The Role of Micro-Mechanics in Soil Mechanics

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Summary

It is suggested that observations of the changing microstructure of soils will permit the selection and refinement of relevant micro-mechanisms which control soil behaviour. A few micro-mechanical models are introduced to demonstrate how they might shed light on shortcomings in the classical continuum approach to soil mechanics. The ultimate objective is to reduce the number and enhance the physical meaning of parameters required to describe soil behaviour, so as to raise the confidence of geotechnical engineers in the constitutive modelling of soils.

1 Introduction

In the first quarter of the 20th century soils were treated by engineers either as linear elastic or as bodies in limiting equilibrium mobilising their shear strength on possible slip surfaces. The influence of pore water was a source of some confusion, though Atterberg's classification of fine-grained soils started to provide a sound empirical underpinning.

"Soil mechanics" became recognised as a new discipline in the second quarter of the century, following Terzaghi's creation of the concept of effective stress, and the resolution of the mechanisms of pore pressure generation and dissipation in one-dimensional consolidation. Advanced direct shear test apparatus were developed, and used by Hvorslev and others to reveal the importance of the degree of overconsolidation, though undrained tests could only by conducted approximately due to difficulties with water control at boundaries. Taylor drew attention to the link between dilatancy and strength in the shearing of soils.

In the third quarter of the century, the development of the triaxial test by Bishop led to the creation of a growing database of the non-linear behaviour of soil elements. This supported new elasto-plastic soil models developed by Roscoe, Schofield, Burland and others. Variants of the Cam-Clay mathematical model were based on the precepts of metal plasticity coupled with volumetric hardening. Critical states of unlimited shearing at constant volume were found to be related to states of normal compression through geometrically similar yield surfaces. The progress of triaxial stress path tests could be seen in terms of an initial elastic response, followed by yielding, with either dilation and softening or contraction and hardening towards an ultimate critical state.

The final quarter of the 20th century saw the proliferation of constitutive models based on plasticity, but extended in an attempt to comprehend more complex features of behaviour such as hysteresis and anisotropy, cyclic ratchetting and liquefaction, creep and ageing. Unfortunately, few models were able to match comprehensive test data, and those that did accurately represent a wide range of behaviour often involved dozens of parameters which had to be selected by curve-fitting. The proponents often challenged the validity of each others' models, and engineering decision-makers were left with little confidence in the process of constitutive modelling. The supposed validation of a particular model against the behaviour of a particular soil, using a large suite of expert element tests, left end-users uncertain about the validity of applying the model to their own soil. To be practical, a large number of parameter values would have to be left unchanged and fresh values fitted only to those parameters which could economically be evaluated from the few standard tests they had time to conduct in the progress of their own ground investigation. Since the parameters in these advanced models generally had no discernible physical significance, the end-user often had little to guide them.

The growing gulf between constitutive modellers and engineering decision-makers has permitted the latter to adhere unnecessarily to unhelpful interpretations and attitudes regarding soil characterisation, testing and analysis. Many engineers continue to use values of linear elastic modulus, and continue to select inappropriate soil cohesion and friction values, which would have been considered poor practice even 25 years ago. It is not uncommon, for example, to find communities of engineers who would still believe today that clay "being a cohesive medium, does not possess internal friction" and who are surprised to be asked to measure it. Most clays near the surface are over-consolidated and dilatant, so that their drained, frictional strength is inferior to their undrained strength in applications such as slopes, excavations, tunnel headings and shallow foundations. Yet what is the frequency of measurement of effective stress parameters compared to undrained strength measured in quick compression tests?

However, two parallel advances have been made in the last 25 years that promise a great deal in future if the current impasse over constitutive relations can be resolved. First, there are now many readily available finite element analysis programs which can accept various soil models and which can simulate a variety of geotechnical processes and activities. Our capacity to simulate the behaviour of soil bodies is often quite good, if only we can correctly characterise the soil. Second, there has been a huge investment in centrifuge testing facilities around the world, so that physical models can be taken through ever more realistic sequences of construction and loading at small scale but at correct stress levels. Special actuators have been developed in various centrifuge centres to simulate eccentric or cyclic loading, pile driving, tunnelling, earthquake effects, grouting etc., and these facilities are now readily available in over 70 centres spread over most regions of the world.

Of course, physical models suffer from exactly the same predicament in soil characterisation as do numerical models: how is an engineer to construct a soil model which will behave like the real thing? This may seem easy with a physical model, in so far as real soils might be sampled and used directly. However, the problem of excessive particle size in relation to model size always requires that large stones from the field are removed, and may require the complete substitution of a model soil with finer grading. Furthermore, there has been relatively little progress in constructing physical models with realistic soil fabrics, and correspondingly little research has been done on the significance of testing soils which are generally too uniform in grain size, too homogeneous, and too "young". And yet, if we do not make centrifuge tests we will often find ourselves using finite element or other analyses with no validation in the physical world whatsoever.

This paper proposes that micro-mechanics offers the appropriate tool to overcome the difficulty over soil characterisation. The technology of the 21st century includes fast particle size analysis using lasers which discriminate from 0.04 microns to 2.5mm, $400 digital cameras which record 3.3 million pixels per picture, optical microscopes with fast computerised image processing which can recreate three-dimensional microstructures, and facilities for lab-bench X-ray, computerised tomography, and MRI. Our agenda should be to observe and quantify soil microstructure as it changes under load, and to establish reasonably economical methods of routine evaluation which can usefully supplement conventional test data.

Macroscopic events in the field, in a centrifuge test or a triaxial test, should be correlated with micro-mechanisms. This should lead to the identification of mechanisms for "stiffness degradation", "shear rupture", "friction", "yield", "compressibility", "hardening", "creep", "ageing", "cracking" etc, and therefore to the elucidation of the (presumably few) independent physical properties of soil grains and pore fluids which control these phenomena. The challenge for constitutive modellers would then be seen in two parts – the identification of micro-mechanisms and associated dimensionless groups of physical parameters, and the prediction of evolving micro-structure including distributions of the size and orientation of both grains and pores. Such a clarification would inevitably lead to the theoretical integration of phenomena which are presently seen as unconnected, and to the discarding of theories which invoke inappropriate mechanisms – of internal energy dissipation, for example. Furthermore, "problem soils” would much more easily be re-categorised in proper mechanical terms. There is therefore an anticipated pay-off in terms of scientific understanding and the prediction of soil behaviour.

There should also be a pay-off in terms of engineering application. If we can supplement macroscopic stress-strain curves with microscopic evidence of changes in soil structure, this will underpin our selection of constitutive relations for FE analysis, or model soil types for centrifuge models. If different microstructures respond in different ways this can be addressed; micromechanical benchmarking is bound to improve the confidence of our ultimate clients in our advanced methods of prediction. In every other branch of materials science, engineers are first shown micrographs to establish that the material conforms to some accepted category, and then handed test data – we can surely aim do the same. Of course, aims are different than deliverables. We have to learn how to apply new technologies to maximum effect before we can attempt to do the key research referred to above. The aim of this paper is to show a few reasons for mobilising our resources, and a few possible avenues for advancement.

2 What is wrong with continuum soil mechanics?

2.1 Challenges

Before pointing to deficiencies in the current state of geo-mechanics it seems necessary to mention some of the challenges that geotechnical engineers are required to face. First, there is an extraordinary diversity in the *materials* we are asked to study, arising from the variety of natural processes which created and modified them. Each elementary grain has a mineralogy arising from its own life history, and a glance at any textbook of geology or geochemistry would show the plethora of mineral types which can be found in the earth’s crust. The differences in texture between glacial moraine, fluvial sands, alluvial muds, marine clays and other sedimentary clastic materials, let alone biogenic deposits such as limestone, coal or peat, and evaporites such as salt and gypsum, are no less remarkable. We must then recall the effects of temperature and the chemistry of suffusing groundwater in possibly solidifying sediments, or weathering them back into individual peds or grains once they have been cemented. Finally, disturbances which include tectonic strains and man’s previous construction activities can fissure or fragment them once again.

Natural materials observe no quality assurance criteria. They have to be *characterised* and tested during a ground investigation, and their presence as discontinuous strata on any particular site obviously compounds the problem. Characterisation needs to be extensive to ensure that significant local features are not missed. Assessments of particle size distributions from disturbed samples of coarse-grained soils, and the conduct of Atterberg plasticity tests on fine-grained soils, are often used not only to correlate and log strata but also to estimate their mechanical properties through empirical correlations. Undisturbed sampling for the purposes of later high quality laboratory tests can be an almost insuperable problem, especially in sands and gravels. On the other hand, rotary coring techniques have improved a great deal in recent years, and piston sampling is available for soft fine-grained soils. Methods of probing without sampling, such as the SPT and CPT, disturb soil so thoroughly that it is necessary to use ancillary geophysical methods (with seismic measurements, etc) if independent inferences of undisturbed soil properties are to be made. There may, however, be possibilities of observing soil fabric in situ if imaging systems were developed and proved first in the laboratory.

Even if perfect samples can be extracted and returned to the laboratory, it is not easy to decide which of the many classes of soil and rock behaviour should be investigated by *testing*. Soil mechanics is an intimate mixture of solid and fluid mechanics. The stiffness and strength of the skeleton, and the permeability and porosity of the pore space, are all highly significant and variable. In particular, soil permeability may vary over 6 or 7 orders of magnitude from sands to clays found within a few centimetres of each other in a soil profile. Furthermore, the words "stiffness" and "strength" do little to convey the range of compressive, shear and tensile phenomena which are of interest (normal compression, swelling, cyclic straining, plastic flow at a critical state, shear rupture, tensile cracking), nor the range of strain magnitudes (from 10^{-6} to 1), nor the range of time periods (from a fraction of a second to a hundred years).

