

Homogenization method for numerical analysis of improved ground with cement-treated soil columns

K.Omine & H.Ochiai

Department of Civil Engineering, Kyushu University, Fukuoka, Japan

M.D.Bolton

Department of Engineering, Cambridge University, UK

ABSTRACT: A two-phase mixture model for evaluating stress-strain relationship of mixtures with different elastic materials is proposed based on the consideration of stress distribution. As a homogenisation method in numerical analysis of composite materials, this approach is applied to improved ground with pile-shaped columns for obtaining average elastic moduli and yield stresses of the improved ground. A series of model tests of the improved ground with cement-treated soil columns was conducted and vertical settlement and horizontal displacement under inclined loads was measured. From comparison between the numerical analysis and test results, it is confirmed that the proposed method is effective for predicting the coefficients of subgrade reaction and bearing capacities of the improved ground.

1 INTRODUCTION

Mechanical properties of many kinds of mixtures are needed to be clarified in geotechnical engineering. Improved ground made by the Deep Mixing Method is a composite ground with pile-shaped columns and it is regarded as one of the mixtures. For obtaining average behaviour of the improved ground, it is required to apply any homogenisation method.

In this paper, a homogenisation method for two-phase mixtures is discussed and applied to the improved ground with cement-treated soil columns. First, the two-phase mixture model is proposed based on the evaluation of stress distribution. Stress-strain relationship of the mixtures with different elastic materials is derived from the proposed model. As an application of this homogenisation method, average elastic moduli and yield criterion of the improved ground are obtained. In order to confirm the validity of the proposed method, 1-g model tests of the improved ground with cement-treated soil columns were conducted and the test results are compared with the results calculated by FEM analysis.

2 TWO-PHASE MIXTURE MODEL

2.1 Previous study

Some homogenisation methods for two-phase mixtures have been proposed. Estimation of elastic moduli of the mixtures is a classical problem in micromechanics. Pioneer works on the estimation of elastic moduli of the mixtures have been performed

by Voigt and Reuss. Voigt (1889) assumed that all the elements of the mixture are subjected to the same uniform strain and Reuss (1929) assumed that all the mixture elements are subjected to a uniform stress equal to the applied stress. The Voigt and Reuss approximations give upper and lower bounds of the elastic moduli of the mixtures, respectively. Hashin (1962), Hashin and Shtrikeman (1962) have derived upper and lower bounds of the elastic moduli of the mixtures based on new variational principles. On the other hand, Eshelby (1957) first pointed out that the stress disturbance in a mixture can be simulated by an eigenstress (internal stress) caused by an inclusion, and elastic moduli of mixtures with ellipsoidal inclusions are derived from this method. From the different point of view, the authors has proposed a homogenisation method for evaluating stress-strain relationship of two-phase mixtures with random spherical inclusions (Omine et al., 1992). The model cannot be applied to mixtures with various shaped inclusions. Hence, it is important to use an appropriate method suited for the individual mixture.

2.2 Stress-strain relationship of two-phase mixtures

Two-phase mixture consists of two different materials, namely matrix and inclusion. Stress-strain relationship of the two-phase mixtures is discussed based on the stress distribution in the mixture.

First of all, average stress, $\bar{\sigma}$, and average strain, $\bar{\epsilon}$, of the mixture are defined as

$$\bar{\sigma} = f_s \bar{\sigma}_s + (1 - f_s) \bar{\sigma} \quad (1)$$

$$\bar{\varepsilon} = f_s \bar{\varepsilon}_s + (1 - f_s) \bar{\varepsilon} \quad (2)$$

where f_s is volume fraction of inclusions and subscript 's' and superscript '*' mean inclusion and matrix, respectively. Stress-strain relationships of the inclusion and the matrix are represented as,

$$\bar{\varepsilon}_s = C_s \bar{\sigma}_s \quad (3)$$

$$\bar{\varepsilon} = C^* \bar{\sigma} \quad (4)$$

where C_s and C^* are compliances of inclusion and matrix, respectively. In order to evaluate the stress distribution in the mixture, the following relationship between stresses of the inclusion and matrix is introduced,

$$\bar{\sigma}_s = b \bar{\sigma} \quad (5)$$

where the stress distribution tensor, b , is an extension of the stress distribution parameter which was used in the previous study (Omine et al., 1998). The value of the stress distribution tensor depends on a shape of the inclusion, so that determination of the stress distribution tensor is mentioned in the next section. If the stress distribution tensor are known, average stress-strain relationship of the mixtures is represented from Eqs.(1)–(5) as follows,

$$\bar{\varepsilon} = \{f_s C_s b + (1 - f_s) C^*\} \{f_s b + (1 - f_s) I\}^{-1} \bar{\sigma} \quad (6)$$

where I is unit tensor.

