed to aid use (Krumbein 1941).

Sphericity and roundness are measurements of two, very different, morphological properties - sphericity is related to "form" as defined by Barrett, whilst roundness relates to angularity and texture. Sphericity is essentially most sensitive to elongation - i.e. that increasingly elongated grains have lower values of sphericity, whilst many other features are lost. Conversely, roundness is largely sensitive to the sharpness of angular protrusions from the particle, giving low values for rough particles and higher values for smooth ones. Hence, particles which may be rather elongated, or triangular will still have rather high values of roundness if the surface texture is smooth, whilst roughened spherical particles may receive low roundness values.

It is the common use of the words "sphericity" and "roundness" in everyday language which may cause confusion when applied in engineering practice. Also, the use of chart methods is time consuming and may add a subjective element to the process of grain characterisation. Computer analysis of shape has improved greatly in recent years and standard applications may include such measurements as aspect ratio and "roundness" (typically given as the square of the ratio of the perimeter of the particle to its area, which is different to Wadell's definition). This may remove the subjectivity inherent in the chart methods, however, there still remains the issue of clarity regarding the use of non-standardised terms.

In powder technology, Shape Factors have been suggested as a method of particle shape characterisation, albeit with limited success. Heywood (1954) suggested a shape coefficient "f/k", which used measurements of surface area relative to volume to place particles in categories such as "angular, tetrahedral", "sub-angular" and "rounded". This method could not distinguish, however, between complex shapes or clusters of shapes. Staniforth and Rees (1981) suggested an additional parameter to determine re-entrant angles "e/n", where the number of downward pointing projections in a collection of particles is divided by the number of particles. However, it could only successfully characterise highly stylised shapes. Poczeck (1997) has suggested the use of a Shape Factor "NS" , which involves measurements of the deviation from standard shapes in a matrix form. This method is limited by its inability to define irregular shapes and distinguish relative roundness.

Fractal analysis (Vallejo 1995) has been presented as a possible alternative to the above methods. It has the advantage of objectivity in that computer analysis is used to determine the fractal parameters. However, it allows measurement of texture rather than the particle morphology, and may need to be used in combination with other parameters to totally define a particle.

Fourier Methods

An alternative method of grain characterisation is to use a Fourier mathematical technique. Much research has been undertaken in the field of powder technology and geology to produce mathematical relations that may characterise the profiles of individual particles (Beddow 1989; Clark 1981).

One possibility is the (R,θ) Fourier method in closed form (Ehrlich and Weinberg 1970), where the profile of a 2D soil particle or flake is traced out as shown in Fig. 2:

The profile equation is:

$$R(\theta) = a_0 + \sum_{n=1}^{N} \left(a_n \cos n\theta + b_n \sin n\theta \right)$$

Equation 1

Where R(8) is the radius at angle θ

N is the total number of harmonics

n is the harmonic number

a, b are coefficients giving magnitude and phase for each harmonic

This method is favoured in geology research, predominantly for the analysis of roughness and textural features of granular soils. However, one **difficulty** with this method lies with re-entrant angles where a particle profile “doubles back” upon itself, with the type of shape given in Fig. 2. In this case, two possible values of R(8) can exist - this may be problematic with carbonate sands, such as Dog’s Bay which has concave shell-like particles. In addition, the centroid of the particle must be accurately found initially in order to perform the analysis, which can prove difficult with complex shapes.

Another possibility, which was introduced by Clark (198 1) in his review of quantitative shape analysis, is the Fourier descriptor method. This method was later investigated in geology research, (Thomas et al. **1995),** where the authors **recognised** its use in the appraisal of sedimentary processes (i.e. by classifying particles into groups according to spikes of individual harmonic frequencies). However, the use of this method to provide morphological and textural commentary to the mechanical behaviour of soil populations has not previously been considered.

In Complex Fourier analysis, the boundary of the particle is circumnavigated in the complex plane at constant speed. The step size is chosen such that one circumnavigation takes time 2π and the number of steps is 2^k (e.g. in this research, 2’ = 128 steps are used). A complex function is obtained:

$$x_m + iy_m = \sum_{n=-N/2+1}^{+N/2} (a_n + ib_n)\left[\cos\left(\frac{2\pi nm}{M}\right) + i\sin\left(\frac{2\pi nm}{M}\right)\right] \qquad \textbf{Equation 2}$$

Where x, y are coordinates describing the particle
N is the total number of descriptors
n is the descriptor number
M is the total number of points describing the particle
m is the index number of a point on the particle
a, b are **coefficients** for each descriptor
i denotes an imaginary number

The Fourier descriptor method was chosen over other methods to describe the individual particles due to its flexibility and ease of use. It does not suffer from the **re-entrant** angle problem of the **(R,θ)** Fourier method and accurate location of a centroid for each particle is not required.

The number of points chosen also dictates the number of descriptors gained from the Fourier analysis, and therefore the level of detail described. For example, for 128 points, **N=128** possible Fourier descriptors are produced, that is **-63<=n<=+64.** The complex nature of Equation 2 means that the lower order descriptors **(n=+1** to **+4** and n=-1 to -4) will describe the overall particle morphology and generally have the greater coefficients due to the larger described features. The values generally decay toward the descriptors at **+64** and -63 as the features become smaller, For general **use** as descriptors for a body of sand however, only the magnitude of the descriptor coefficient is required, i.e. $\sqrt{a_n^2 + b_n^2}$.

In general, it has been found that the first 10 to 15 descriptors are usually sufficient for full character description of a highly complex shape (Sonka et al. 1993). In the case of a sand particle however, 3 terms have been found to be enough to quantify the approximate morphology.

Morphology - General

Standard shapes were used to extract features of the descriptors. The shapes were a circle, a square, an equilateral triangle, an isosceles triangle, an oval and a rectangle, as shown in Figs 3(a) to (f). Roughened versions of the square and triangle (Figs. 3(i) and (j)) showed the effects of random noise or rough features on the gross morphological descriptors. A square and a triangle with rounded edges (Figs. 3(g) and (h)) were analysed to determine the difference between "angular" and "rounded" particles. The results of the complex Fourier analysis of the first seven descriptors is also given in Fig.3.