Geo-mechanics is therefore riven with uncertainty and difficulty regarding the sampling, characterisation and testing of these natural materials. Perhaps as a result, design methods which may seem to depend on theoretical principles and mechanical properties are often actually based on *empirical* correlations and previous experience. On the other hand, the industry is rapidly developing new technologies (soil nailing, deep mixing, bio-remediation, tunnel boring, pipe jacking etc) which must be optimised without the benefit of prior experience, and which may need to be tailored to the ground conditions. And notwithstanding the newness of the technology used in construction, there is no possibility of making a few prototype structures prior to the production run; Civil Engineering has generally got to work first time.

No single new idea will resolve these many difficulties. The challenge for micro-mechanics is principally to get the physics of geo-phenomena clarified – to improve empirical correlations, to relate previously unrelated mechanisms, to define appropriate tests and to assist in the proper definition of parameters – by providing a fresh stream of micro-structural data.

2.2 Complications

The calculations carried out by most geotechnical engineers today would be familiar to their counterparts of 50 years ago, notwithstanding the intervening advances in soil modelling. Elastic analyses are carried out for settlements of foundations, using some elastic modulus E. Plastic analyses are carried out to check for limiting stability, using some soil cohesion c or angle of internal friction ϕ. The main sources of errors in these analyses come from the fact that soils are not simple elastic or plastic materials, and can not easily be segregated into "cohesive" and "granular" materials with unique parameters. The parameters E, c, and ϕ are complex functions of state. Since strength is simply the integrated effect of tangent stiffness multiplied by successive increments of strain, it will be necessary only to remind the reader of the complexity of fixing a local value of stiffness.

Soil stiffness E is a Young's modulus which disguises independent variations of the fundamental stiffness parameters – G (shear modulus in distortion at constant volume) and K (bulk modulus in volumetric change at constant shape). Values of stiffness vary over about two orders of magnitude between soft plastic clays and pure sands, so the description "silty clay with some silty sand seams" does little to guide the engineer to an empirical estimate of settlement possibilities. Furthermore, the bedding of sedimentary soils always seems to create significant inherent *anisotropy* in stiffness. And for each soil type, stiffness varies strongly with the magnitude of the mean effective *stress*. At low stress levels the stiffness increases as stress increases, but at some higher value of stress the soil *yields*, and its tangent stiffness reduces usually by a factor of 5 to 10, before increasing again if stresses are raised still further.

The influences of soil type, orientation, and stress level are therefore highly significant. They are, however, compounded by even more significant variations with soil *strain* even where the mean effective stress level is held constant. Soil starts with a high elastic stiffness at strain levels below an elastic threshold about 10^{-4} to 10^{-5}, and this then deteriorates significantly as strain increases. The tangent shear modulus drops to zero, of course, as the soil approaches peak strength. If strains are *cycled* beyond the elastic threshold, hysteresis is evident. At each reversal of strain direction, the subsequent response is stiff and the stiffness then deteriorates with increasing strain, as before, until the next reversal. But the hysteresis loop of cyclic strain is not perfectly closed. Depending on the stress conditions, there can be a residual strain which accumulates cycle by cycle, leading perhaps to compaction of the soil – or possibly to dilation and ultimate softening. Neither is this residual strain per cycle a constant, it reduces cycle by cycle for soils which are compacting and increases cycle by cycle for soils which are dilating.

Finally, there are *time* effects. Soil consists of solid grains and intervening fluids in the pores. If stresses are applied there is likely to be an eventual change of pore volume, but this can only be achieved once pore fluids have escaped to the boundaries. The *primary* phase of consolidation therefore consists of excess pore pressure generation followed by dissipation due to the hydraulic gradients. The *secondary* phase, which is often called creep, can occur even in the absence of excess pore pressures. It rate seems to reduce according to the logarithm of time. It is uncertain how creep is related to *ageing*, the enhanced stiffness and brittleness of soils which have lain undisturbed for a long time in comparison with the behaviour of the same soils after they have been remoulded and brought back into the same state of stress and density. However, their more rock-like manner has led to aged soils being described as *structured* but without much evidence to show how this may have arisen.

Brittleness is not an issue only for aged or cemented soils, of course. Most dense sands dilate at peak strength, and then rupture on shear bands within which they soften to a lower strength as they approach a critical state. Heavily overconsolidated clays do the same, but can also manifest a further strength reduction in which the angle of friction in the slip bands reduces below the critical state friction angle by a further factor of up to 2, to the so-called *residual* friction angle. These episodes of negative stiffness imply that engineers should be applying fracture mechanics principles rather than plastic analysis to the failure of many soil bodies, though this is never done. In stead, designers usually rely on factors of safety.

Research workers have often sought to explain the byzantine complexity of soil stress-strain behaviour by referring to micro-structural changes. Differences between sands and clays are attributed to the relative flakiness of clays. Anisotropy is attributed to the purely vertical body force acting on a settling particle, which comes to rest mainly by inducing vertical reactions from the bed on which it comes to rest. Residual friction is attributed to particle reorientation after significant relative sliding on a slip surface. There is some evidence in each of these cases to support the contention, but their visualisation is far from routinely possible. Other effects – of stress level, strain level, cycles, creep etc – are considered more mysterious, and micromechanical insights would be highly desirable.

2.3 Incomplete theories

Fig 1 Treating a soil element as a system

Figure 1 shows the conventional block diagram of a system responding to an input. In deriving constitutive relations for a material we will treat the soil element as a system and seek to know the complete function f in an expression such as:

$$\text{output} = f\,(\text{system, input}) \tag{1}$$

The input will involve all the influences to which we aim to subject it, and the output encompasses the behaviour in which we are interested. Note that our focus here is on material behaviour, rather than on the behaviour of a soil body. Boundary value problems require further knowledge regarding the spatial satisfaction of equilibrium and continuity equations.

Let us investigate just one of the "complications" referred to in the preceding section, to assess how unsatisfactory our current state of knowledge might be. Consider the stress effect on soil voids ratio as discovered in a one-dimensional compression test. Here, we will ignore time effects and focus on the voids ratios achieved just as imposed changes of stress become 100% effective on the soil skeleton. The class of oedometer data sketched in Figure 2 is perhaps the first representation of soil behaviour shown in an introductory soil mechanics course, but it is also included in all constitutive models:

$$\text{voids ratio} = f\,(\text{applied effective stress}) \tag{2}$$

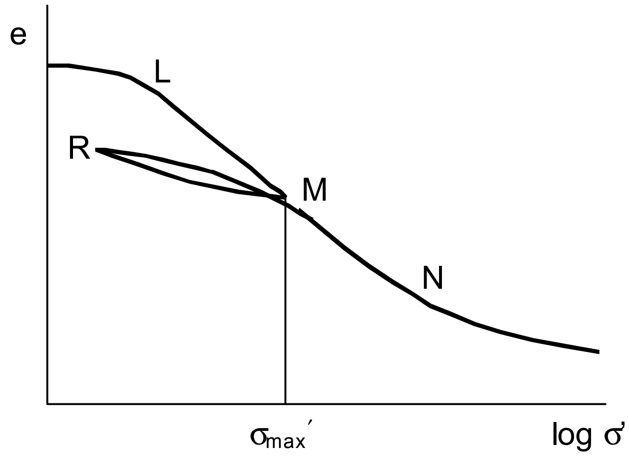


Fig 2 The compression of soils

Different forms of the function f are preferred by different research workers. The Cambridge school prefers:

$$e_{nc} = e_1 - \lambda \log \sigma' \qquad \text{for states of normal compression LMN, } \sigma' = \sigma_{max}'$$
$$(3)$$
$$e_{oc} = e_{nc} + \kappa \log (\sigma_{max}'/\sigma') \qquad \text{for states of over-consolidation MR, } \sigma' < \sigma_{max}'$$

Whatever form is chosen, equation 2 itself is *dimensionally inconsistent*, and therefore fundamentally flawed from the point of view of physical understanding. The reason for the inconsistency is that the applied stress σ' should have been normalised by dividing it by whatever system parameter is operative in resisting the applied stress. Without this normalisation, equation 2 is apparently able to make a mockery of equation 1 by representing the physical phenomenon of void compression without involving any inherent system parameter at all. The corrected form of equation 2 is given below:

$$e = f\,(\sigma' / X) \tag{4}$$

where the normalising parameter X represents the resistance of the solid skeleton to deformation, and has the same units as σ'. Candidates for X must certainly include the shear stiffness G_p and the crushing strength σ_p of the particles.

At first sight, the form of equation 3 applied to the over-consolidation loop MR seems to overcome the problem since σ' is normalised by σ_{max}'. While this is true, the equation still fails to include any system parameters. The way the soil tester discovers σ_{max}' is by looking in the records of the test history, where it has been written. But how does the soil sample itself "recall" σ_{max}'? Where is σ_{max}' recorded in the soil's own physical characteristics?

Two attitudes can be taken to these philosophical criticisms. The first approach is purely empirical and consists, as we have seen, of curve-fitting. This is the approach taken by the entire geotechnical establishment at present, and it is also the approach taken by most constitutive modellers following the pattern set by the Cambridge school in the 1960s. The second approach is to seek to understand the micro-mechanisms responsible for void compression so as to derive dimensionally correct compression relations. This latter is the "clastic mechanics" approach that has been adopted over the last 7 years by the author and his co-workers, with some signs of promise. It has been possible both to explain the form of compression data and to suggest which are the underlying fundamental material parameters; further details are given later.