2.3 Determination of stress distribution tensor

For discussing determination method of the stress distribution parameter for various mixtures, the relationship between the stress distribution parameter and Eshelby's tensor is shown in Fig.1. The stress distribution parameter for the laminates or the mixtures with spherical inclusions is represented as a power function of Young's modulus ratio, E_s/E^* , of inclusion and matrix. It has been also found from the results of numerical analysis that the adequate value of n is in the range of 1/3–1/6 for the mixtures with pile-shaped inclusions (Adams et al., 1967, Omine et al., 1997). On the other hand, Eshelby has proposed elastic moduli of the mixtures with ellipsoidal inclusions. This method has a limitation to mixtures with large volume fraction of inclusions. However it is applicable to mixtures with various shaped inclusions because an ellipsoid with radiuses of a_1 , a_2 and a_3 changes into various shapes, for example, sphere ($a_1=a_2=a_3$), cylinder ($a_1=\infty$) and plate ($a_1=a_2=\infty$). The values of Eshelby's tensor are given for the following mixtures as,

$S_{1111} = 1$; Horizontal Laminate

$S_{1111} = 0$; Vertical Laminate

$S_{1111} = \frac{7-5\nu^*}{15(1-\nu^*)}$; Spherical inclusion

$S_{1111} = \frac{5-4\nu^*}{8(1-\nu^*)}$; Pile-shaped inclusion

Type of mixtures	Horizontal laminate	Mixture with spherical inclusions	Mixture with pile shaped inclusions	Vertical laminate
Assumption	Constant Stress	Constant Strain energy	Approximation based on numerical analysis	Constant strain
Stress distribution parameter b	$\left(\frac{E_s}{E^*}\right)^n = \left(\frac{E_s}{E^*}\right)^0 = 1$ where, $n = 0$	$\left(\frac{E_s}{E^*}\right)^{\frac{1}{2}}$ where, $n = 1/2$	$\left(\frac{E_s}{E^*}\right)^{\frac{1}{3} \sim \frac{1}{6}}$ where, $n = 1/3 \sim 1/6$	$\left(\frac{E_s}{E^*}\right)^1$ where, $n = 1$
Eshelby's tensor S_{1111}	1	$7/15 \sim 9/15$ (for $\nu^*=0-0.5$)	$5/8 \sim 3/4$ (for $\nu^*=0-0.5$)	0
$1 - S_{1111}$	0	nearly 1/2	$3/8 \sim 1/4$	1

Figure 1. Relationship between stress distribution parameter and Eshelby's tensor.

As shown in Fig.1, a power, n , of the stress distribution parameter (in the i -direction of applied stress) is approximately related to Eshelby's tensor as

$$n \approx 1 - S_{1111} \quad (7)$$

Hence, the stress distribution parameter of the mixture with ellipsoidal inclusions is given using Eq.(7) as follows,

$$b = \left(\frac{E_s}{E^*}\right)^n = \left(\frac{E_s}{E^*}\right)^{1-S_{1111}} \quad (8)$$

In the condition of $\bar{\sigma}_{11} \neq 0$ and all other stresses are zero, the Young's modulus of the mixture is represented by substituting Eq.(8) into the following equation

$$\bar{E} = \frac{(b-1)f_s + 1}{\frac{f_s b}{E_s} + \frac{(1-f_s)}{E^*}} \quad (9)$$

Figure 2 shows comparison between the results of numerical analysis by Adams et al. (1967) and the calculated results calculated from Eq.(9) for pile-shaped inclusions. The average Young's modulus of the mixtures normalized by that of the matrix increases with increase in Young's modulus ratio of, E_s/E^* , and the volume fraction of inclusions, f_s . Although Eshelby's method is not applicable to the mixtures with large amount of inclusions, the proposed equation estimates well Young's modulus of the mixtures with various volume fractions of inclusions in the range of E_s/E^* less than 100.