From the analysis of the standard shapes (Figs. 3(a) to (f)), particular features or "Signature Descriptors" may be obtained. Descriptors n=O, -1, -2 and -3 give Radius (a similar measure to particle size), Elongation (i.e. aspect ratio), Triangularity and Squareness features, respectively. Descriptor n=+l gives a measure of Asymmetry or irregularity, such that the square, rectangle, oval and equilateral triangle (i.e. all "regular" shapes with their centres of gravity equidistant from any corners) have values of 0.00, whilst the isosceles triangle and the roughened equilateral triangle and square both have non-zero values. Descriptor n=+2 is a "second order" elongation term and n=+3 a second order triangularity term.

Figs. 3(g) and (h) show the effect of rounding the edges of standard shapes. In Fig. 3(g) the reduction in Triangularity coefficient, n=-2, is marked for the rounded triangle (coeff.=O. 1748) compared with the standard equilateral triangle (coeff.=0.2500), however, it is still high compared with other values, indicating that this shape is generally triangular in morphology. The results are echoed for the rounded square for the Squareness coefficient, n=-3 in Fig. 3(h) (coeff.=0.0642) compared with the standard square (coeff.=0.1113). This shows that the Signature Descriptors are sensitive to the sharpness of the protrusions from a particle, or its overall "angularity". Note that the second order triangularity term, n=+3, is high for the sharp triangles, Figs. 3(e) and (f) (coeffs.=0.0625, 0.0496), but low for the rounded triangle, Fig. 3(g) (coeff.=0.0068). The second order terms, n=[+2, +3] give additional information, but the Signature Descriptors, n=[-1, -2, -3], and Asymmetry descriptor n=[+l], are in general sufficient for approximate morphological description.

Examination of Figs. 3(i) and (j) shows that, whilst the coefficients of descriptors for noisy or very rough particles are affected slightly (reducing the higher values and increasing the lower values), the results do not obscure the overall signature of the particle morphology, i.e. Squareness, n=-3, (coeff.=0.113) Fig. 3(d) and (coeff.=0.0816) Fig. 3(j); Triangularity, n=-2, (coeff.=0.250) Fig. 3(e) and (coeff.=0.217) Fig. 3(i).

Method of Image Capture and Analysis of Sand Grains

Image Analysis

Groups of individual sand grains were electron micro-photographed using a constant **SEM** tilt angle of 20^{o}. Grams tend lie with their most minor axis flat on to the vertical - i.e. in the most stable configuration. Tilting the SEM allows better quantification of the overall shape by including a glimpse of this minor axis and produces electron "shadows" that helps with grain differentiation by computer. Various authors have discussed the effects that SEM tilt produces upon the statistical results of (R,θ) Fourier analysis (Dowdeswell 1982; Tilmann 1973), and it has been found that apparent morphology can vary with tilt angle. The extreme case here is with the Dog's Bay sand whose shell-like particles (Fig. 5(d)) might appear either rounded and flat or extremely thin and elongated, dependant on the perspective. However, the overall effect is simply to mask the difference between particle shapes by reducing the difference between the coefficients of the Signature Descriptors.

Taking a statistical mean of the data produced with a constant angle of tilt was felt to reduce potential errors associated with perspective and give a good overall picture of particle morphology. The average number of particles used for this study was 260, though the minimum number was 91 for Type C which had the lowest values of standard deviation for each Signature Descriptor. In general, it is felt that a minimum of 200 particles should be assessed to ensure reasonable statistical analysis (see Table 1), although t-Tests may be undertaken on smaller sample sizes than this.

The digital images were input to a commercial image processing package, Image Pro-Plus 4.0 (Media Cybernetics, Silver Springs, MA, USA) in grey-scale. Outlines of the grains were produced by the programme according to the threshold of grey chosen - some manual corrections needed to be made to ensure that the grains were correctly defined and did not overlap. After this analysis, each grain had associated with it a series of coordinates in the x,y plane which defined the particle shape. The coordinates given by the programme were randomly spaced apart dependant on the shape and roughness variation around the grain (i.e. a rough edge would have more coordinates to define it than a smooth one, see Fig. 4a). These were output to a text file.

A simple in-house programme in C++ was used to determine the Fourier descriptors for each sand grain. In order for the Fast Fourier Transform (FFT) to be correctly carried out for each grain, its perimeter is required to be broken into steps of equal length. Therefore, the length of each grain perimeter was assessed and this was broken into 128 equal lengths to produce 128 new coordinates. In order to reproduce grains in the real-space domain from the FFT, both the real and imaginary elements of the frequency domain must be maintained as separate - i.e. both the magnitude and the phase angle are required. Fig. 4 shows a typical profile of a sand particle as reconstructed using Fourier descriptors in this manner. Fig 4(a) gives the original profile as output by the image analysis programme whilst Figs 4(b) to (f) show the effect of reducing the information by suppressing greater numbers of the higher order descriptors. The fine detail is increasingly lost but the overall morphology is retained.

The data for this was output to a text file for each sand type. The coefficients were further analysed using a spreadsheet to produce statistical data, including mean and variance for each sand type, with two-tailed t-Tests being performed in comparative studies between populations of sand grains.

Sand Types

Three types of silica sand were compared with one carbonate sand. The silica sands were laboratory-use white or "silver" Leighton Buzzard silica sand, denoted "C" (particle sixes between 212-300mm) and "E" (particle sizes between 90-150mm) and unprocessed silica sand obtained from the Lower Greensand formation in the South of England, denoted "HW" (this has a slight brown colour, indicating the presence of impurities such as iron oxide). Figs. 5(a) to (c). The carbonate sand was Dog's Bay, denoted "DB", and was comprised largely of shell fragments, Fig S(d).