A similar micro-mechanics agenda is recommended for all the other "complications" of continuum soil behaviour. Note that the aim should certainly not be to subvert the acquisition of data, nor necessarily to supplant the engineer's favourite method of curve-fitting, but rather to superimpose on the data a physical understanding of mechanical processes at the scale of the grains.

A similar agenda faced thermodynamics in the second half of the 19^{th} century, when empirical macroscopic knowledge concerning the states of gases was converted into a theoretical molecular scheme – the kinetic theory of gases. Whereas Boyle had observed that "ideal gases" obeyed the "law"

$$p\,v = m\,R\,T \tag{5}$$

Maxwell and Boltzmann developed the kinetic theory of gases that explained R in terms of the statistical distribution of the kinetic energies of colliding molecules. This led directly to an understanding by van der Waals of "imperfect" gases whose molecules were finite in size and mutually attractive, and to linkages between empirical facts of nature that had previously been thought to be unrelated, such as internal energy, viscosity, condensation, evaporation, ionisation, molecular diffusion, thermal conductivity and dielectric constant. Although engineers continue to use thermodynamics to make calculations of heat and work, Boltzmann's statistical mechanics simplified and integrated the system of definitions in a wide branch of physics, and was therefore indispensable to clear thinking and to progress. Essentially, *we* are seeking a framework for the statistical mechanics of mineral grains in statical contact.

2.5 Opportunity

We can see that;

- empiricism is based on soil types such as "sand" or "clay" but real soils are mixtures;
- classical mechanical parameters such as E, c and ϕ are not constants, but vary with soil state;
- classical analyses based on these parameters can seem to give any desired answer;
- while soil test technology can be excellent, data reduction amounts to curve fitting;
- many approaches to curve fitting are dimensionally incorrect;
- fundamental mechanisms of behaviour are not understood;
- every facet of soil behaviour is plotted separately and gives rise to new parameters;
- practising engineers have little confidence in the plethora of curve-fitting parameters;
- unnecessarily wide bounds for parameter values lead to wide variations in predictions;
- good analytical methods are available, but are under-used due to lack of faith in parameters.

This presents research workers with a clearly defined opportunity: to raise confidence in soil constitutive relations and continuum analysis by elucidating the basic micro-mechanisms of all facets of soil behaviour, providing an independent stream of micro-structural observations, and improving the techniques of test data reduction by deducing more appropriate and dimensionally consistent curve-fitting techniques.

The scientific underpinning of this effort should include:

- measurements of grain and void size distributions;
- characterisation of fabric – shapes, orientations and spatial distributions;
- characterisation of particle contact force variation;
- characterisation of fluid velocity variation;
- measurements of particle intrinsic parameters – density, elastic stiffness;
- measurement of particle (in fluid) extrinsic parameters – crushing strength, friction;
- computer simulation of particulate aggregates using Distinct Element Modelling (DEM)
- computer simulation of intergranular flow and dispersion.

4 Micro-mechanisms of stress and crushability

4.1 Stress - load paths

In continuum mechanics stress and strain are regarded as uniform over a region that is small in relation to the significant sizes of the loaded boundaries. In micro-mechanics, we allow variations in stress and strain at the scale of significant microstructure. So in granular media we need to accept that stress is extremely high at points of loaded contact, rather small away from such points, and equal to the pore pressure in the voids between particles. If micro-mechanisms are to be understood, it must first be accepted that the effective stress tensor devolves to some pattern of inter-granular contact forces. Research by de Josselyn de Jong and Verruijt on the photoelastic analysis of glass balls has shown that the major stress is carried in strong load paths through chains of particles which happen to enjoy favourable contact normals. Figure 3 shows the contact force response at a given instant due to moderate vertical compression; the thickness of the contact force lines indicates their magnitude. These strong load paths switch around suddenly as the deviatoric stress is increased, so that many particles may take turns in carrying an unfair proportion of the overall load. This work has been extended using DEM simulations by Cundall and Strack (1979), and more recently by Thornton (2000).

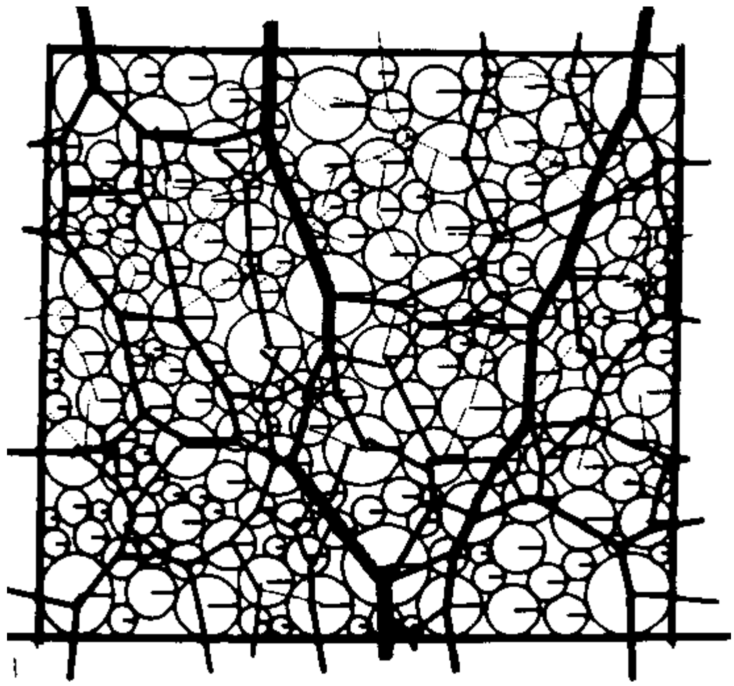


Figure 3 Stress visualised as a network of contact forces

4.2 Clastic mechanics

A fresh approach to soil mechanics has been developed recently, under the heading of "clastic mechanics": McDowell et al (1996), Bolton and McDowell (1997), McDowell and Bolton (1997), McDowell (1997), Robertson and Bolton (1997), Robertson, Bolton and McDowell (1997), McDowell and Bolton (1998), McDowell and Bolton (1999), Robertson (2000). Clastic grains are said to be those that fracture or crush before they deform plastically. Clastic mechanics therefore considers an aggregate of grains whose interaction mechanisms are:

- contact elasticity;
- particle sliding and rotating;
- particle crushing and fracture.

The novel element has been the use of the notion that particles break, creating smaller fragments which can fit into the existing voids, generating irrecoverable soil "strains" from a continuum perspective. If the soil element is compressed beyond the point at which grains crush, void-filling leads to plastic soil compression. Soil plasticity is intimately related to particle crushing.

4.3 Statistical models of particle fracture: GRANALOGY and CRUSH

These ideas were first explored using very idealised 2D numerical simulations of soil aggregates, embodied in two programs GRANALOGY and CRUMBLE, Robertson (2000). These simulations were based on an extension of Weibull's statistical theory for the fracture of brittle materials such as ceramics. At some instant, the probability of survival P_s of a (plane) grain of size d carrying a stress σ is said to be given by:

$$P_s = \exp \left[- (d / d_o)^2 (\sigma / \sigma_o)^m (C - 1)^{-a} \right] \qquad (6)$$

where d_o is some characteristic grain size for which σ_o is its characteristic crushing strength (for which 37% of a large sample of similar size break), C is the number of contacts, and exponents a and m (Weibull modulus) are modelling parameters. For simplicity, it was assumed that the stress σ applied to an element comes to act equally on every grain; this represents a time average of all the various possible contact force networks, such as those visualised in Figure 3, covering some small interval of strain.

Consider a soil of initially identical grains. Some grains crush, at random, as the effective stress is slowly increased. What happens next? The surprising conclusion of McDowell and Bolton was that while similar grains continue to crush as the stress continues to rise, so do the broken fragments of the first victims of splitting. A grain that splits is likely to keep on splitting. Crushing of the original grains continues, but as the survivors turn into islands floating in a sea of smaller fragments, they become relatively invulnerable.

If a grain is surrounded by a swarm of smaller supporters it will tend to be in a state of isotropic compression, as it would be if it were bathed in a high pressure liquid. There has to be a tension field if brittle materials are to fracture. In the absence of tensile interactions between grains, cracks will tend to grow in regions of a particle that are suffering high shear and low mean stress, so that tensile stresses are generated in some directions. An efficient nut-cracker consists of two hard platens converging on a nut, which causes the nut to split, typically opening up longitudinal cracks starting at a point on the equator. In order for the nut to split easily, it must be free to deform by extending its equator. So it is the small grains, trapped between two larger neighbours, which tend to split. The algorithm chosen to represent this propensity to split featured the coordination number C, in a function $(C - 1)^{-a}$. A particle with zero or 1 contact can have no chance of splitting; putting C = 2 gives the maximum chance, with the likelihood reducing as C increased thereafter.

Figure 3 suggests that the majority of large particles are incorporated in a strong chain of contact forces at every stage. However, it also implies that it will be the smallest particles trapped in the chain that will suffer the greatest internal stresses, since they have to carry the same forces as all their colleagues. Since the strong chains will shift around as particles crush or shift, every small particle will presumably have a high chance of splitting at some stage. So an alternative algorithm for the selection of particles for splitting might be based simply on relative size, such as $(d_o/d)^A$ where d_o is the original largest particle and d is the particle whose relative chance of splitting is being calculated.