Elastic compliances of inclusion and matrix in general form can be expressed by Young's moduli of these materials as $C_{1111}^* = 1/E^*$ and $C_{s1111} = 1/E_s$. Young's modulus ratio, E_s/E^* , is represented as a compliance ratio, C_{1111}^*/C_{s1111} , of inclusion and matrix. In order to generalise the stress-strain relationship of mixtures with ellipsoidal inclusions,

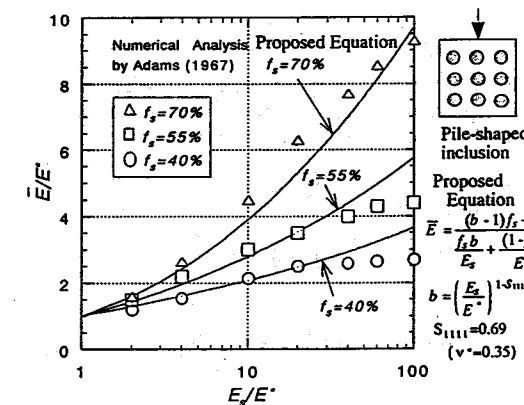


Figure 2. Comparison between the results of numerical analysis and the proposed equation.

the stress distribution parameter of Eq.(8) is extended into stress distribution tensor in a simple manner as

$$b = b_{ijkl} = \left\{ \left(\frac{C_{ijij}^*}{C_{sijij}} \right)^{1-S_{ijij}} \right\}_{ijkl} \quad (10)$$

Eshelby's tensor, S_{ijij} , of the mixtures with ellipsoidal inclusions is represented in a general expression as a function of radiuses of ellipsoid (a_1 , a_2 and a_3) (Mura, 1982). The summation convention is not used for i, j, k and l in the parenthesis, $\{\}$, of the Eq.(10). All other non-zero components of the stress distribution tensor are obtained by the cyclic permutation of (1, 2, 3). The components which cannot be obtained by the cyclic permutation are zero (for instant, $b_{1112}=b_{1223}=b_{1232}=0$).

3 APPLICATION TO IMPROVED GROUND

3.1 Average elastic moduli of the improved ground

Improved ground by the deep mixing method (DMM) is a composite ground with pile-shaped columns as shown in Fig.3. Such improved ground is regarded as anisotropic material with different elastic moduli for horizontal and vertical directions. The elastic behaviour of the improved ground is represented by the following stress-strain relationship

$$\begin{Bmatrix} \bar{\varepsilon}_{11} \\ \bar{\varepsilon}_{22} \\ \bar{\varepsilon}_{33} \\ \bar{\varepsilon}_{12} \\ \bar{\varepsilon}_{13} \\ \bar{\varepsilon}_{23} \end{Bmatrix} = \begin{bmatrix} \frac{1}{\bar{E}_h} & \frac{-\nu_{hh}}{\bar{E}_h} & \frac{-\nu_{hv}}{\bar{E}_v} & 0 & 0 & 0 \\ \frac{-\nu_{hh}}{\bar{E}_h} & \frac{1}{\bar{E}_h} & \frac{-\nu_{hv}}{\bar{E}_v} & 0 & 0 & 0 \\ \frac{-\nu_{hv}}{\bar{E}_h} & \frac{-\nu_{hv}}{\bar{E}_h} & \frac{1}{\bar{E}_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\bar{G}_{hh}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\bar{G}_{hv}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\bar{G}_{hv}} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_{11} \\ \bar{\sigma}_{22} \\ \bar{\sigma}_{33} \\ \bar{\sigma}_{12} \\ \bar{\sigma}_{13} \\ \bar{\sigma}_{23} \end{Bmatrix} \quad (11)$$

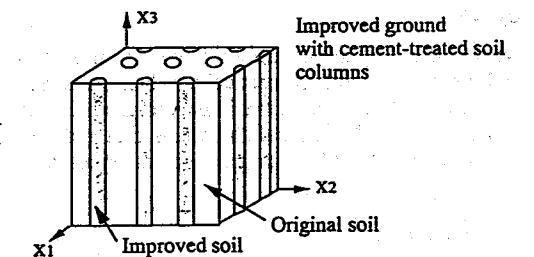


Figure 3. Improved ground with pile-shaped columns.

