

**Complementary Shear Stresses
in Dynamic Centrifuge Modelling**

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Complementary Shear Stresses in Dynamic Centrifuge Modelling

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Abstract:

The semi-infinite extent of the natural soil medium needs to be simulated accurately while attempting to model stress wave propagation or dynamic soil-structure interaction problems on a centrifuge. In this internal report results from two centrifuge tests conducted to study the effects of complementary shear will be discussed. The normal stress at the base of a horizontal sand bed and the shear stress generated at the base of the centrifuge model were generated using Stroud cells (Stroud, 1971). The **complementary** shear stress along the vertical face of the duxseal boundary was measured by means of a thin duraluminium sheet coated with sand particles and which was fitted with strain gauge ϵ at three different levels.

It was shown that it is indeed possible to measure the normal and shear stresses at the base of dynamic centrifuge model using standard Stroud cells. A simple De Alembert's analyses suggested that about two-thirds of the shear stress is transmitted into the model and remaining shear stresses would result in a pressure wave generation along the vertical face of the boundary. The complementary shear stress measured by the strain gauged dural sheets agreed satisfactorily with the observed base shear stress magnitudes. The present study lists the specific centrifuge model tests which need to be conducted in future.

1 Introduction

Centrifuge modelling of earthquake shaking of horizontal ground requires care with boundaries of specimens. Natural soil strata extend to large distances in the lateral directions Fig. 1a. Under earth's gravity a uniform horizontal layer of soil exerts a uniform vertical pressure on a horizontal rock surface. When the rock shakes in an earthquake, Fig-1b, there will be a

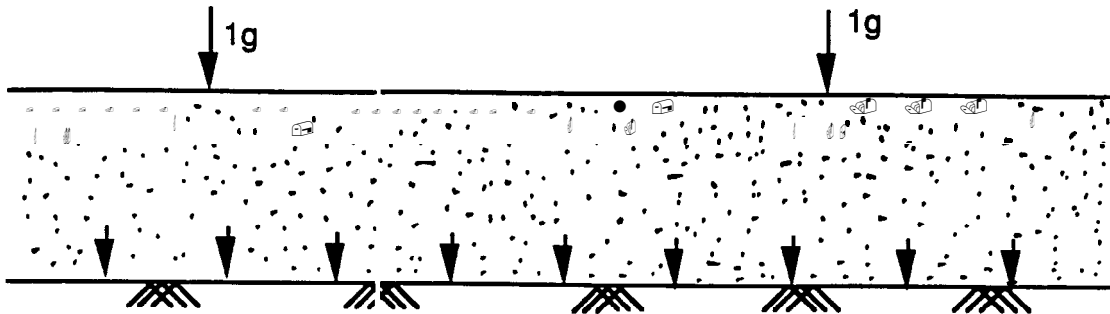


Fig.1a Horizontal sand layer overlying rigid bed rock

vertically propagating horizontal shear wave in the ground. If the ground is in effect a very stiff elastic layer the wave will propagate very quickly and there will be a linear variation of horizontal shear stress with depth. There will also be complementary shear stresses on all vertical

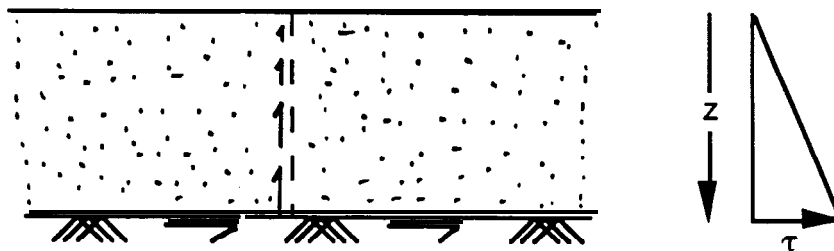


Fig.1 b Shear stresses during an earthquake

planes and those stresses will **decrease** uniformly with depth 'z' in the layer. In a model test it is important to have correct complementary shear stresses on end walls of a specimen Fig.1c.

end walls which are distance 'l' apart. If the walls were **smooth** and there were no **complementary** shear stresses these "overturning moments" could be generated by variation of vertical pressures on the container **base, Fig. 1f**; if the base were smooth and there were no shear stresses on the base there could be accelerations of the specimen due to difference between lateral pressures at either end wall of the container, Fig. 1g. However these would be incorrect model conditions, since in the field there is no reason for the vertical and lateral pressures at any point in Fig. 1a to vary in a positive or negative direction at any time during vertical propagation of a horizontal shear wave.

A technique of absorbing 'dulseal' end boundaries between the rigid end walls of a container and the specimen was developed at Princeton by Coe et al (1983) and has been used extensively in modelling **boundary** value problems at Cambridge University, Fig. 1h. The performance of dulseal boundary in absorbing the stress waves under lg conditions was studied by **Steedman** and Madabhushi (1991). It was observed that at least 65% of the incident stress

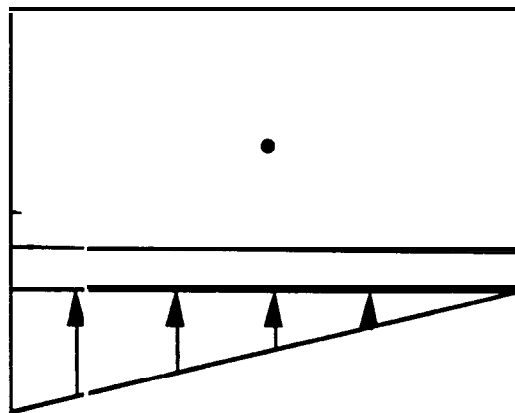


Fig. 1f Vertical pressure distribution

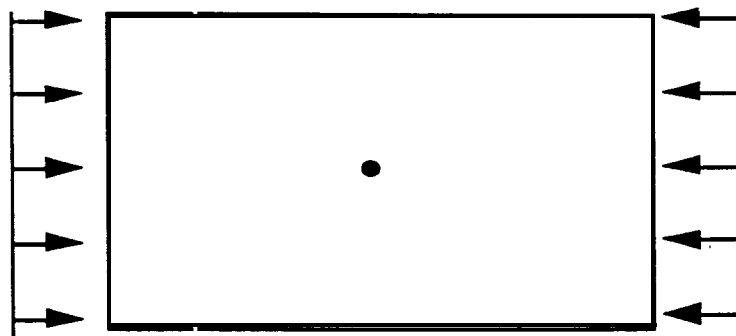


Fig. 1g Pressure on end walls

In addition load cells **were** installed in the container base to measure variation of shear force and normal force on the **base**. The experiments were of a preliminary nature. It is proposed to continue them with further experiments in which there are load cells on the rigid end walls of the container and in which there are more accurate complementary shear stress measurements in the rough dural sheets. Additional costs will be incurred in these further tests and this is simply a preliminary internal report for the purpose of generating such funds.

In this report results from two centrifuge tests BET- 1 and BET-2 (Boundary Effects Test) will be discussed. In these centrifuge tests the boundaries of the model constituted of a duxseal layer with a thin dural sheet with **sand** glued on it as shown in Fig.1.lc. The configuration of the tests and the instrumentation used will be discussed in the next section. The results from the tests will be presented in section 3. A simple De Alembert's analysis to estimate the shear stress transmitted into the model will be presented in section 4. Measurement of complementary shear using the strain gauged dural sheets will be discussed in section 5. The phase relationship of the normal stress-time histories will be considered in section 6. The conclusions from this investigation and the future work **that** must be carried out will be presented in section 7.

2 Centrifuge Model Tests

For both the centrifuge tests BET-1 and BET-2, the model configuration consisted of a horizontal sand bed. Two rectangular blocks moulded from duxseal were placed on either side of the model (see Fig. 1. lc). In between the vertical sides of duxseal layer and the sand layer two thin dural sheets were placed. The face of the dural sheet adjacent to the sand layer was glued with a thin layer of sand.

2.1 Configuration

A schematic section of the model for these tests is shown in Fig.2.1. Several accelerometers were placed within the model to measure the acceleration-time histories at different locations. The positioning of the accelerometers is shown in Fig.2.1. Three **Stroud** cells which can measure both normal and **shear** stresses were placed at the base of the model as shown in the figure. The stress cells are **designated** as PPT 101,102 and 103. Each of the vertical sheets of dural on either side of the model **was** strain gauged at three levels as indicated in the figure.