Statistically, a coordination algorithm and a relative size algorithm would be bound to give similar results since high coordination is correlated with large relative size. Since the coordination algorithm offers the possibility of preventing the splitting of small particles with only one contact, and which can therefore not be under stress, it was preferred.

4.4 Void filling

Following grain splitting, or crushing, there must be a rearrangement of the particles. The extra degrees of freedom of the fragments mean they can pack better. In isotropic or one-dimensional compression, for example, we would expect the volume of the voids ultimately to reduce. In sequence, the fracture of a grain carrying a large effective contact force must lead instantly to the induction of an excess pore pressure in the associated void, to maintain equilibrium. If the soil is undrained, the excess pore pressure will be spread around and equalise, producing a small decrement in effective stress throughout the sample. This will cause a small amount of elastic swelling of those contacts not involved in the fracture, which has to compensate for the irrecoverable reduction of volume in pores which are closing and filling with fragments. If there is a drainage boundary, however, the fluid expelled from a collapsing void can ultimately escape from the sample.

In the process of void-filling, the size of fragments matters. Voids are usually much smaller than neighbouring grains, so split particles will usually create relatively large fragments which remain in effective contact with neighbours and carry effective stress almost at once. However, if fragments are smaller than neighbouring voids they may be temporarily "lost", and be unavailable to carry further effective stress until the void they reside in collapses. Lost fragments leave the soil temporarily weaker, and maybe unstable. This raises the question of whether particles split or crush. Figure 4 shows silica sand grains before and after being taken to a stress of 4 MPa in a triaxial test; Bowman (1999). This stress level is too small to induce particle splitting., but there has clearly been spalling at contact points, leading to the creation of very small fragments. Nakata (ibid) describes a sequence in which scratching of grains evolves to crushing or spalling at contacts and then to particle splitting. We should not expect all grain fracture phenomena to lead to the same macroscopic behaviour.

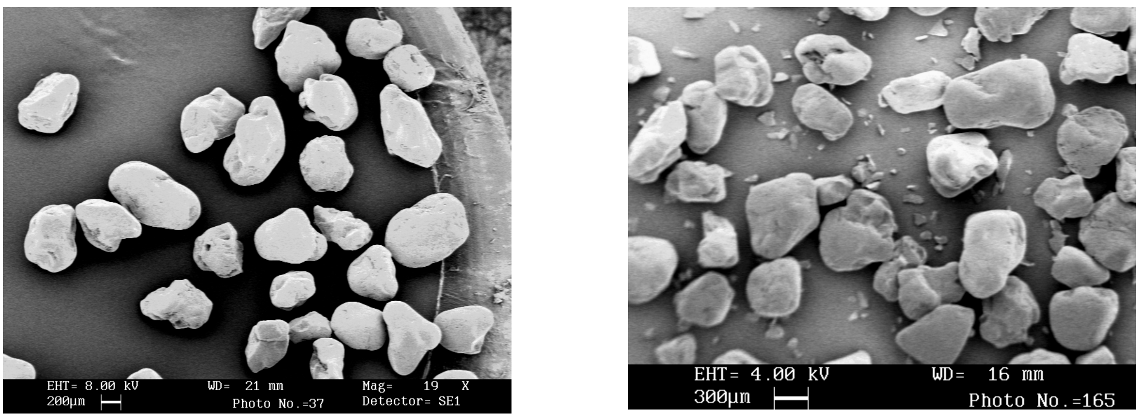


Fig 4 Damage to quartz sand at moderate stresses

4.5 Wiebull hardening

In the GRANALOGY and CRUMBLE simulations the particles were initially right isosceles triangles of identical size, and they were permitted simply to split into smaller but similar triangles. These smaller particles were taken to be stronger than their "parents", in stress terms, following equation 6 and the usual behaviour of brittle materials as described by Griffith. A Weibull modulus m in the vicinity of 5 is often found to fit the statistics of individual particle crushing on quartz sands. This was shown to ensure that the rate of increase of strength with reduction of particle size was sufficiently gentle not to influence the rule that small particles are more vulnerable. Wiebull hardening after particle splitting was, however, seen to be the cause of "plastic hardening" along the "normal consolidation line" through a process of fractal compression.

4.6 Fractal compression

Figure 5 shows the emergence of fractal structure in the two computational schemes. Figure 5a shows the GRANALOGY structure of neighbourhood relations after clastic compression. The figure is to be treated as a record of particle sizes and contacts, with the voids between particles being implicit. Figure 5b shows the attempt in CRUMBLE to respect voids explicitly; fragments are permitted to drop or slide into voids beneath them. In each case, a quasi-fractal structure emerged in which some large particles always survived by being surrounded by smaller grains which could support them, this occurring at every scale. Normal compression then becomes associated with the evolution of a fractal structure as previously split particles split again.

Every halving of the critical grain size of the smallest particles increases grain strength, and therefore applied stress. If there is a simple Wiebull relation, the increase in stress will be by the same factor for each generation. Every successive wave of splitting also reduces voids ratio. And if the same proportion of the sample is effected by splitting at the formation of each new generation of fragments, the voids ratio will also reduce by the same factor. This logic suggests that fractal compression should lead to a power curve for the "normal consolidation line":

$$(e / e_1) = (\sigma / \sigma_1)^L \tag{7}$$

where (e_1, σ_1) is any point on the line. This will give a fractal compression line which is linear on a log-log plot of voids ratio versus effective stress. This is not the form usually assumed, which plots as linear on a linear-log plot of voids ratio – see equation 3. However, it is the form selected by Pestana and Whittle (1995) to fit the high-stress data of many sands.

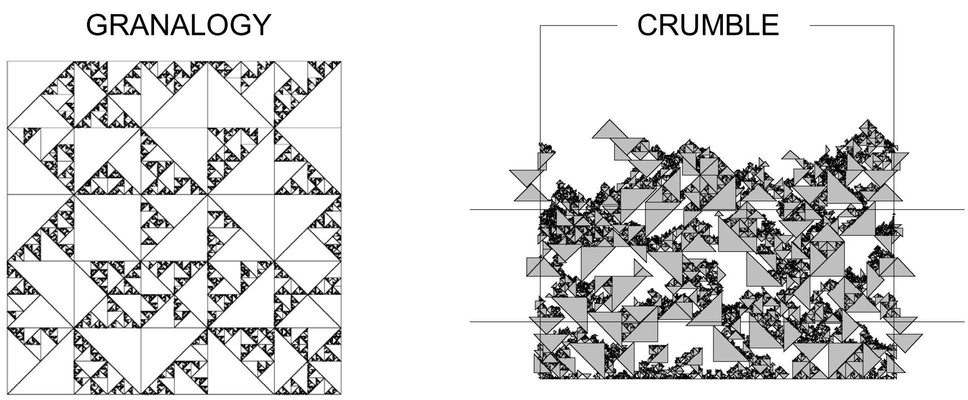


Figure 5 Fractal structures emerging from numerical simulations

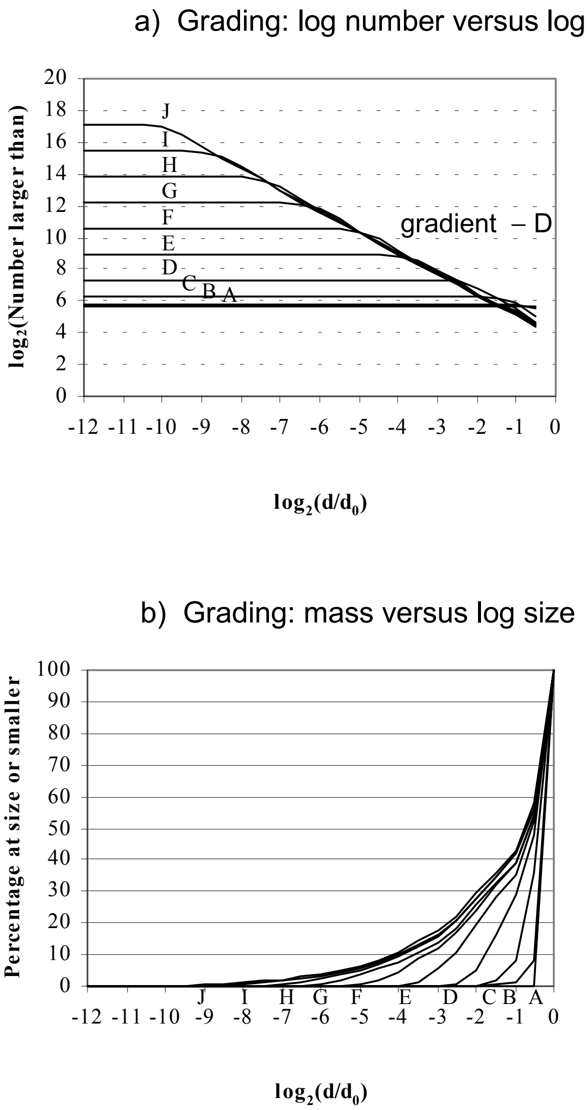


Figure 6 Grading curves after equal log increments of stress A to J in GRANALOGY

Figure 6 shows how the size distribution of particles evolves into a limited fractal. At first (A to B) there is little crushing induced by stress-doubling. Then a consistent trend develops in which a fractal dimension D can be used to describe the development of new generations of particles produced after each doubling of stress (B to J). This is seen in Figure 6a as a straight line of slope –D on the plot of log $N_{L>d}$ versus log d, lying between limits of initial d_o and current d_{min}.