In order to investigate morphological changes to sands related to such construction processes as pile driving, Type E and Type HW sands were also slightly crushed and analysed. Crushing was accomplished under uniaxial compression in a steel cylinder with 5 cycles of loading and unloading up to 23MPa for 2 minute periods, to ensure the same energy input to both sand types. The particle size distributions are given in Fig. 6 for the sands tested. In order to ensure the whole range of particle sizes within each sand-type was assessed with sufficient resolution, each sand was sieved and the sieve fractions were electron micrographed individually.

Results & Discussion

Fig. 7 shows coefficients for all 128 descriptors used to describe a typical particle. Descriptors diverging from approximately +/-8 describe increasingly fine detail, the precision of which may be gradually lost due to a trade-off being taken between describing a large number of particles and having a high image resolution. Nyquist's rule states that twice as many points around the soil grain must be known as descriptors extracted - otherwise signal aliasing takes place. Therefore, for the resolution of the images analysed for gross morphology only, if descriptors are only assessed up to say n=+3 and -3, the theoretical minimum number of evenly spaced points required to describe a grain would be 14 (i.e. 7 multiplied by 2), whilst in this research, 100 points are typically used to describe each particle. However, for assessing textural detail, a fuller range of Fourier descriptors is required. Closer up grain images taken for this purpose yielded a minimum of 130 points per grain which limited meaningful textural analysis to a maximum of +32 and -32 descriptors. Hence descriptors from *+J-33* to *+J-64* are ignored.

Morphology - Sands

For the purposes of this research upon shape, the coefficients of descriptors -1, -2 and -3 are focussed upon and are referred to as "Signature Descriptors", whilst descriptor +1, denoted "Asymmetry", provides additional commentary to the process of particle breakage. Table 1 gives comparative data of these descriptors for the silica sand types C, E, E crushed, HW, HW crushed and DB.

The carbonate sand, DB, is shown to be most elongate, triangular, square and asymmetric of all the sand types based on the mean of the data, although the variance is high. Further analysis - comparing with Type E crushed (the closest in terms mean values of coefficients) shows that the difference in morphology is highly significant (t-Test Elongation; t=6.041: P=O.OOO, Triangularity; t=3.304: P=0.001, Squareness; t=1.521: P=0.129; Asymmetry; t=5.415: P=0.000).

Examination of the coefficients for the laboratory sands shows that although C and E are from the same source, the average particle shapes are quite different, that is, E is significantly more elongate, triangular and square (t-Tests - Elongation: t=2.773, P=0.006; Triangularity: t=2.398, P=0.018; Squareness: t=2.285, P=0.024) than C. It was noted in the analysis of sieved sand fractions that smaller particles in Fraction E were yet more elongate and triangular (not shown in the Table).

The data also indicate that the morphology of Type E clean uncemented sand and HW slightly cemented natural sand are, in fact, quite similar despite their apparent difference to the naked eye (t-Tests - Elongation: t=1.048, P=0.295; Triangularity: t=0.602, P=0.547; Squareness: t=1.274, P=0.203; Asymmetry: t=1.048, P=0.295). However, crushing Type E and Type HW produced quite different results. Crushed Type E has higher mean coefficients and greater standard deviation values than intact Type E for the three signature descriptors, implying more elongated, more triangular and more square-cornered particles are produced upon crushing. The difference between results is however, slight, and it may be considered that the results are not statistically significant (t-Tests - Elongation: t=0.588, P=0.557; Triangularity: t=0.538, P=0.591; Squareness: t=1.234, P=0.218) although they do indicate a trend. The authors believe that the energy used to crush these particles may need to be greater to produce a result which is considered "significant".

Conversely, crushed Type HW results in a significantly lower coefficient for Elongation than intact Type HW, implying a rounding of particles is taking place (t-Test Elongation: t=3.242, P=0.001), whilst Squareness reduces slightly, though not significantly (t-Test Squareness: t=1.022, P=0.307) and Triangularity is little changed. This implies that a different crushing mechanism is at work. It is suggested that whilst breakage occurs across grains for Type E, producing greater quantities of flakey particles, breakage for Type HW occurs preferentially at cemented contacts, producing rounder individual particles from conglomerates (see Fig. 5(c)). This is supported by the particle size distribution changes for Type E and HW, as shown in Fig. 6. Crushing Type E produces an increase in fines with little change in numbers of larger particles, whilst crushing Type HW produces both an increase in fines but also a decrease in the number of larger particles.

Examination of Asymmetry yields an interesting result, in that whilst the natural sand, HW, is far more irregular than the clean sand Type E, both produce lower coefficients of Asymmetry as a result of crushing. In the case of Type E, the result is not statistically significant (t-Test Asymmetry: $t=0.725$, $P=0.469$), but in the case of HW it is highly significant (t-Test Asymmetry: $t=2.605$, $P=0.009$). Both results indicate a trend however, of removal of irregular or asymmetric particles (in the case of Type E, in spite of a trend in increasing angularity). This is in keeping with the suggestion by McDowell (1996) that large co-ordination numbers are less "helpful" to more angular particles in reducing the induced tensile stress of particles under high load.

Textural Detail

Using the R(0) method of Fourier description, Meloy (1977) found a relationship between the higher order coefficients, such that a plot of the log(order) against log(amplitude) gives a straight line above order 8. The possibility that this phenomenon might exist for the Fourier descriptor method was investigated.

Three sands are presented to show differences in texture - two silicaceous, Type C laboratory sand and Type HW natural sand, and one carbonate, Dog's Bay. Type HW crushed was also examined to compare with intact HW. Type E laboratory sand and Type E crushed were also examined, with results similar to Type C - however, for clarity the data are omitted.