2.2 Instrumentation

2.2.1 Accelerometers:

Miniature piezo electric accelerometers manufactured by **D.J.Birchall**, **A23** type where used. These devices can measure the acceleration with an accuracy of $\pm 5\%$ within a range of 20 Hz to 2 kHz.

Calibration :

The accelerometers need to be calibrated carefully. The procedure adopted for these tests was as follows. The **accelerometers** was mounted on a calibrator which imparts a standard $\pm 1g$ acceleration. The output of the **accelerometer** was observed using an oscilloscope and the peak to peak voltage generated was measured. Half of this value was taken as the calibration constant in the units of 'Volts/g'. The procedure was repeated before and after each test for all the accelerometers used.

2.2.2 Stroud cells:

The stroud cells were originally designed to measure the normal stress, shear stress and the moment loading simultaneously in a direct shear box test, **Stroud** (1971). In this test series an attempt was made to use these **stress** cells to measure the temporal variation of the normal and shear stresses under dynamic loading, for the first time. The stress cells work on the principle of subjecting four very thin elastic webs of the cell to direct loads so that they suffer measurable linear strains. The strain gauges glued onto these webs will detect a change in the electrical resistance on straining, which is **measured** using a simple Wheatstone bridge circuit. A schematic diagram showing the construction of the cell is shown in Fig.2.2. A detailed description of the stress cells was presented by Bransby (1973).

Range:

The range of these stress **cells** was limited to 0-360 **kPa** for both normal and shear stresses due to the dynamic loading. It was expected that a peak normal stress of 120 **kPa** will be generated at the base of the model at 80 g, during both these tests. Further, the maximum shear load expected under a 50% earthquake is only 30 **kPa** during these tests.

Calibration:

The principles of design and strain gauge **circuitry** were explained in detail by Bransby (1973). The calibration of the **Stroud** cells was carried out following the standard **procedure described** by Bransby. The calibration matrix for each of the transducers containing 9 elements was determined by applying standard loads in each of the load directions (normal, shear and moment

The schematic section of the model is shown in Fig.1. 1c. The required strong box was thoroughly cleaned. The three stress cells were fixed rigidly to a small channel section as shown

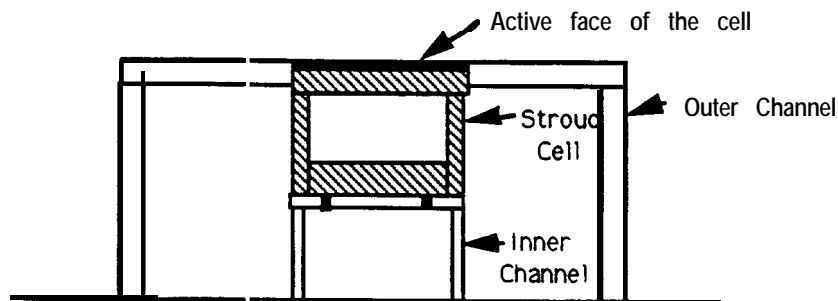


Fig.2.4 Section showing the mounting of the Stroud Cell

in Fig.2.4. This channel was to fix the stress cells relative to the ~~base relative to the base~~ and allow for the passage of output leads to the junction box. A second larger channel was used to protect the body of the stress cell from sand as shown in Fig.2.4. Three square holes of dimensions 33 X 33 mm were machined in the outer channel to allow the active face of the transducers to be flush with the base of the channel.

The sand used in these two centrifuge tests was Leighton Buzzard sand 52/100. This is a medium dense sand with rounded and sub rounded particles and was extensively used in the previous centrifuge tests by many research workers. This sand was placed in a hopper suspended from a gantry and was rained at a predetermined rate into the strong box to achieve the required density. The hopper can be moved along the gantry to achieve a uniform layers of sand. When the sand level was flush with the base of the channel the duxseal blocks were placed on the ends of the model and the shear sheets were placed carefully. When the sand level reached the required level to place the accelerometers the pouring of sand was stopped and the sand surface was vacuum levelled. The accelerometers were carefully placed at the required positions and pouring of sand was continued. On reaching the final level of sand layer (100 mm) the surface was vacuum levelled and the model was loaded on to the blue end of 10 m diameter beam centrifuge.

The centrifuge was started and the model was accelerated to 80 g in steps of 20 g. Four earthquakes were fired using the bumpy road actuator (Kutter,1982) and the strength of the

earthquake in successive earthquakes was increased-The soil parameters for the two centrifuge tests are tabulated below.

Table 1 Soil Parameters

Test Id.	Model	Void ratio	Bulk Density Kg/m ³	Relative Density %
BET-1	Dry	0.78	1486.9	50.1
BET-2	Dry	0.70	1556.0	69.6

3 Presentation of Results

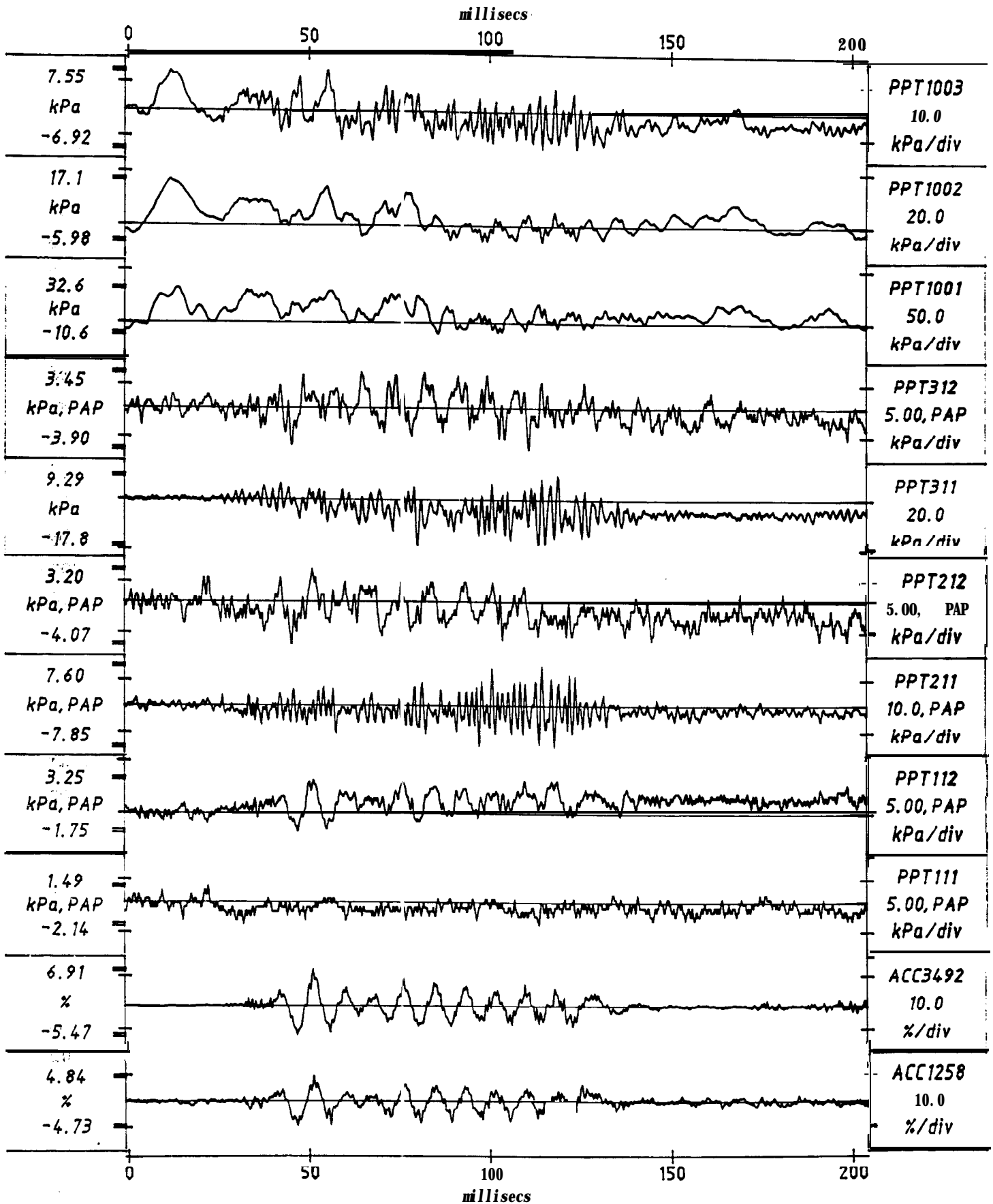
Two centrifuge tests were conducted with the soil parameters shown in Table 1. During each test the output from the transducers was recorded on a 14 channel Racal tape recorder. The analog signals were digitised using FLY 14 suite of programs, Dean and Edgecombe (1988).