McDowell and Bolton (1998) used a Cam Clay work equation modified to include surface energy to derive an expression for λ (see equation 3) in the form:

$$\lambda = \text{(friction factor) x (shape factor) x (toughness factor)} \quad (8)$$

where each factor is properly dimensionless. This was based on the proposition that there is a unique probability applicable to the fracture of the finest current particle in a soil sample subject to ongoing compression. The outcome was consistent with the results of GRANALOGY simulations. In comparing (3) and (8) with (7), note that different, perhaps equally reasonable, assumptions lead to subtly different theoretical forms of the compression curve.

4.7 Over-consolidation cycle

McDowell and Bolton (1999) also used the size dispersion evident in the foregoing analysis of clastic compression to create typical hysteretic soil behaviour. Adjacent load paths, e.g. passing through a single, solid, large particle, and passing through a surrounding matrix of many fine particles, have quite different stiffnesses. This provokes relative slippage and variations in internal contact force distributions, even due to isotropic unloading and reloading inside the clastic yield surface. The phenomena of hysteresis (kinematic hardening), and of compaction due load cycling, are introduced as a consequence of clastic hardening.

4.8 Clays

Since evidence of the evolving microstructure of clays is rather harder to acquire than evidence of the crushing of sands, it is natural to ask whether the ideas of clastic mechanics can be extended to include them. Figure 7 shows a micrograph of Mexico City clay, taken from Mesri et al (1975).

Figure 7 Biogenic fragments in Mexico City clay

It is, at least, tempting to ascribe the notorious compressibility of this material to the crushing of the prominent silt-sized "shell" fragments. However, as with other clays, it is possible to reconstitute a dispersed clay slurry, consolidate the clay, then macerate it, remix with water and dispersing agent, and reconsolidate with similar results as before. This implies that agglomerates of clay platelets may form distinct crushable grains which can participate in clastic compression, and then be capable of being dispersed and re-aggregated once again. It might be presumed that each aggregate grain is held together by attractive inter-platelet forces, the supposed origin of "true cohesion" in clays. In this model of clay "true cohesion" resides inside skeletal aggregates, and acts in lieu of the rather stronger inter-atomic forces holding a sand grain together. The definition of the boundary of a clay agglomerate is based, rather like the boundary of a tectonic plate, on a history of relative sliding creating a zone of disorder and weak bonding.

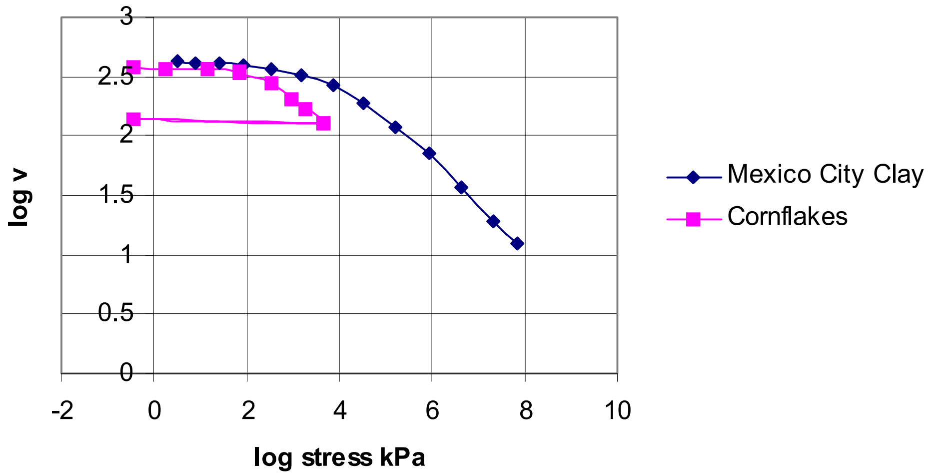


Figure 8 The comparative behaviour of clay and cornflakes

To reinforce the case that clastic mechanics applies over a wider range of materials, consider the data of one-dimensional compression for Mexico City clay and for Kellogs Cornflakes, plotted on natural logarithmic axes in Figure 8. Coincidentally, the voids ratios of loosely poured cornflakes and the freshly sedimented clay are very similar. The implied crushing strength of the clay grains is about four times greater than the cornflakes but the plastic compression index of the two materials is, again, very similar. There is no doubt at all about the clastic origins of the plastic compression of cornflakes. Every fracture is clearly audible, and (in a transparent plastic cylinder) visible. It was also verified that a fractal distribution of grain sizes was obtained, creating a generous amount of multiply-fractured fragments familiar to anyone who pours out the last portion of this breakfast cereal. Of course, cornflakes can not be reconstituted.

It seems to be reasonable for the purposes of creating hypothetical constitutive models to treat clays as clastic materials comprising crushable aggregates of clay platelets, and to apply the principles of clastic mechanics to their behaviour.

4.9 Discussion

The clastic mechanics initiative produced some apparently successful micro-mechanical models to provide a commentary on compression behaviour: see Figure 9.

The missing parameter X in equation 4 has been proposed to be the crushing strength σ_o of a representative grain of diameter d_o, at least in terms of irrecoverable compression. But self-similarity on the fractal (normal) compression line excludes the introduction of σ_o into the constitutive equation of compression, since the representative grain is now understood to be reducing in size as stress increases. This leaves much of the conventional interpretation of soil behaviour in tact while suggesting that plots of log e versus log σ might turn out to be at least as viable as e or v versus log σ. But it also opens up many avenues for research aimed at an integrated understanding of phenomena. Some of these are mentioned below.

i) The magnitude of irrecoverable strain in the first stage of virgin compression, AB, is greater than in subsequent reloading over the same stress interval, FG. This arises from the probability that the grains at A have more diverse crushing strengths than those at D which have been conditioned by the process of clastic compression CD. This truth is independent of any supposed disturbance in the sampling of soil. If nothing is known about the loading history of the soil at A it is incorrect to view turning point C as necessarily indicating either diagenesis or pre-load: consider the cornflakes.

ii) Anisotropy following clastic compression can be understood as the simple consequence of the preferential crushing of grain contacts transmitting vertical stress compared with those carrying the smaller lateral stress. Lateral loading then gets a softer response.

iii) An understanding of the reduction of grain size with increasing stress leads to the possibility of including permeability changes due to fractal compression.

iv) Cycles of loading such as DEFGH can create extra plastic compression due to the overloading of fine particles which lie close to large particles. A similar mechanism may apply to liquefaction following cyclic shearing.

v) Secondary consolidation or creep, such as may occur at point I in Figure 7, can be imagined as the delayed fracture of grains. In primary consolidation many grains are fracturing, causing neighbouring spurts of excess pore pressure as voids collapse. These spurts merge to maintain excess pore pressures and create outward hydraulic gradients which permit fluid to escape at the boundaries. In secondary consolidation occasional grains fracture after a period of time during which micro-cracks extend, perhaps due to hydrolysis at crack tips. Individual spurts of excess pore pressure dissipate into the surrounding matrix of elastic contacts. As fragments fall into the collapsing void there is sufficient room for local elastic relaxation, so the process of secondary consolidation is independent of external drainage. The puzzle of void migration at zero effective stress, even while the soil macroscopically is carrying large stresses, is perhaps explained by the extremely heterogeneous distribution of contact forces shown in Figure 3.

vi) Ageing may relate to the reorganisation of loose clay platelets which are initially "washing around" the macro-pores. If their random motions lead to them eventually to aggregate more strongly, the soil matrix will become stiff and brittle, while the macro-voids are effectively enlarged so that permeability also increases.