It was found that descriptors from +/-8 to +/-32 give a measure of the finer textural features of each sand-type as shown on a $\log_2$-$\log_2$ plot in Fig. 8. Log to base 2 is used to reflect the self-similar, fractal nature of the textural features. Each increase in a power of 2 is a doubling of the frequency of navigation of the particle, so that the same features will contribute, albeit, to increasingly fine detail. Statistical analysis of many particles then ensures that no spike occurs at any particular frequency, giving a straight line on the log-log plot.

It is interesting to note that Type HW and HW crushed give very similar results - in other words the average particle roughness does not change upon crushing. Type C roughness is in general, less than Type HW with a near-parallel linear plot at a lower intercept. Dog's Bay sand shows a different trend from the silica sands, with rougher features at the larger scale, reducing to smoother features at the very fine scale.

It is noted that particular care should be taken when comparing the results of an examination of fine detail. For example, when comparing sands of different overall particle size, the level of "fineness" described by the descriptors will be that compared with the size of each particle, given that 128 (or 2^k) steps are taken around each particle irrespective of the size. That is, each descriptor is normalised by the first descriptor value, which is a measure of the radius of the particle. Hence for a truly comparative study of fine detail, the image magnification should be fixed to that which will give a good description of all the features to be captured (i.e. following Nyquist's Rule), irrespective of the sand analysed.

That the higher order descriptors provide a linear $\log_2$-$\log_2$ relationship in Fig. 8 is felt to be due to the fractal nature of the roughness features. The offset of the plot gives a measure of the comparable roughness of each sand type, whilst the gradient gives a measure of the decay of the roughness to a smaller scale which may be related to the crystalline nature of the constituent material.

Conclusions

A method is presented here, whereby particle morphology and texture may be **quantifiably** and accurately described by use of complex Fourier analysis, whilst the use of scanning electron microscopy and computer techniques allows statistical analysis to be undertaken upon groups of grains.

Four Fourier descriptors are shown to provide a signature of morphology, n=+l, -1, -2 and -3 and these correspond to measures of Asymmetry, Elongation, Triangularity and Squareness, respectively. Further analysis of higher order descriptors (divergent from **n=+/-8)** is shown to provide a summary of the textural features of groups of particles. The linear relationship of log (amplitude) against log (descriptor number) can be used to quantify roughness features by its slope and intercept. A higher intercept shows a greater degree of rough texture and a steeper gradient shows a greater decay of roughness toward the smaller scale.

Using this technique, two standard laboratory sands, C and E, which come from the same source, are shown to have quite different morphological signatures but similar texture. The textural gradient of the silica sands, C, E and HW, is found to be very similar, whilst the standard laboratory sands are generally smoother than the natural, unprocessed sand, HW. The carbonate sand, DB, has a different textural gradient than the silica sands, being rougher at a large scale and becoming smoother as the scale reduces. Breakage of particles by crushing is shown to affect the morphological signature differently depending on the type of sand, though it does not significantly alter texture.

Acknowledgements

This study was made possible by support from the EPSRC **(GR/L97025),** to whom the authors are grateful. The authors also wish to thank Mr S. Chandler and Mr T. Ablett of the Geotechnical Group for technical advice and assistance, Mr A. Heaver of the Materials Group for help with the electron microscopy and Dr M.D. **Bolton** for many thoughtful suggestions and comments.

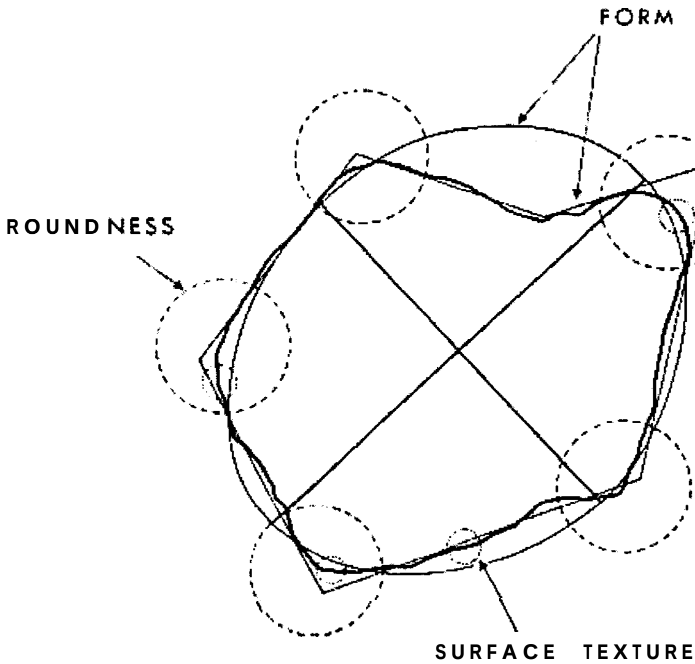


Fig. 1 Particle shape terminology as defined by Barrett, 1980. The term “Morphology” is used in this paper as a combination of Barrett’s roundness and form. Texture is as Barrett’s definition.