3.1 Test BET-1

The schematic section through the model for test BET-1 is shown in Fig.3.1 together with the placement of the transducers. The vertical stress cells are designated as PPT 111,211 and 311 and shear stress cells are designated as PPT 112,212 and 312. Similarly, the shear stress cells on the left hand side sheet are designated as PPT 1001,1002 and 1003 at the base, middle and top of the sand layer respectively. On the right hand side sheet they are referred to as PPT 2001,2002 and 2003 respectively.

The acceleration-time histories observed at various accelerometer positions within the model are presented in Fig.3.2 to 3.10. The strength of the earthquakes from 4.78 % in first earthquake to 23.6 % in the fourth earthquake. In Fig.3.6 and 3.8, the shear stress observed at the base of the model (PPT 112,212 and 312) in all the stress cells varied as the input base motion given by ACC 3492. The stress cells on either side of the model recorded a net increase in the vertical pressure after the end of the earthquake both in earthquakes 3 and 4 (see Fig.3.6 and 3.8).

1024 data points per transducer, plotted after 2 smoothing passes



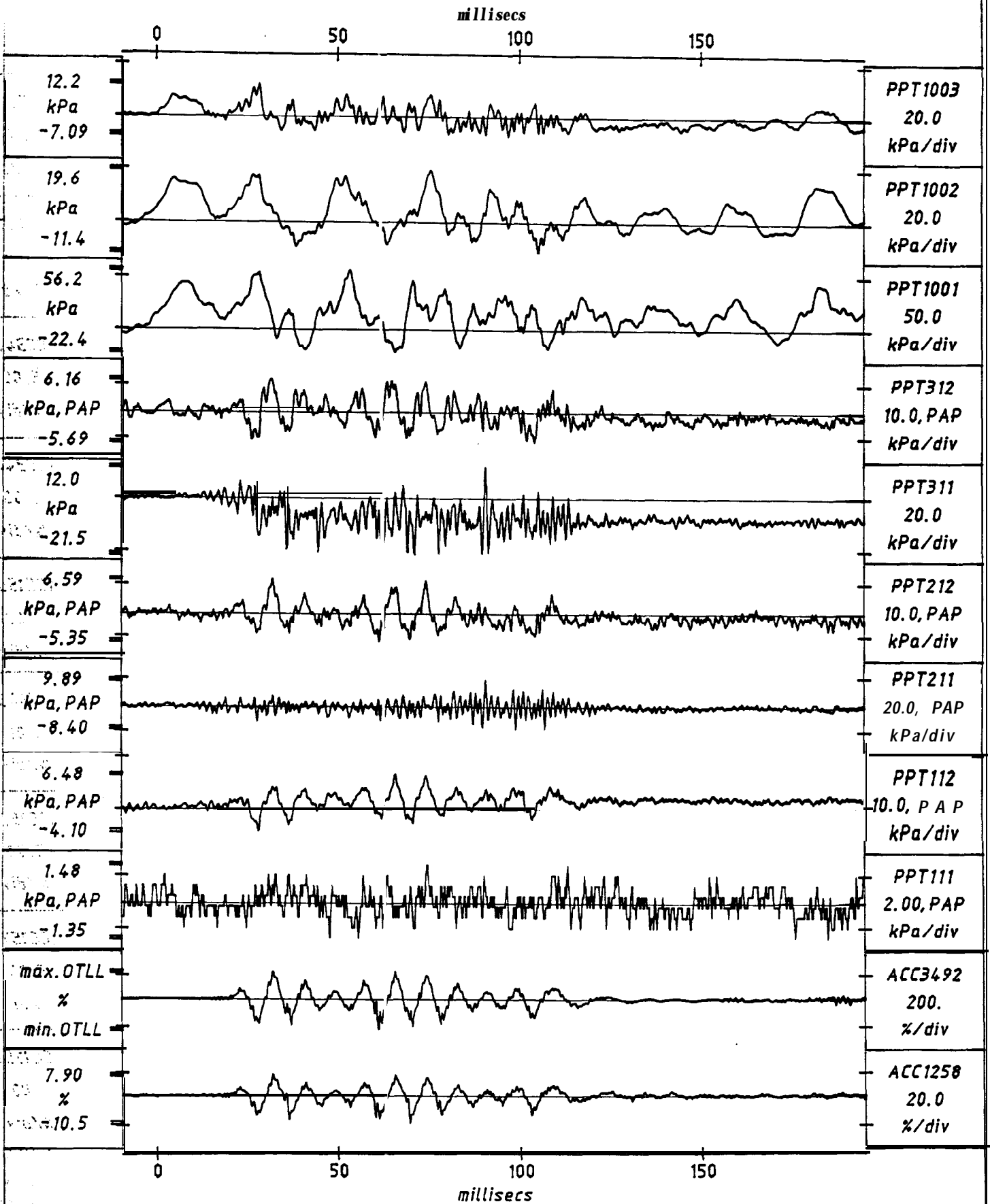
Scales : Mdel

TEST BET1
MDEL DRY EQ1
FLIGHT 1

SHORT-TERM
TIME RECORDS

G = 80.0g FIG. NO.
Km = 4.78% 3-2
Kp = 4.84%

1024 data points per transducer, plotted after 2 smoothing passes



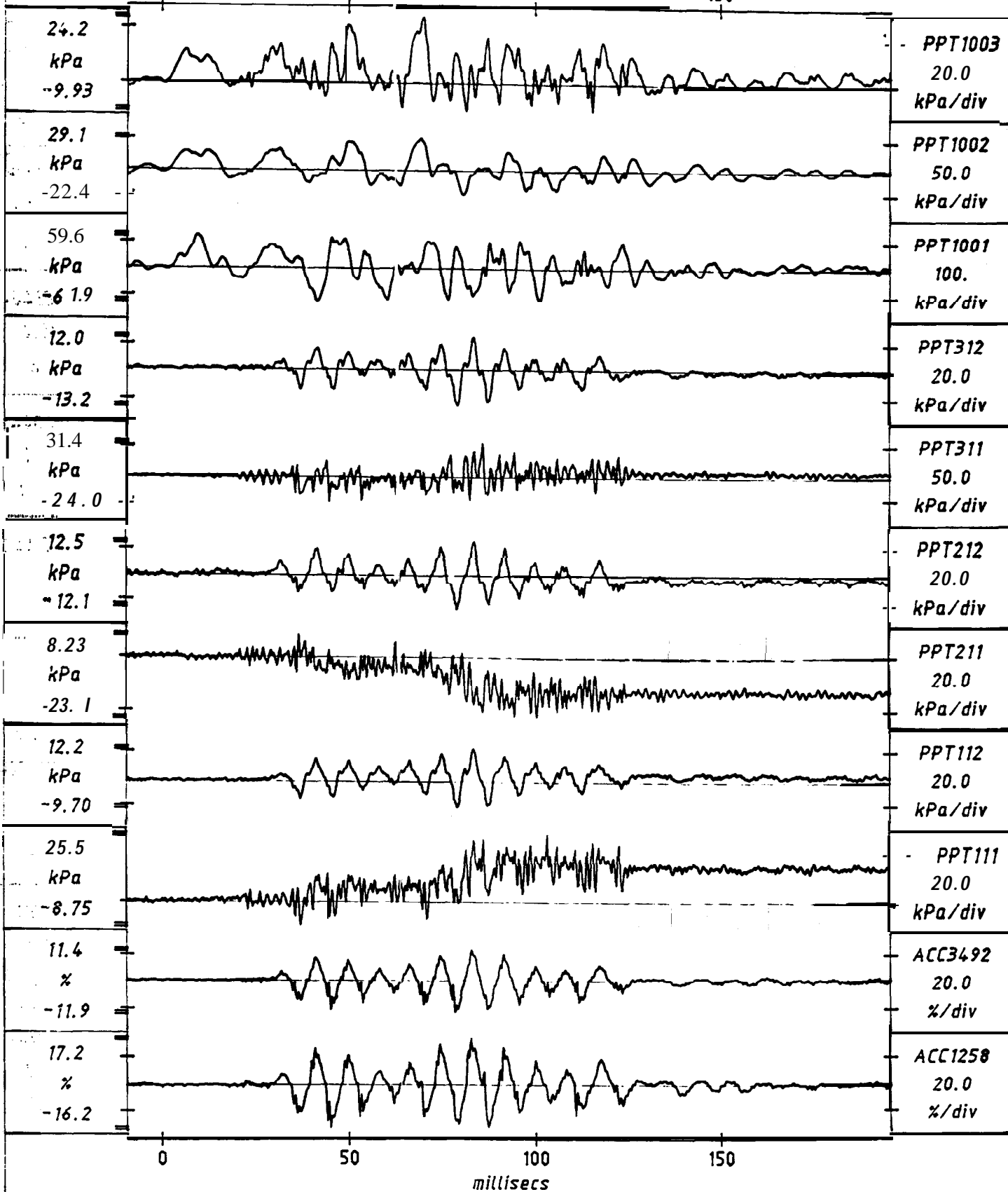
Scales : Model

TEST BET1 MODEL DRY FLIGHT 1	EQ2	SHORT-TERM TIME RECORDS	G = 80.0g Km = 9.21% Kp = 10.5%	FIG. NO. 3-4
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1024 da ta points per transducer, plotted after 2 smoothing pastes

millisecs

0 50 100 150

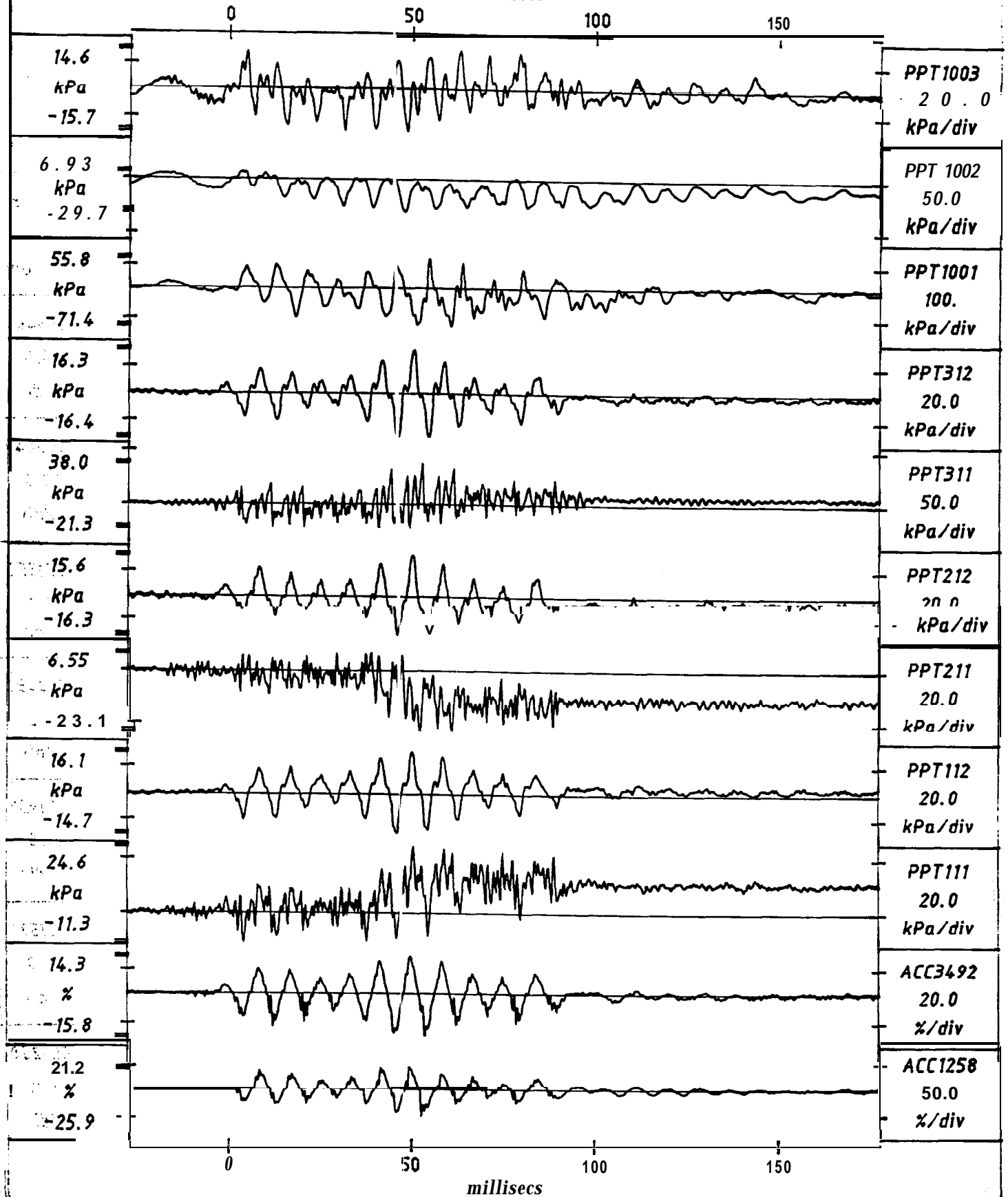


Scales : Model

TEST BET1 MODEL DRY FLIGHT 1	EQ3	SHORT-TERM TIME RECORDS	G = 80.0g Km = 16.7% Kp = 17.2%	FIG. NO. 36
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1024 data points per transducer, plotted after 2 smoothing passes