where $\bar{E}_h = \bar{E}_1 = \bar{E}_2$, $\bar{E}_v = \bar{E}_3$, $\bar{G}_{hh} = \bar{G}_{12}$, $\bar{G}_{hv} = \bar{G}_{13} = \bar{G}_{23}$, $\nu_{vh} = \nu_{31} = \nu_{32}$, $\nu_{hh} = \nu_{12}$ and subscripts of 'h' and 'v' mean horizontal and vertical components, respectively. The average Young's moduli, Poisson's ratios and shear moduli of the improved ground are given from the proposed two-phase mixture model as

$$\bar{E}_i = \frac{(b_i - 1)f_s + 1}{\frac{f_s b_i}{E_s} + \frac{(1-f_s)}{E^*}} \quad (12)$$

$$\bar{\nu}_{ij} = \frac{f_s b_i \frac{\nu_s}{E_s} + (1-f_s) \frac{\nu^*}{E^*}}{\frac{f_s b_i}{E_s} + \frac{(1-f_s)}{E^*}} \quad (13)$$

$$\bar{G}_{ij} = \frac{(b_{ij} - 1)f_s + 1}{\frac{f_s b_{ij}}{G_s} + \frac{(1-f_s)}{G^*}} \quad (14)$$

Thus, the average elastic moduli of the improved ground are obtained from those of the improved soil and the original soil. The stress distribution tensors in Eqs.(12)–(14) are expressed as follows,

$$b_i = \left(\frac{E_s}{E^*} \right)^{1-S_{iii}} \quad (15)$$

$$b_{ij} = \left(\frac{G_s}{G^*} \right)^{1-S_{ijj}} \quad (16)$$

where $b_i = b_{iii}$ and $b_{ij} = b_{ijj}$ and Eshelby's tensors for the mixtures with pile-shaped inclusions are $S_{1111} = S_{2222} = 3/4$, $S_{3333} = 0$ and $S_{1212} = S_{1313} = S_{2323} = 1/4$ (Mura, 1982).

3.2 Yield criterion of the improved ground

It is assumed that the improved soil and the original soil are both linear elastic perfectly plastic material as shown in Fig. 4. If the frictional angles of the improved soil and the original soil are zero in undrained condition, the yield stresses of those soils are represented as

$$\sigma_{ys} = 2c_{us}$$

$$\sigma_y^* = 2c_u^*$$

where c_{us} and c_u^* are cohesions of the improved soil and the original soil, respectively. When horizontal stress, $\bar{\sigma}_{11}$, is applied to the improved ground, it is considered that the original soil yields first and then the average stress of the original soil is represented as

$$\bar{\sigma}_{y11} = \bar{\sigma}_y^* = 2c_u^* \quad (17)$$

The average stress of improved soil at yield of the original soil is represented using the stress

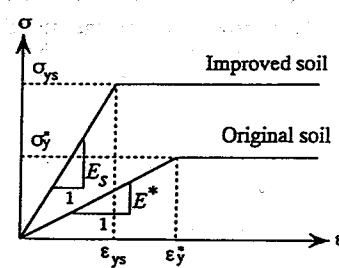


Figure 4. Linear elastic perfectly plastic model for improved and original soils.

distribution tensor, $b_h (=b_{1111})$, as follows

$$\bar{\sigma}_{s11} = b_h \bar{\sigma}_{y11} = 2b_h c_u^* \quad (18)$$

Hence, the yield stress, $\bar{\sigma}_{yh} (= \bar{\sigma}_{y11})$, of improved ground in the horizontal direction is given as an average stress of the improved soil and the original soil (Eqs.(17) and (18)),

$$\bar{\sigma}_{yh} = 2(f_s b_h + 1 - f_s) c_u^* \quad (19)$$

In a similar manner, the yield stress of the improved ground in the vertical direction is obtained from the yielding condition of the improved soil as

$$\bar{\sigma}_{yv} = 2\{f_s + (1-f_s)/b_v\} c_{us} \quad (20)$$

On the other hand, it is considered that the original soil yields at first for the shear stress, $\bar{\sigma}_{12}$. The yield stress, $\bar{\sigma}_{yhh} (= \bar{\sigma}_{y12})$, of the improved ground in the horizontal shear component is given as an average stress of the improved soil and the original soil,

$$\bar{\sigma}_{yhh} = \frac{2}{\sqrt{3}} (f_s b_{hh} + 1 - f_s) c_u^* \quad (21)$$

where von Mises criterion is used. Furthermore, the yield stress, $\bar{\sigma}_{y hv} (= \bar{\sigma}_{y13} = \bar{\sigma}_{y23})$, of the improved ground in horizontal-vertical shear component is also obtained as

$$\bar{\sigma}_{y hv} = \frac{2}{\sqrt{3}} (f_s b_{hv} + 1 - f_s) c_u^* \quad (22)$$