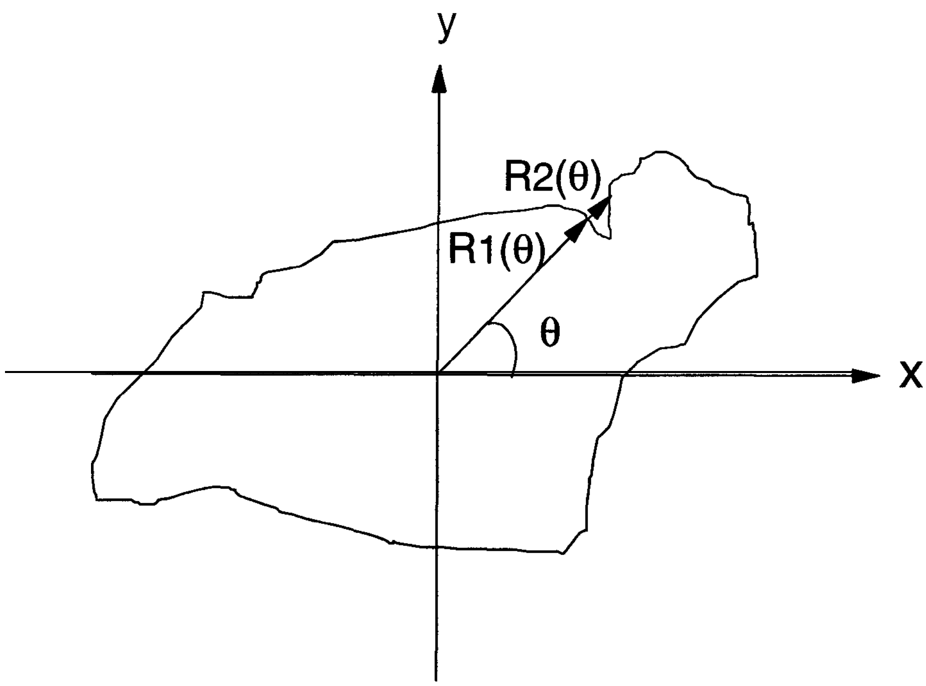


Fig. 2 Use of Fourier analysis in closed form and difficulties encountered with re-entrant angles.

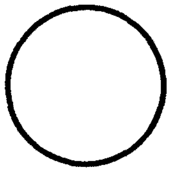
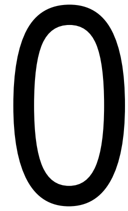
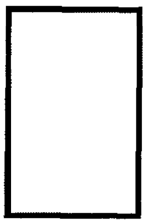
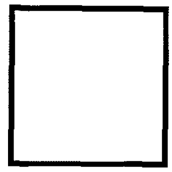
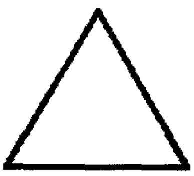
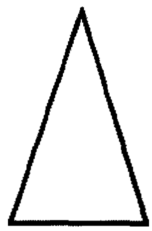
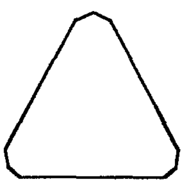
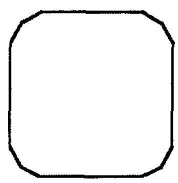
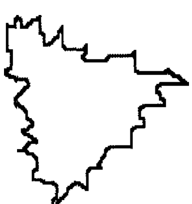
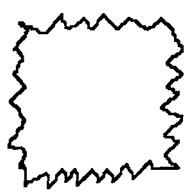
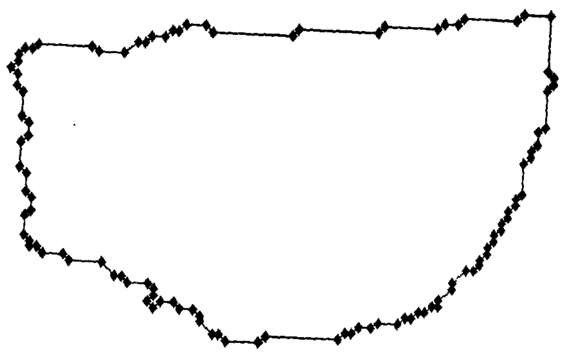
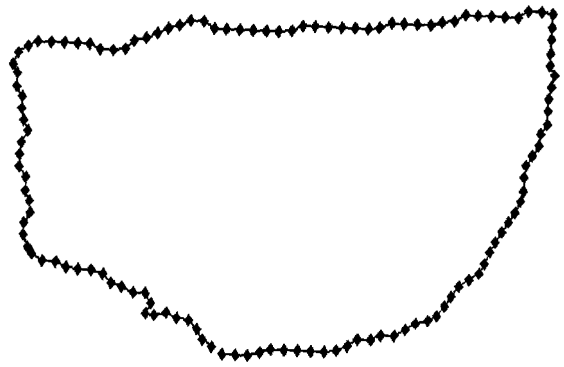
Shape	Descriptor	Coefficient	Shape	Descriptor	Coefficient
 (a)	0	-	 (b)	0	
	-1	0.0000		-1	0.2607
	+1	0.0000		+1	0.0000
	-2	0.0000		-2	0.0000
	+2	0.0000		+2	0.0749
	-3	0.0000		-3	0.0137
	+3	0.0000		+3	0.0000
 (c)	0		 (d)	0	
	-1	0.2680		-1	0.0000
	+1	0.0000		+1	0.0000
	-2	0.0000		-2	0.0000
	+2	0.0814		+2	0.0000
	-3	0.0814		-3	0.1113
	+3	0.0000		+3	0.0000
 (e)	0		 (f)	0	
	-1	0.0001		-1	0.2314
	+1	0.0001		+1	0.0346
	-2	0.2500		-2	0.2286
	+2	0.0001		+2	0.0467
	-3	0.0001		-3	0.0281
	+3	0.0625		+3	0.0496
 (g)	0		 (h)	0	
	-1	0.0043		-1	0.0000
	+1	0.0015		+1	0.0000
	-2	0.1748		-2	0.0000
	+2	0.0019		+2	0.0000
	-3	0.0035		-3	0.0642
	+3	0.0068		+3	0.0000
 (i)	0	-	 (j)	0	
	-1	0.0483		-1	0.0214
	+1	0.0490		+1	0.0799
	-2	0.2170		-2	0.0279
	+2	0.0268		+2	0.0142
	-3	0.0555		-3	0.0816
	+3	0.0692		+3	0.0285

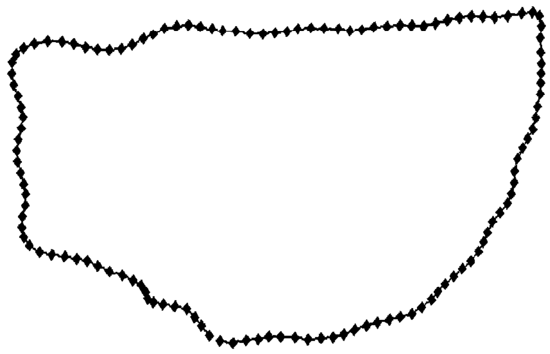
Fig. 3 General shapes used to investigate Fourier shape descriptors for morphological description. 0 Radius, -1 Elongation, -2 Triangularity, -3 Squareness, +1 Asymmetry, +2 Second Order Elongation, +3 Second Order Triangularity. (Note that n=O is not given as in this case the result is arbitrary).