millisecs



Scales : Model

TEST BET1 MODEL DRY FLIGHT 1	EQ4	SHORT-TERM TIME RECORDS	G = 80.0g Km = 23.6% Kp = 25.9%	FIG. NO. 3-8
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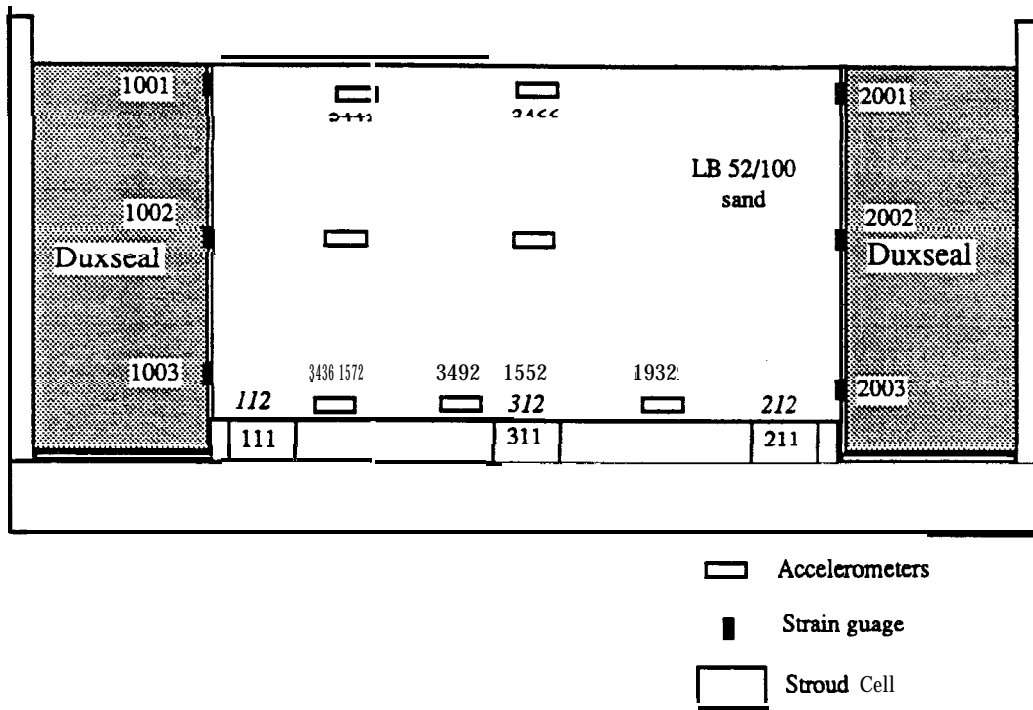
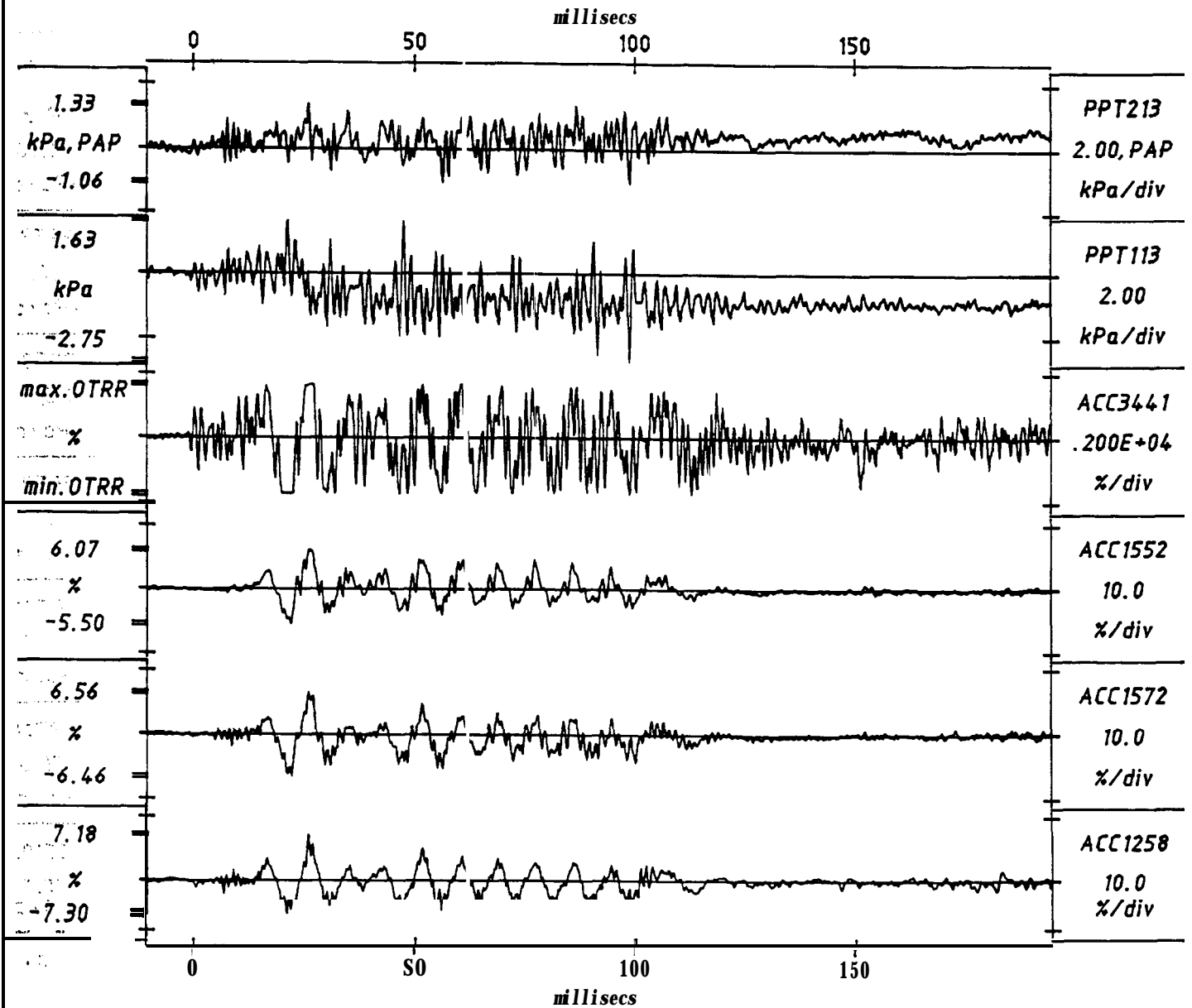