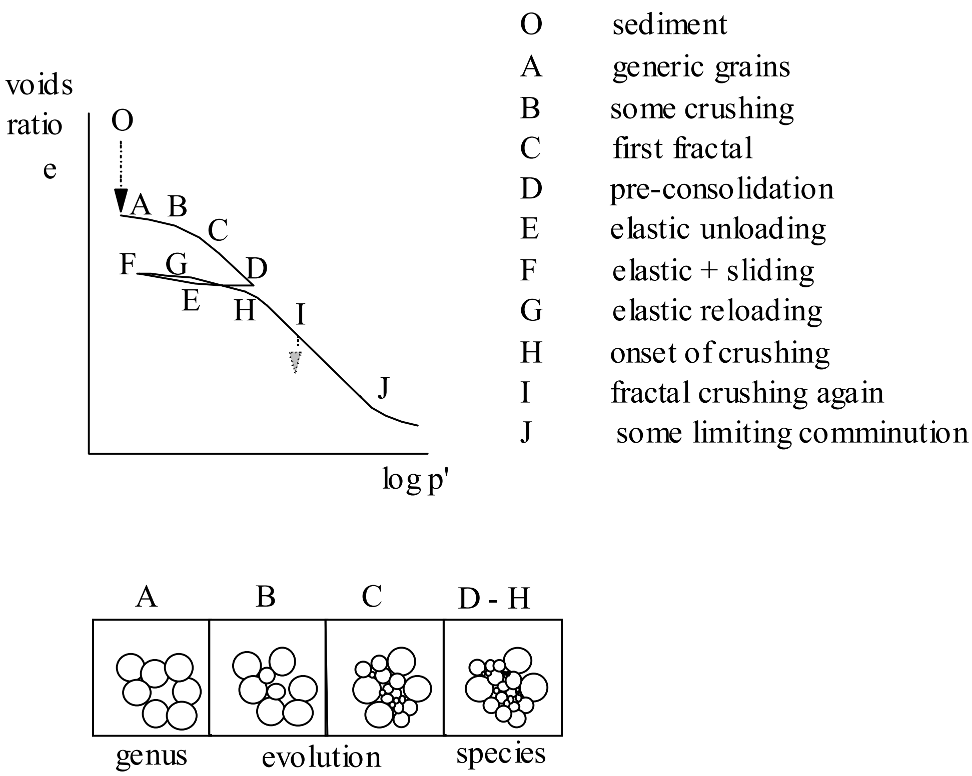


Figure 9 A clastic mechanics interpretation of soil compression

5 Techniques for validation

5.1 Grain crushing tests

It is necessary to test the crushing and fracture strengths of soil grains if we are to understand the micro-mechanics of granular interactions. Sufficient grains of each significant size should be tested to make a reliable statistical model: Lee (1992), see Figure 10. Grains can be tested in compression between parallel platens and their tensile strength related to their crushing strength: McDowell and Bolton (1998). Experience shows that local crushing and spalling usually occur before grain splitting. Since there are size effects it would be useful to identify not only the load at which there is a fracture event, but also the size of the fragment. The initial grain can be identified by its mass. On a plot of applied force versus displacement it will be possible to recognise local fracture events as well as grain splitting. These events could all be recorded on video, and the fragments removed for weighing, so that the size of each fracture event can be logged. This should continue at least until the grain has lost half its mass. Nakata et al (1999) and McDowell (ibid) describe recent studies in which grain damage is carefully assessed.

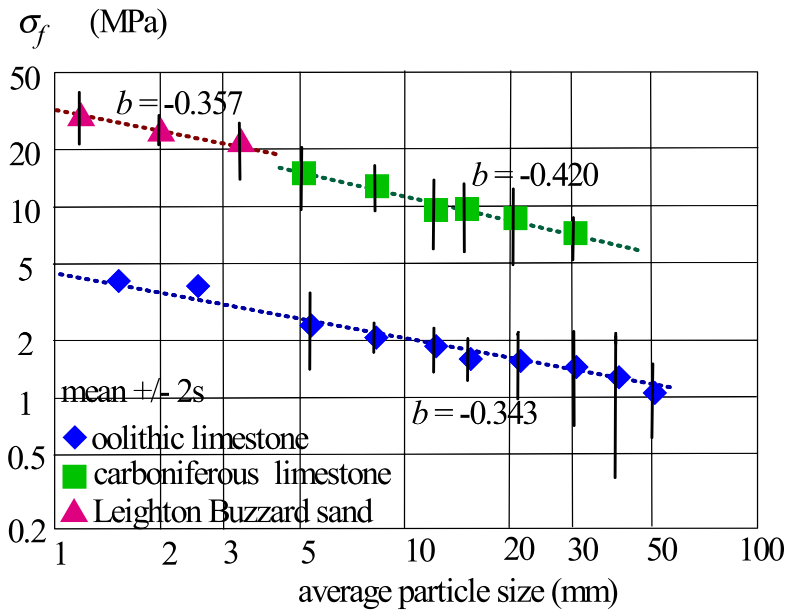


Figure 10 Size effect on statistics of grain crushing, after Lee (1992)

5.2 Distinct Element Modelling

Although DEM is now well-established, the propensity of grains to crush or split has not been considered until recently. Robertson (2000), working with the author, has used the PFC3D program (Itasca Computing) to simulate crushable grains by forming regular agglomerates of elementary spheres, and bonding them. Figure 11 shows the computer simulation of the crushing of such an agglomerate. Figure 12 shows the plan view, in which it can be seen that the grain initially split into two on a “vertical” diametral plane, and then split again when the applied force came to bear on the right-hand hemisphere. Figure 13 shows the statistics of grain strength achieved when agglomerates are purposely flawed through the random omission of 5% to 25% of the elementary spheres. The strength distributions are approximately consistent with Wiebull distributions, with the modulus m decreasing from 5 to 2.5 as the percentage of flaws increases. This is all entirely characteristic of real grain crushing behaviour.

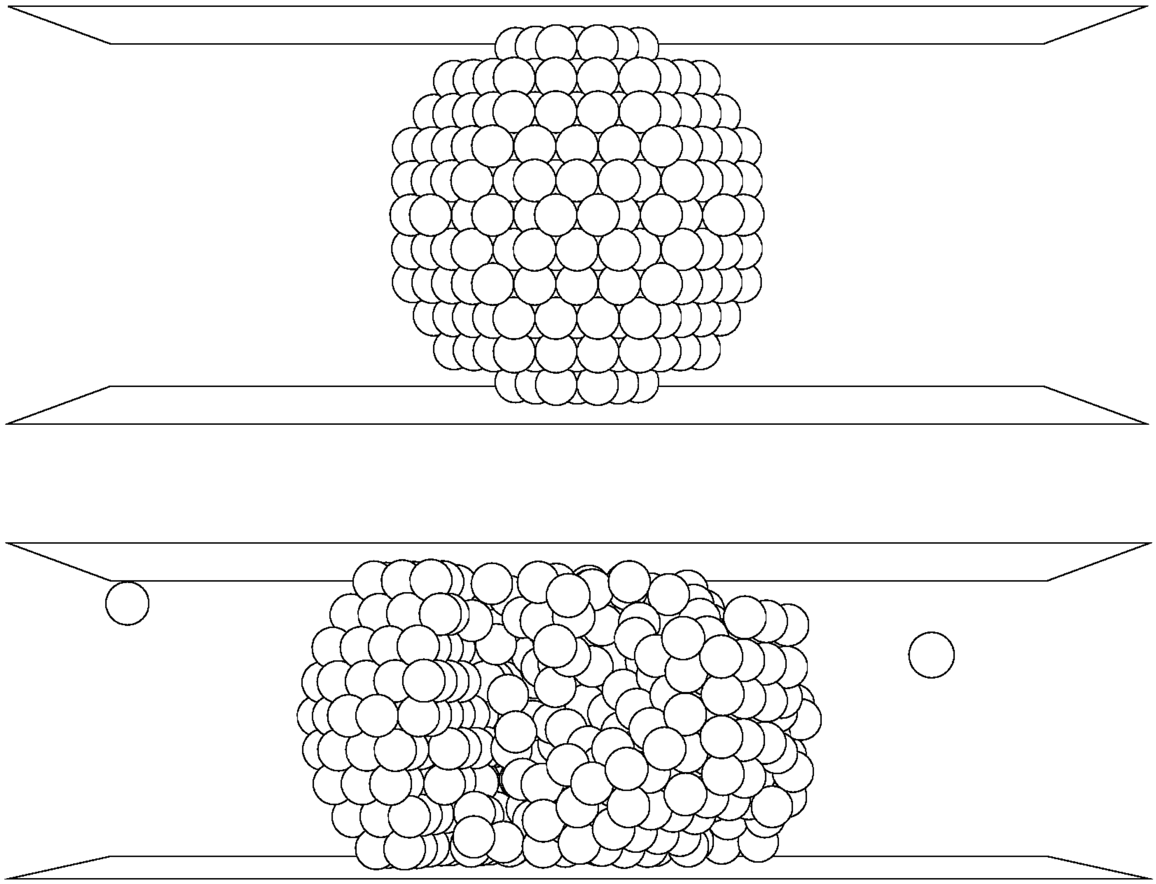


Figure 11 Agglomerates modelling crushable grains, after Robertson (2000)