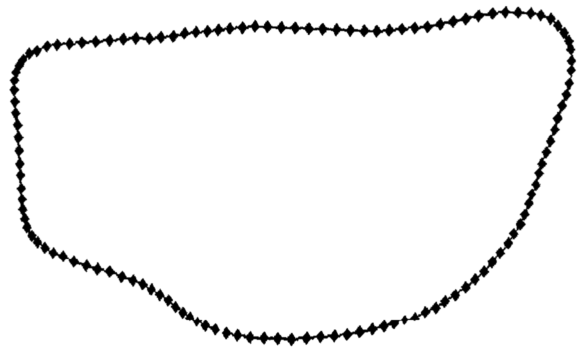
(a) Original digitised outline of particle. The x & y coordinates are taken as real and imaginary respectively, for the complex Fourier analysis'.



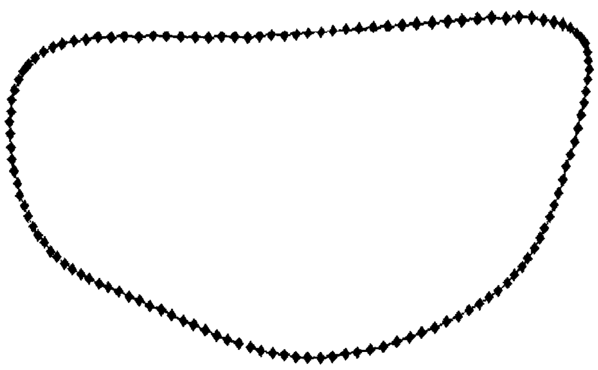
(b) +/- 64 Fourier descriptors



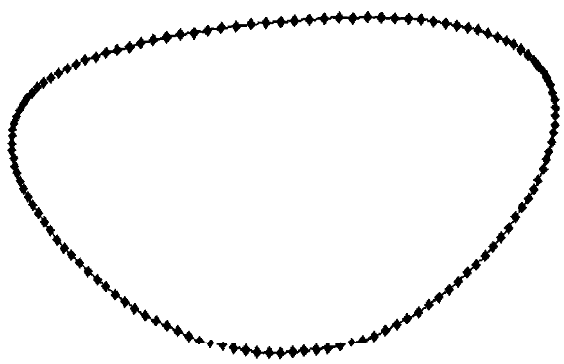
(c) +/- 24 Fourier descriptors



(d) +/- 8 Fourier descriptors

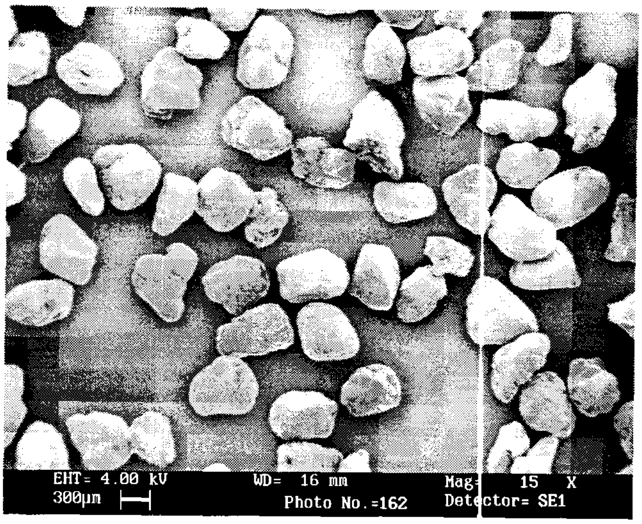


(e) +/- 5 Fourier descriptors

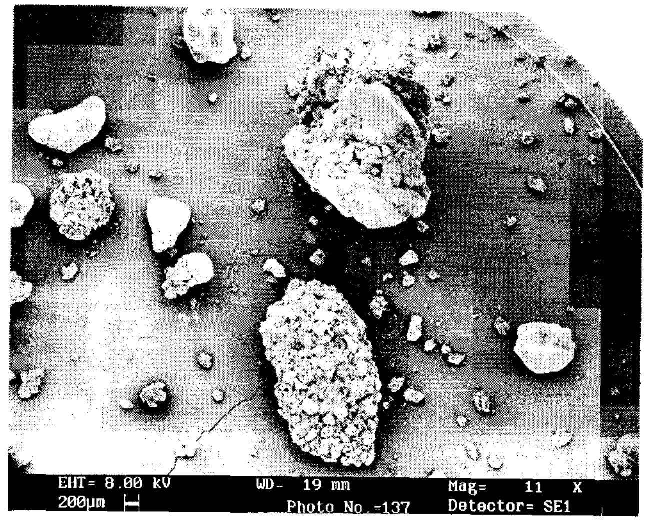


(f) +/- 3 Fourier descriptors

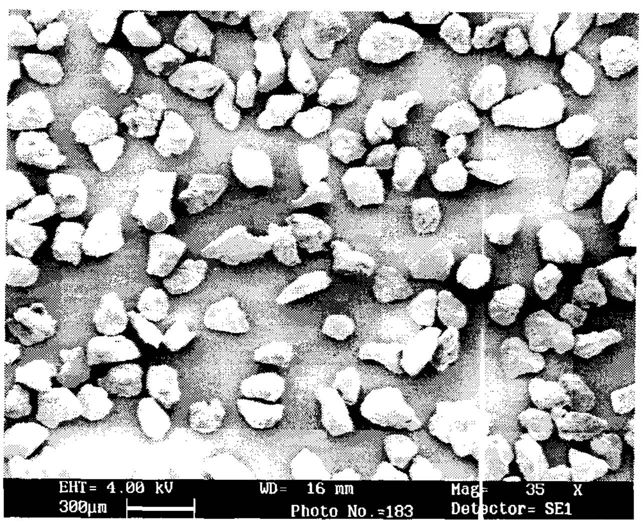
Fig. 4 The effect of reconstruction of a particle from complex Fourier descriptors using successively fewer descriptors.