Fig.3.10 Schematic section of centrifuge model for Test BET-Z

1024 data points per transducer, plotted after 2 smoothing passes



Scales : Mdel

TEST BET2
MODEL d DRY-A
FLIGHT 1

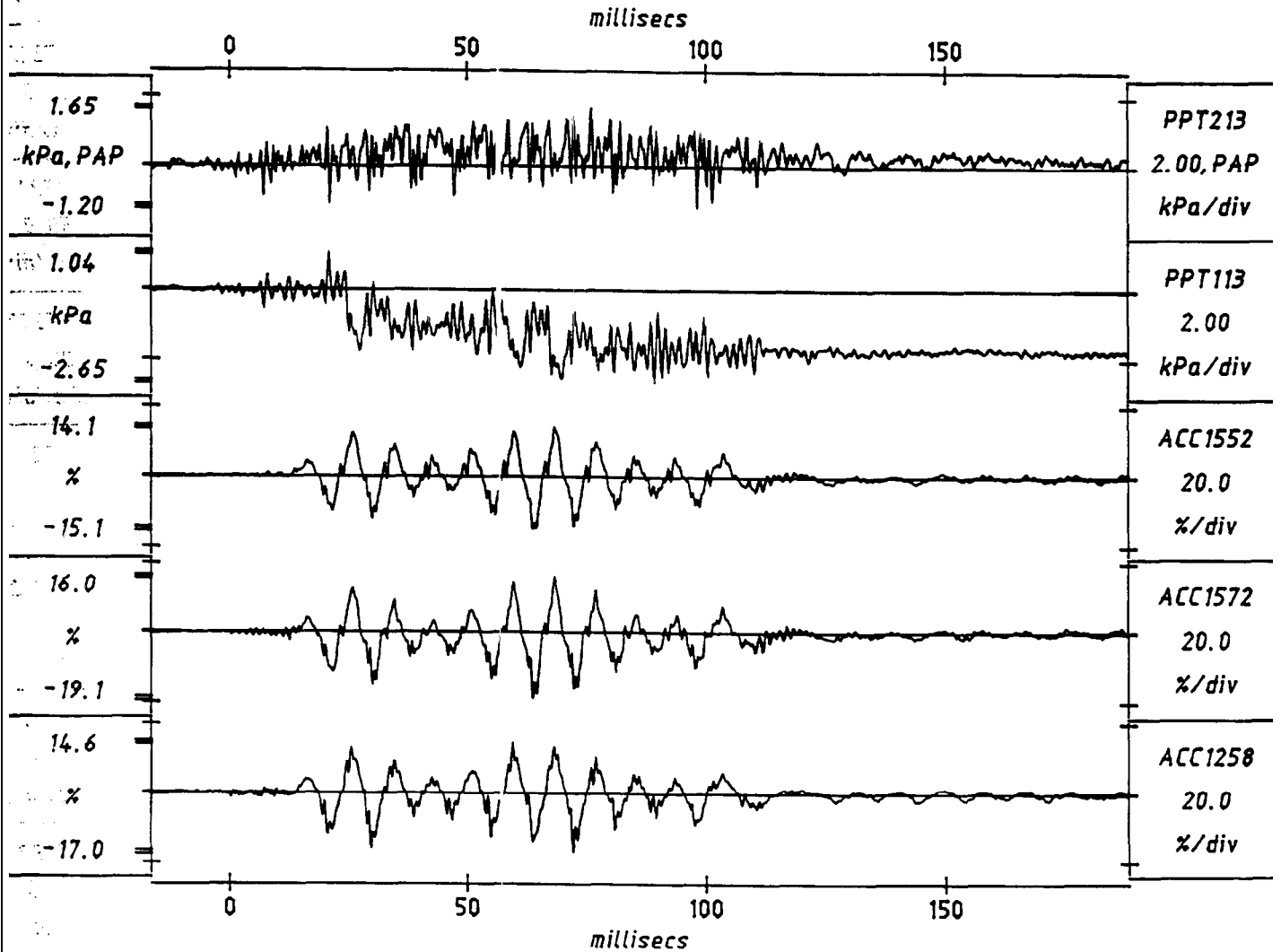
EQ1

SHORT-TERM
TIME RECORDS

G = 80.0g
Km = 7.24%
Kp = 7.30%

FIG. NO
3-12

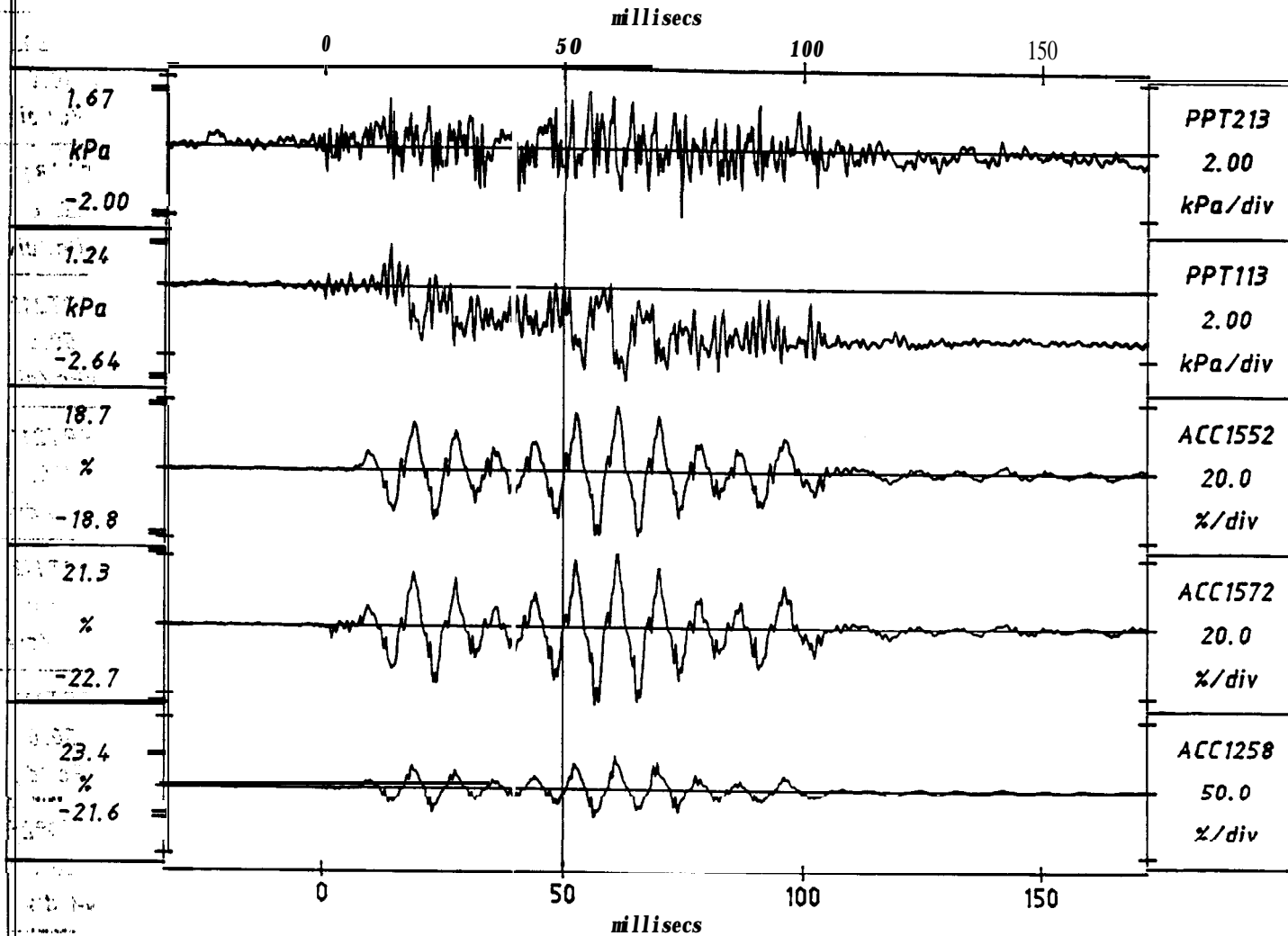
1024 data points per transducer, plotted after 2 smoothing passes



Scales : Model

TEST BET2 MODEL DRY-A FLIGHT 1	EQ2	SHORT-TERM TIME RECORDS	G = 80.0g Km = 15.8% Kp = 17.0%	FIG. NO 3-14
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1024 data points per transducer, plotted after 2 smoothing passes