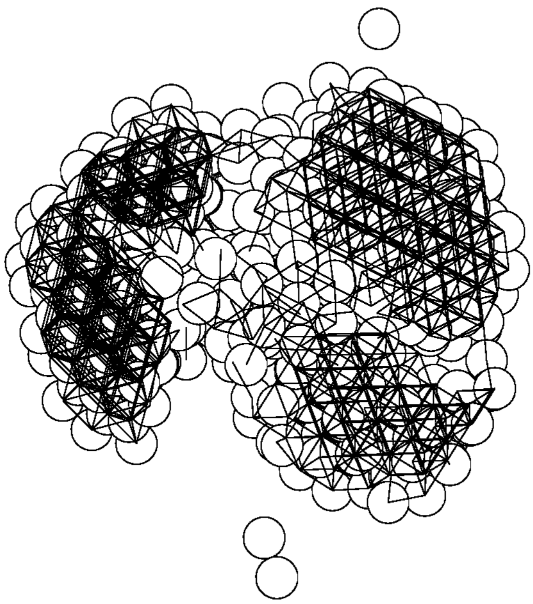


Figure 12 Plan view of crushed grain

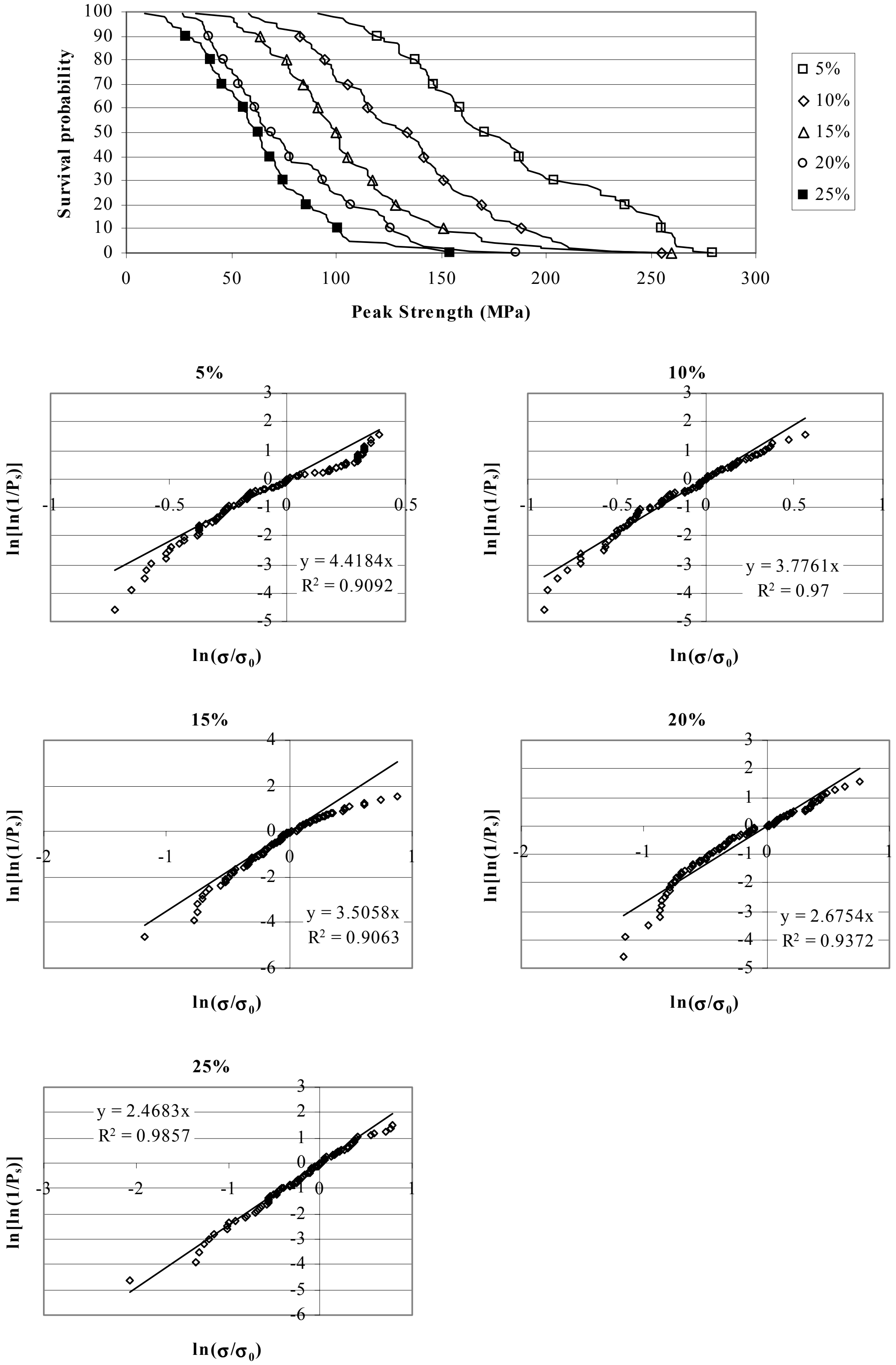


Figure 13 Strength distribution of flawed agglomerates roughly satisfy Wiebull statistics

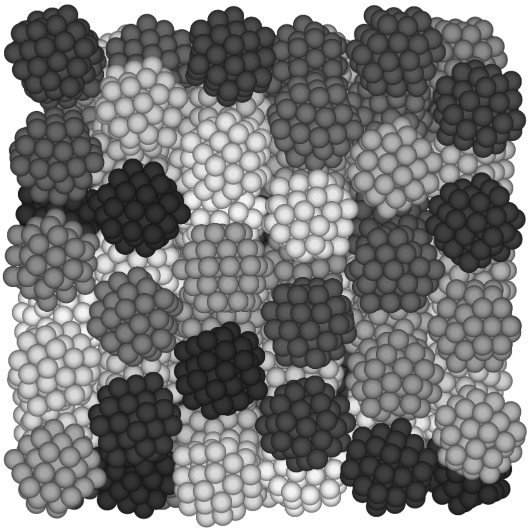
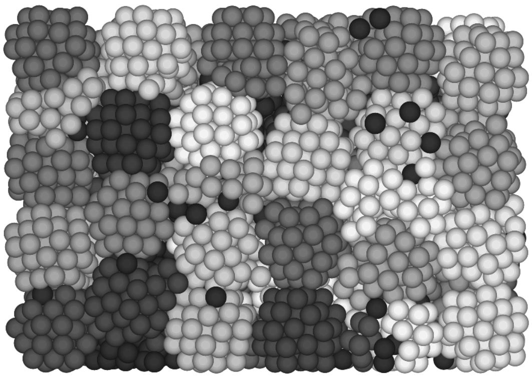


Figure 14 Numerical simulation of an aggregate of crushable grains



Cluster Plots

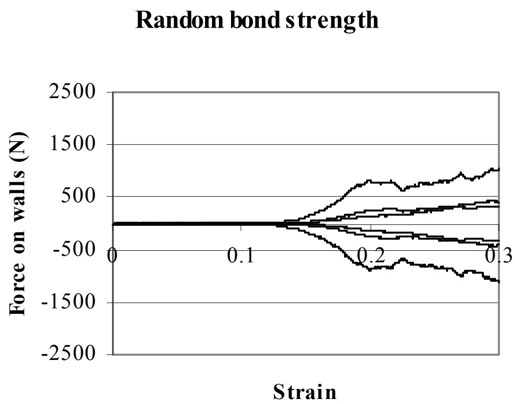


Figure 15 Simulation of the compression of a crushing soil

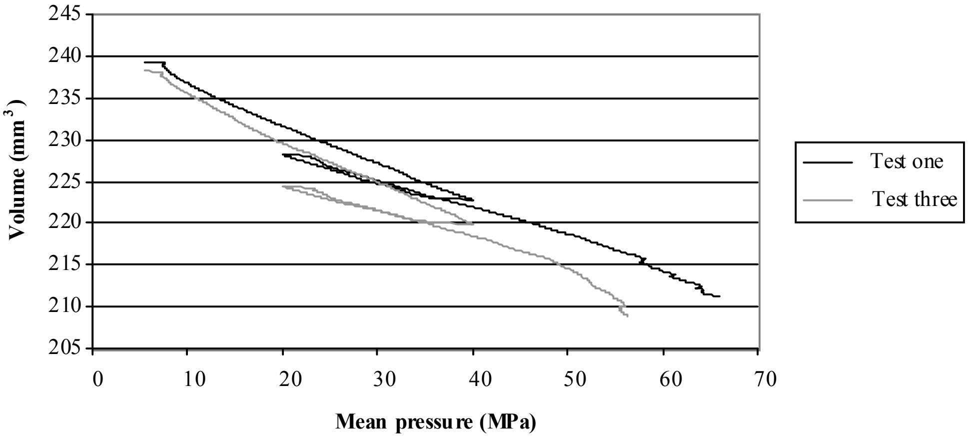


Figure 16 “Elasto-plastic” compression simulated with crushable grains

Figure 14 shows a typical soil aggregate made of Robertson's agglomerates. Figure 15 shows data of the vertical and horizontal stresses, deduced from contact forces on the "walls" of the test space, increasing initially elastically with evidence of contact hardening. Then the stiffness drops, as grains begin to crush, and the ratio of vertical to horizontal stress in this one-dimensional compression simulation rises to $K_o \approx 0.5$. Figure 16 shows the associated compression curve which displays the curious topographical features seen in Figure 2. The author deduces that grain crushing does indeed explain the complex behaviour of soils in compression.

5.3 Granulometry and porosimetry

Various arbitrary definitions of the degree of crushing exist. These usually relate to the increase of fines, either as defined by d_{10} or by the area swept on a standard grading plot, but the practicalities have in the past dictated that fines smaller than 0.06 mm have been ignored. A much preferable alternative is to use the specific surface area as the parameter describing the degree of crushing. In chemical engineering this is defined as total surface area per unit mass (e.g. $m^2\ g^{-1}$), but for the purposes of soil mechanics it would be more usefully defined in a fashion similar to specific volume, as

$$a_s = \text{total surface area of a unit volume of grains} \qquad (9)$$

The advantages are that every particle contributes to the assessment (in a weighted fashion), and that the dimensions (m^{-1}) are more intuitively linked to a characteristic particle size and shape. Modern laser diffraction techniques permit granulometry over particle size ranges from about 10^{-5} mm to 1 mm. This very wide range of sizes certainly facilitates a more accurate determination according to equation 9, but necessitating an arbitrary assumption of grain shape, such as that all grains are spherical. Figure 17 shows the evolution of the granulometry of a calcareous sand, obtained by Joer (1999) – see also Joer (ibid). The initial granulometry extends upwards to the left with a gradient (fractal dimension) of about 2.2. Non of these fines would have been included at all in a conventional analysis of crushing.