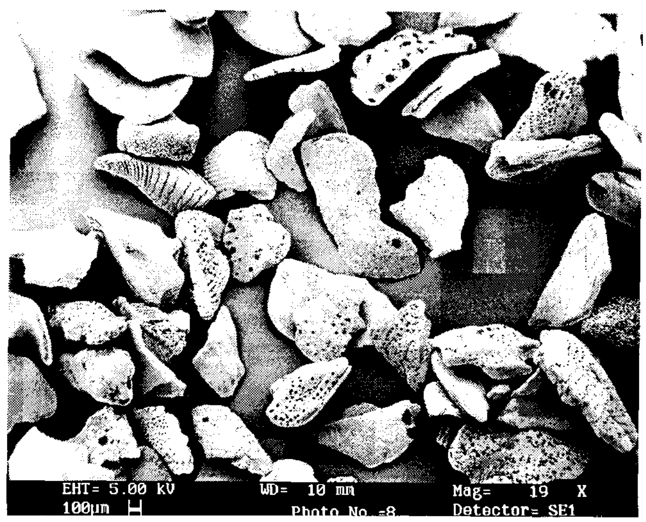
(a) Type C laboratory silica sand



(b) Type HW slightly cemented natural silica sand showing a large conglomerate which to the naked eye appears as a single sand grain



(c) Type E laboratory silica sand



(d) Type DB natural carbonate sand

Fig. 5 Electron micro-photographs of sand Types used in this study.

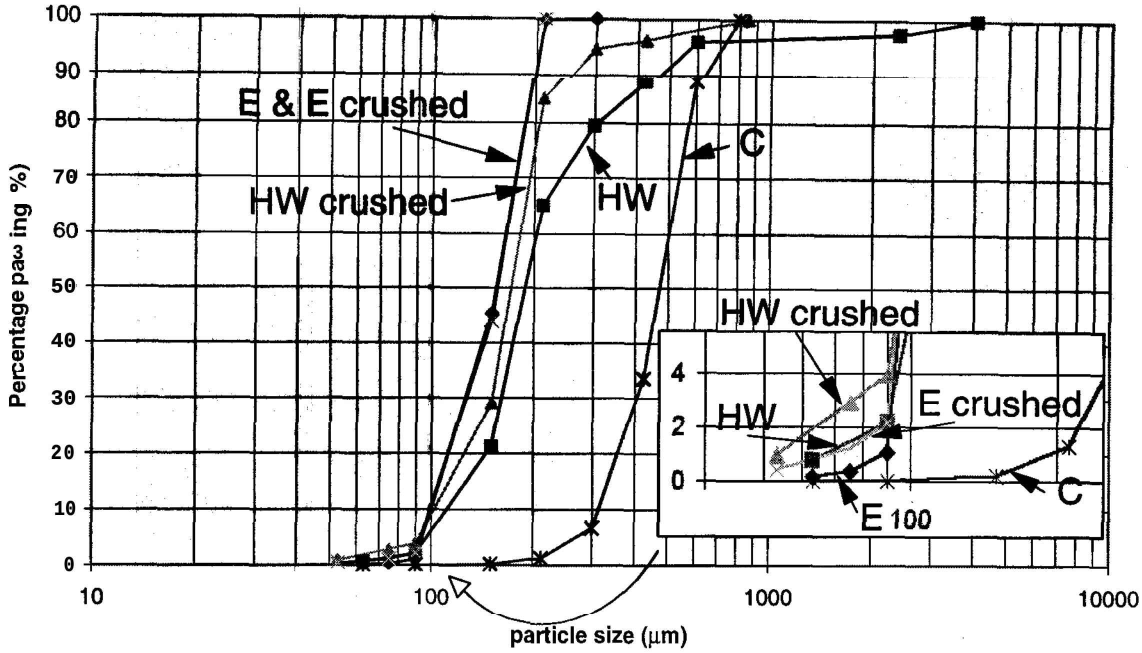


Fig. 6 Particle size distributions for sands analysed.