Scales : **Model**

TEST BET2
MODEL DRY-A
FLIGHT 7

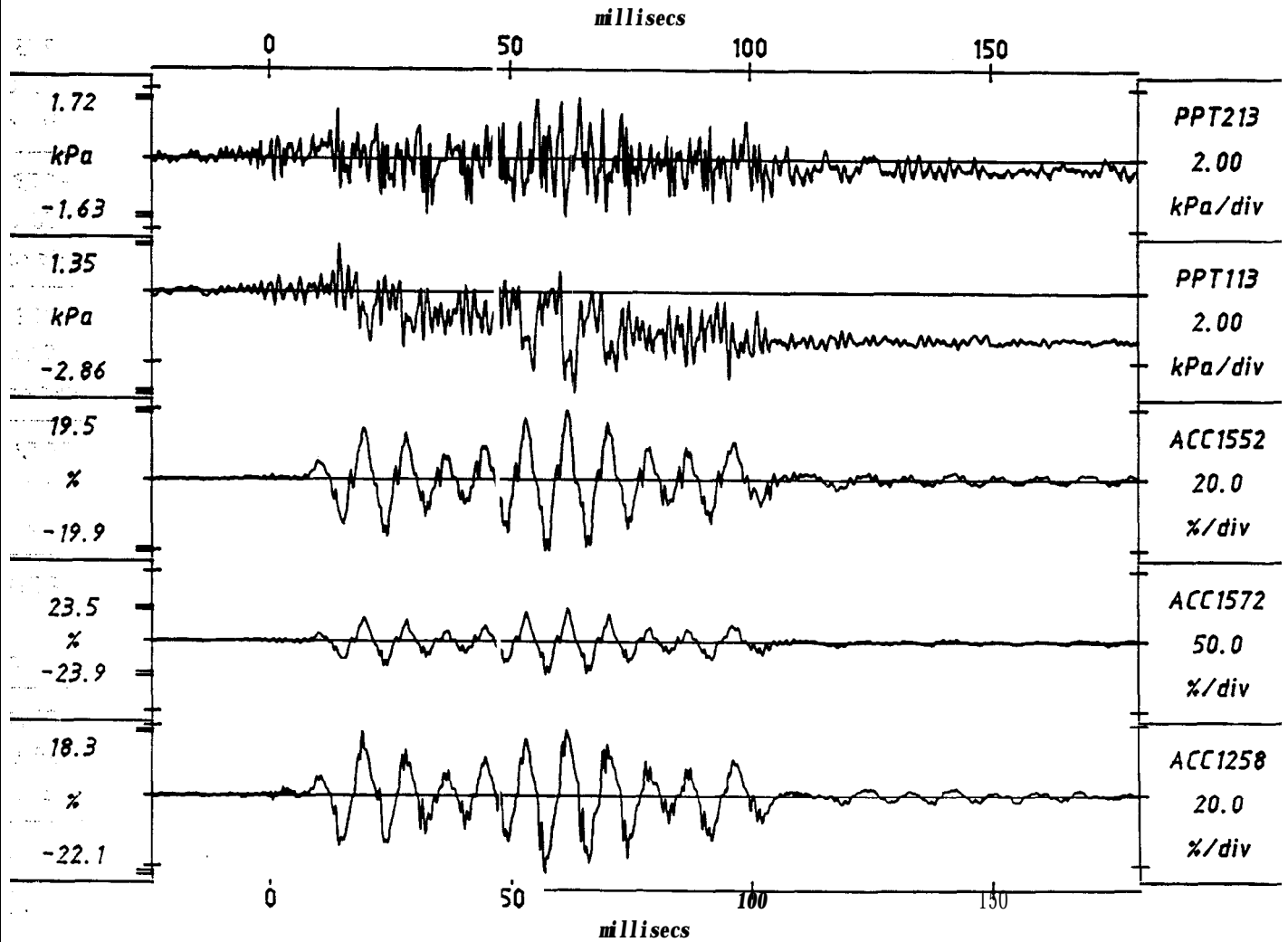
EQ3

SHORT-TERM
TIME RECORDS

G = 80.0g
Km = 22.5%
Kp = 23.4%

FIG. NO.
3-16

1024 data points per transducer, plotted after 2 smoothing passes



Scales : **Mdel**

TEST BET2 MODEL DRY-A FLIGHT 1	EQ4	SHORT-TERM TIME RECORDS	G = 80.0g Km = 20.2% Kp = 22.1%	FIG. NO 3-18
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duxseal-shear sheet boundary system. The observed average shear stress was 3.25 kPa during earthquake 1 of test BET-1. The ratio of observed shear to computed shear is obtained for this case as 63.1%. The peak magnitude of P wave is estimated as $(100-63.1)/100 \times 5.15 = 1.9 \text{ kPa}$ for this case. Using the data obtained in all the tests, the above ratio was evaluated. In all cases the average acceleration were **use 1** in computing the De Alembert's forces. The computation can be tabulated as shown below.

Table 2 Computation of the ratio of observed and computed shear stress

Test Id.	Eq.No	Observed Shear stress kPa	Average acceleration m/s^2	Computed shear stress kPa	$\frac{\tau_{\text{Observed}}}{\tau_{\text{Computed}}}$ %	Peak Magn. of P wave kPa
BET- 1	1	3.25	34.67	5.15	63.1	1.90
BET- 1	2	6.48	58.0	8.62	75.2	2.14
BET- 1	3	12.2	130.6	19.41	62.8	7.21
BET- 1	4	16.1	168.7	25.1	64.2	8.98
BET-2	1	6.41	56.4	8.77	73.1	2.35
BET-2	2	17.7	132.6	21.61	81.8	3.93
BET-2	3	23.9	183.6	28.57	83.6	4.67
BET-2	4	20.5	174.2	27.1 1	75.6	6.61

Based on the estimated ratio of actual shear stress to the computed shear stress in column 6, it can be seen that two-thirds of the shear stress is transferred into the sand. About one-third of the shear stress induces a pressure wave which propagates laterally in the centrifuge model.

Table 3 Computation of Complementary shear stress

Test id.	Eq.No.	Peak complementary shear stress in sand layer (kPa)		
		PPT1001	PPT 1002	PPT1003
BET- 1	1	6.1	3.6	2.0
BET- 1	2	6.6	5.8	2.2
BET- 1	3	14.9	8.0	3.9
BET- 1	4	15.1	12.0	3.2
BET-2	1	5.2	3.0	*
BET-2	2	14.8	5.9	*
BET-2	3	28.0 +	8.0	*
BET-2	4	18.0 ⁺	12.0	*

* Transducer did not function

*The strength of earthquake 4 was less than earthquake 3. This is reflected in the peak shear stress estimates at the base.

In Fig.5.4 and 5.5, the plots for test BET-2, earthquake 1 are presented. In this test the strain gauge just below the **surface** of the sand did not function during the test. The peak complementary shear stresses **observed** during earthquake 1 at the base **and** middle of the sand layer from these figures were 5.2 **and** 3.0 **kPa** respectively. Similar plots were obtained for other earthquakes and the results are as **shown** in Table 3.

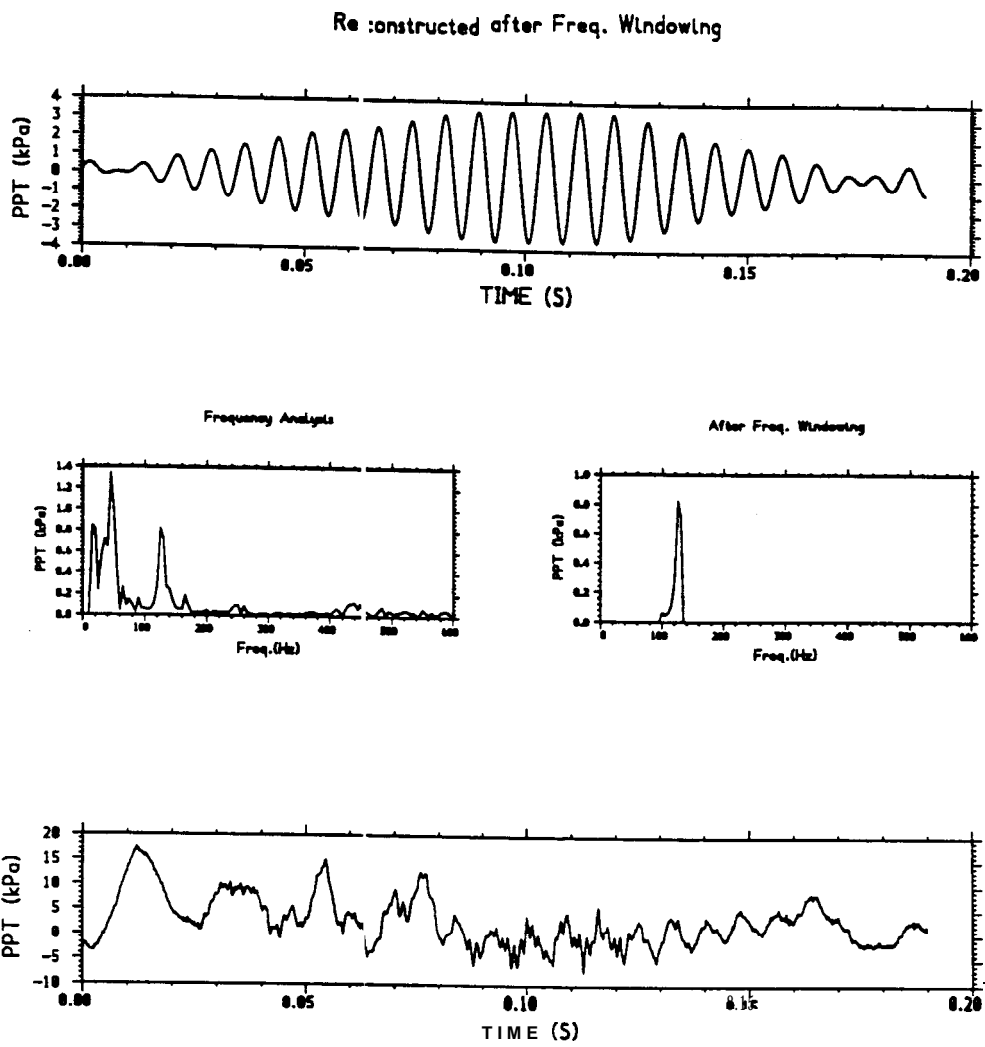


Fig.52 **Frequency** windowing using a Band Pass Filter
 Test BET-I ; Eq.No:l PPT 1002