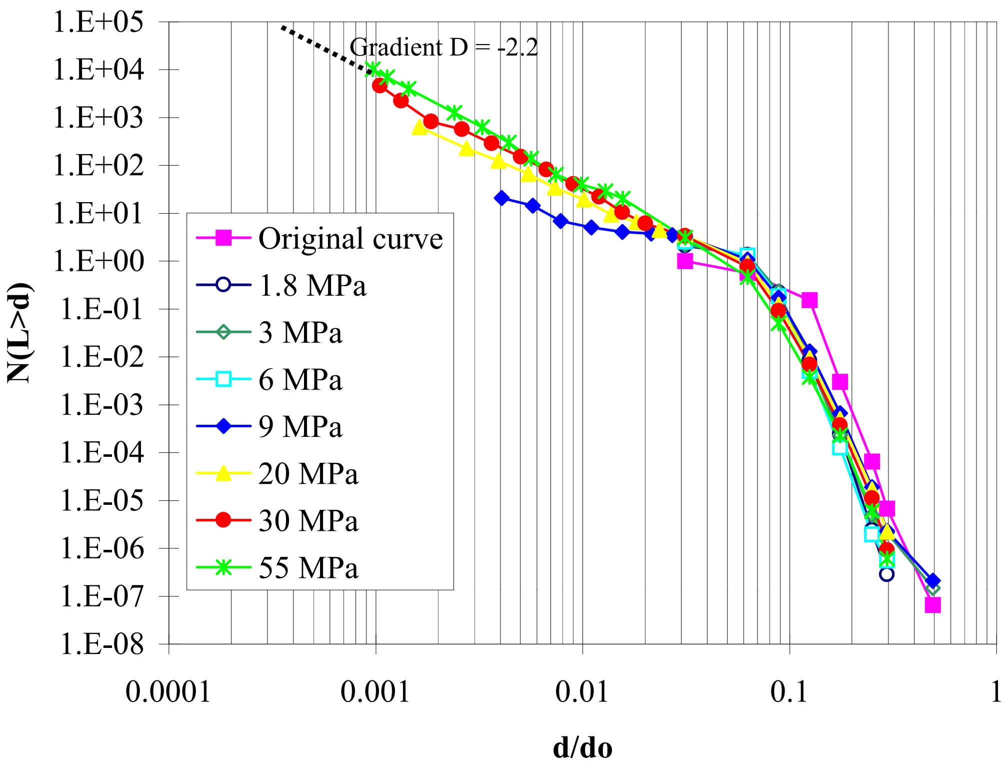


Figure 17 Laser particle size analysis of one-dimesional crushing, after Joer (1999)

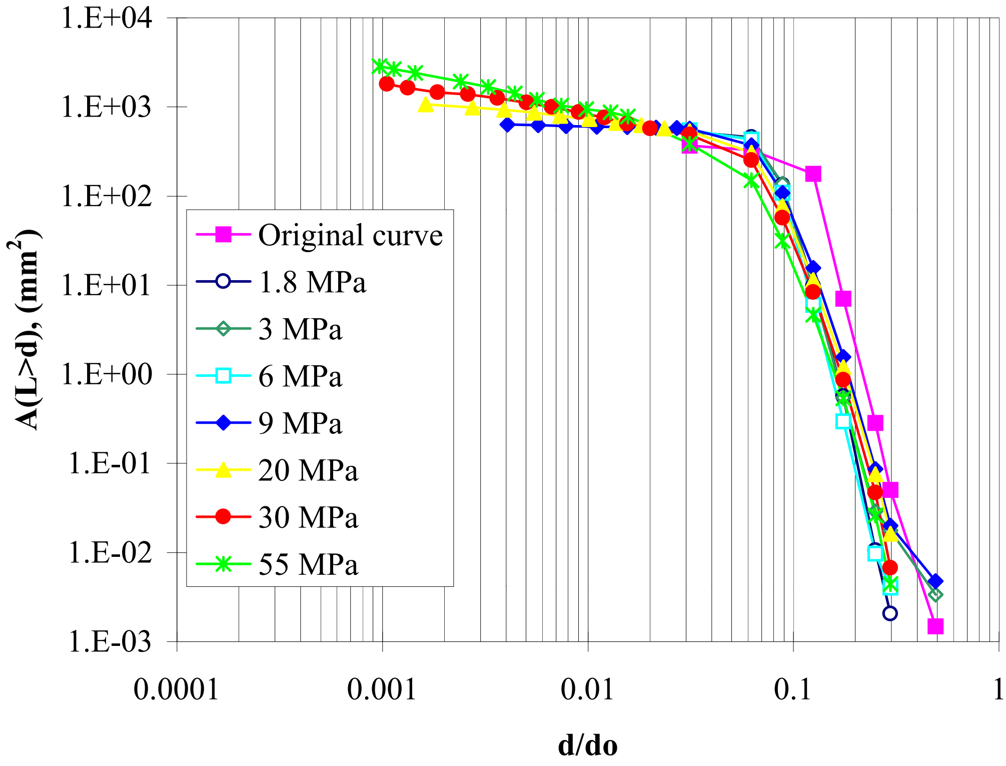


Fig 18 Cumulative surface area evolution

Figure 18 shows the same data expressed as cumulative surface area. It is evident that the fines detectable with the laser device have contributed to approximately a ten-fold increase in specific area due to crushing at a stress of 55 MPa.

Direct test methods for a_s are, however, more promising since all fresh surface due to micro-cracks, spalling or splitting will be recorded automatically. These tests are based on placing a powder sample in a near-perfect vacuum and then deducing the mass of gas sorption sufficient to coat all surfaces with a mono-molecular layer of the gas – nitrogen or krypton for example – permitting surface area to be inferred from a knowledge of molecular size.

We also need, as well as particle sizes, to characterise void sizes – such as by mercury porosimetry in which measurements are made of the increasing pressure required to overcome the surface tension of mercury as it successively invades the finer and finer pores of a dry porous material. Only if we know the sizes of grain fragments and pores will be able to understand the void-filling aspects of soil crushing. Figure 19, from Joer (1999), shows the one-dimensional compression of glass ballotini. When each glass ball fractures it disintegrates into hundreds of tiny fragments, due to the internal stresses locked in when the balls solidify. Since all balls have almost identical strengths, the compression process is quite unstable at 10 MPa as the contact forces from breaking balls try to find alternative load paths through chains of balls which themselves are on the verge of fracture. Also, since the fragments are "lost" in the much larger voids between unbroken neighbours, there can be no clastic hardening until sufficient breakage has occurred to fill these initial voids. Only then can a fractal (normal) compression line emerge, as seen in Figure 19.

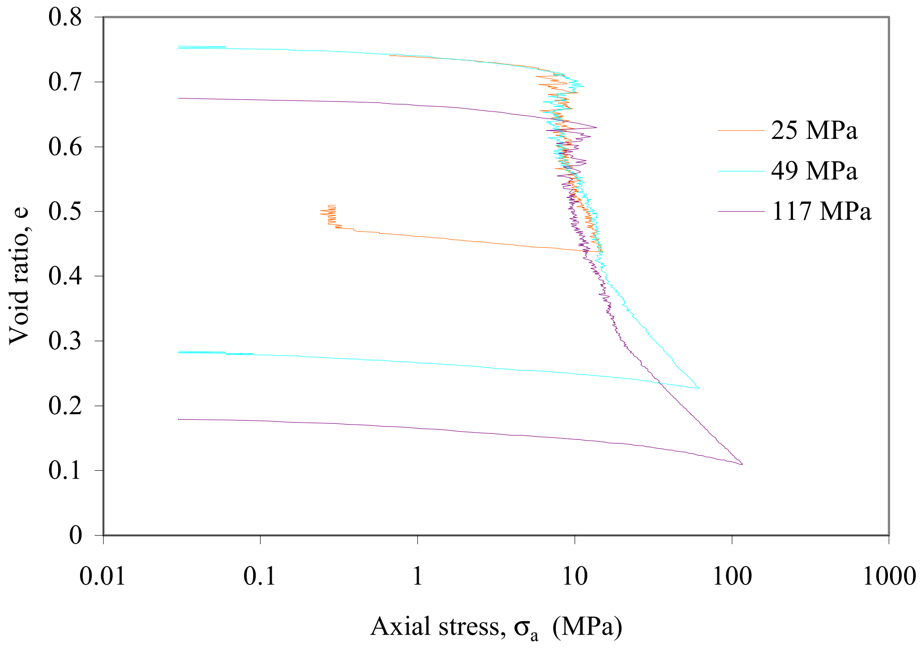


Figure 19 One-dimensional compression of glass ballotini

5.4 Microscopy

While granulometry and porosimetry will certainly enhance our understanding of the evolution of microstructure, they suffer three distinct disadvantages:

i) they are not so meaningful for clays,
ii) they tend to result in the destruction of the sample, and
iii) they can not generally be used for continuous monitoring.

We may seek to overcome these drawbacks by developing direct imaging of the microstructure of soils under test, such as by digital photography or microscopy.

Scanning Electron Microscopy (SEM) has made a strong contribution to the understanding of soil microstructure. However, in standard SEM techniques the free surface has to be freeze-dried and coated with gold to conduct away the electrons. Only the recently developed ESEM (Environmental SEM) techniques, enabling microscopy with about 1 Pa of water vapour in the test chamber, can avoid specimen pre-treatment. Even so, all SEM techniques operate only on a free surface. It is not possible to view the microstructure of soils under load.

Optical microscopy can resolve many features of interest, of course, the main problem to be overcome again is the influence of the transparent window on the behaviour of the soil microstructure adjacent to it. Much more development work is required.

6 Conclusions

Continuum soil mechanics is insufficient. We need to include the statistics of the strength and size of constituent grains in DEM simulations of soil behaviour, in order to improve our understanding and establish some linkage between continuum parameters derived by curve-fitting. We also need to monitor the evolution of microstructure during soil tests, and especially the increase in specific surface or the reduction of permeability either of which would indicate crushing and void filling. It is proposed that brittle fracture of grains or asperities is the essential precursor to the grain rearrangement that we currently describe as “soil plasticity”. A great deal of further effort is required before micro-geomechanics can make its hoped-for contribution – to simplify and rationalise the mechanics of soils and rocks.

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