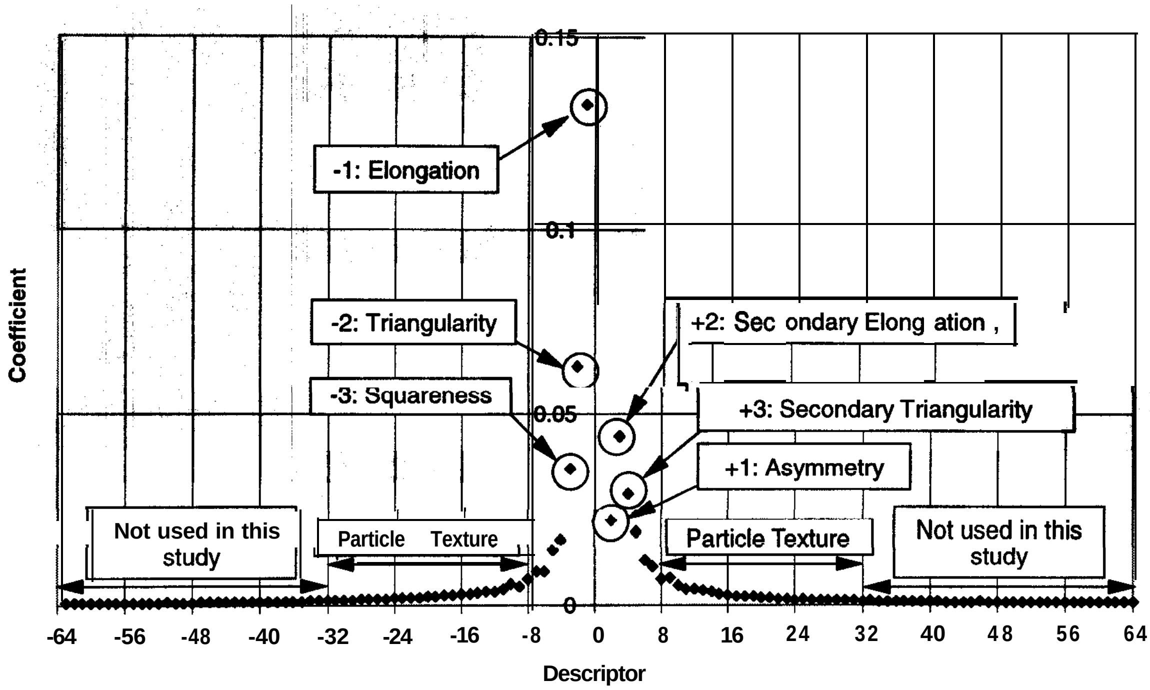


Fig. 7 Descriptor magnitudes for a typical particle. Signature descriptors, “elongation”, “triangularity” and “squareness”, as well as secondary descriptors, as shown. Range of descriptors used for textural analysis, as shown. Descriptors denoted “not used in this study” is due to strict adherence to Nyquist’s rule.

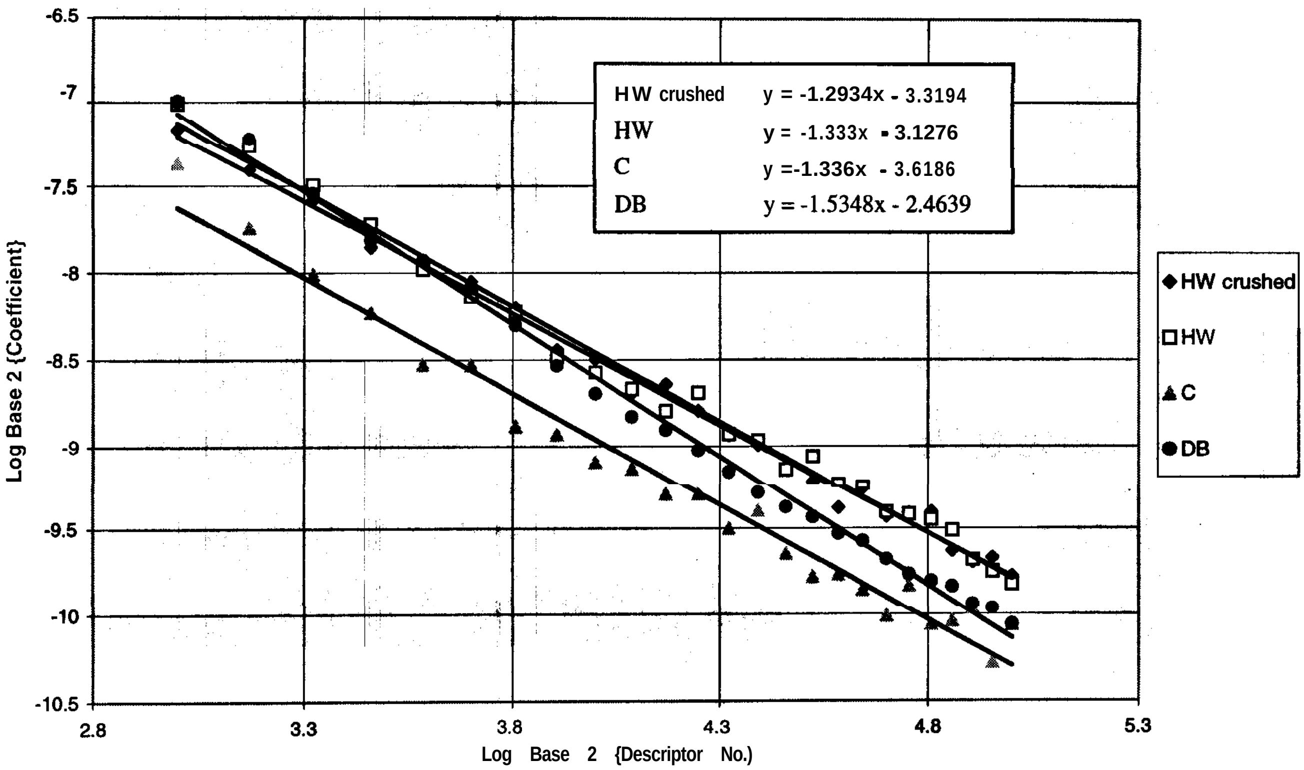


Fig. 8 Log-log plot to base 2 of higher order descriptors showing different slopes and intercepts between types C, HW, HW crushed and type DB. Type E omitted for clarity.

Table 1 Signature Descriptor coefficients for sand types, C, E, E crushed, HW and HW crushed. Numbers of particles, mean values and variance are given for information.

Sand Type	Number of Particles	Sianature Descriptor & Coefficients							
		-1 Elonaation		-2 Triangularity		-3 Squareness		+1 Asymmetry	
		mean	variance	mean	variance	mean	variance	mean	variance
C	91	0.1480	0.00533	0.0522	0.00066	0.0291	0.00021	0.0242	0.00030
E	488	0.1720	0.00800	0.0598	0.00118	0.0331	0.00034	0.0307	0.00051
E crushed	233	0.1765	0.00979	0.0612	0.00115	0.0350	0.00039	0.0293	0.00065
HW	400	0.1653	0.01000	0.0612	0.00124	0.0348	0.00039	0.0324	0.00046
HW crushed	297	0.1421	0.00779	0.0614	0.00127	0.0333	0.00028	0.0284	0.00033
DB	128	0.2763	0.02541	0.0748	0.00155	0.0381	0.00031	0.0492	0.00137

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