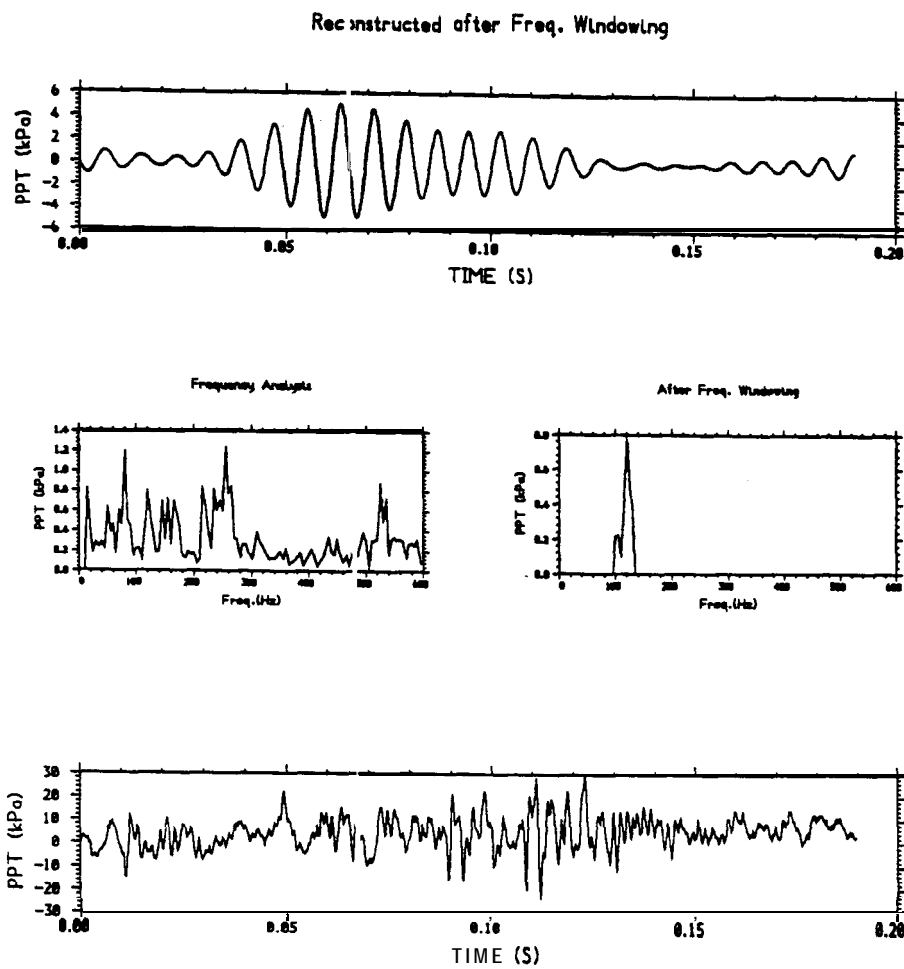


Fig.54 Frequency windowing using a Band Pass Filter
 Test BET-2; Eq.No:1 PPT 1001

6 Phase relationship of normal stress - time histories

In section 1 it was argued that if the duxseal boundary did not carry the complementary shear, a dynamic moment would be induced which acts about the centroid of the centrifuge model. This dynamic moment would cause the vertical pressure to vary along the longitudinal section of the model. The normal stress-time histories for the Stroud cells on either side of the centroid (and equidistant from it) must display a 180 degrees out of phase relationship.

In Fig.3.8, consider the **traces PPT 111 and 211 which indicate normal stresses on either side of the centroid in test BET-1, earthquake 4. It is possible to identify a 120 Hz component in these traces, which are 180 degrees out of phase.** Using the technique of frequency windowing explained in the last section, it is **possible** to frequency window these traces between 100 to 150 Hz, when the out of phase relationship becomes more clear. For this test BET- 1, earthquake 4, the plots of these traces PPT 111 **and** 211 after the frequency windowing are presented in **Fig.6.1.** The vertical line in this figure **indicates** the out of phase relationship.

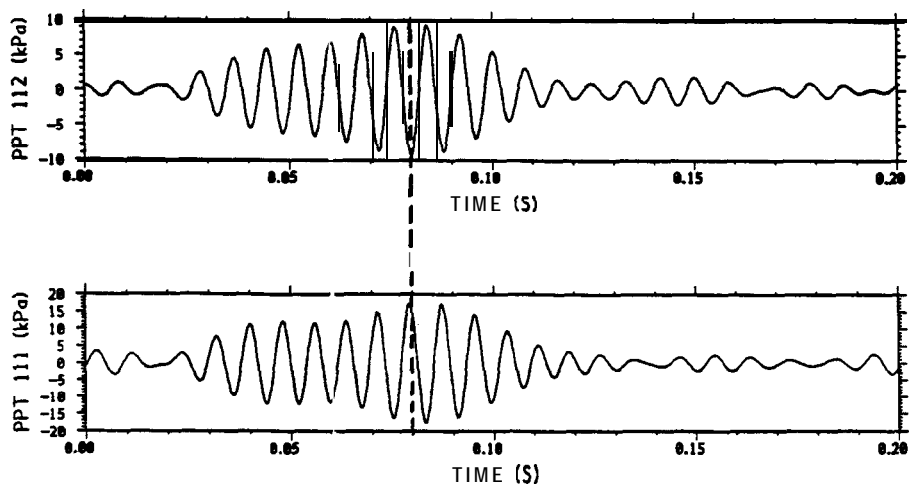
A similar observation can **be** made between traces **PPT111** and 211 for test BET-2 in all the earthquakes in Figs. **3.11,3.13,3.15** and 3.17. For example, for the case of earthquake 2 these traces are presented after **frequency** windowing in Fig.6.2. Again the vertical line shows the 180 degrees out of phase relationship **between** these traces.

7 Conclusions and Future work

In simulating the semi-infinite extent of the soil medium, it is important to consider if the absorbing boundary the complementary shear stress generated during the earthquake. In the two centrifuge tests reported here, an attempt was made to use conventional Stroud cells to measure the normal stress and shear stress at the base of the sand layer during an earthquake. The absorbing boundary in these tests constituted of a 5 inch thick duxseal layer with a thin sand glued dural sheet placed adjacent **to** it to carry the complementary shear stresses. Based on the test results it can be said that the performance of the Stroud cells was satisfactory both in terms of frequency response as well as **the** peak magnitudes of the stresses.

A simple De Alembert's type of calculation indicated that **about** two-thirds of the shear stress that would be generated by **De** Alembert's forces was observed at the base of the model. The other one-third may induce a laterally propagating P wave in the centrifuge model. Further, an attempt was made to measure the complementary shear stress using a **strain** gauged shear sheet. Due to the low voltage **output** of these sheets, a frequency windowing technique had to be

Normal Stress-time traces after Freq. Windowing



Normal stress-time traces before Freq. Windowing

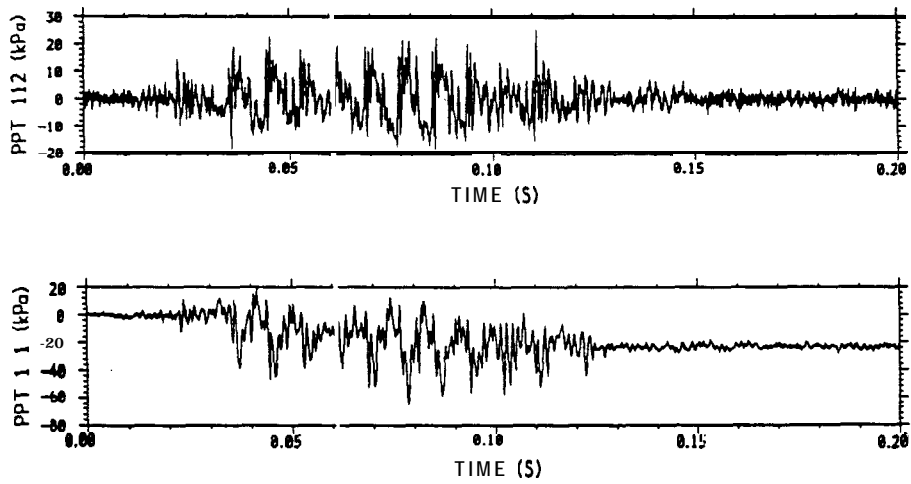


Fig.6.2 Phase relationship between the normal stress-time histories BET-2; Eq.No:2

Steedman,R.S. and Madabhushi,S.P.G.,(1991),' Wave Propagation in Sand medium', **Intl. Conf.** on Seismic **Zonation**, Stanford University, Stanford, California.

Stroud,M.A.,(1971), ' The behaviour of sand at low stress levels in simple shear apparatus', **Ph.D** thesis, Cambridge University, England.