Finally, the following modified von Mises criterion is applied in the consideration of the anisotropic yield condition of the improved ground,

$$\frac{1}{2} \left(\frac{2}{\bar{\sigma}_{yh}^2} - \frac{1}{\bar{\sigma}_{yv}^2} \right) (\bar{\sigma}_{11} - \bar{\sigma}_{22})^2 + \frac{(\bar{\sigma}_{22} - \bar{\sigma}_{33})^2}{2\bar{\sigma}_{yv}^2} + \frac{(\bar{\sigma}_{33} - \bar{\sigma}_{11})^2}{2\bar{\sigma}_{yv}^2} + \left(\frac{\bar{\sigma}_{12}}{\bar{\sigma}_{yhh}} \right)^2 + \left(\frac{\bar{\sigma}_{13}}{\bar{\sigma}_{y hv}} \right)^2 + \left(\frac{\bar{\sigma}_{23}}{\bar{\sigma}_{y hv}} \right)^2 = 1 \quad (23)$$

where the individual yield stress in Eq.(23) is

obtained from Eqs.(18)–(22). This equation is a simple extension of Von Mises criterion and the parameters are decided from the linear elastic perfectly plastic properties of the improved soil and the original soil, namely, E_s , E^* , c_{us} and c_u^* .

4 VERIFICATION OF THE PROPOSED METHOD BY 1G-MODEL TESTS

4.1 Model tests

In order to confirm the validity of the proposed homogenisation method, 1g-model tests of the improved ground with cement-treated soil columns are conducted. A general view of the model test is shown in Fig. 5. Improved soil columns of 30mm in diameter and 200mm in height were made in a mould by mixing Kaolin clay slurry and portland cement. On the other hand, original soft ground with small stiffness and small strength was made by adding a small amount of cement (15kg/m³) to Kaolin clay slurry without consolidation and poring it into the specimen box. The improved soil columns were

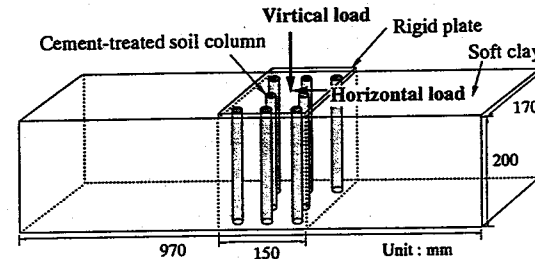


Figure 5. A general view of the model test.

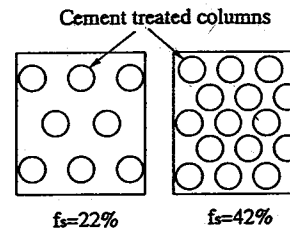


Figure 6. Arrangements of improved soil columns.

Table 1. Test cases.

	Cement content (kg/m³)	Number of columns	Improvement ratio f_s (%)	Inclined load ratio q_h/q_v
Case-1	90	8	22	0.3. 0.6
Case-2	90	15	42	0.3. 0.6
Case-3	250	8	22	0.3. 0.6

installed on the original ground then. Arrangements of the improved soil columns are shown in Fig. 6. The model ground is cured for 3 days and the tops of the improved soil columns are fixed to a rigid plate using plaster. Test cases are shown in Table 1. Loading tests of the improved model ground were conducted under stress controlled conditions. Increments of vertical stress of 0.96kPa and horizontal stress corresponding to the inclined load ratio were applied to the model ground using weights placed every minute.

4.2 Determination of the parameters for numerical analysis

For determining the soil parameters in FEM analysis, unconfined compression tests of the improved soil and the original soil were conducted. These specimens with 38mm in diameter and 76mm in height were made under the same curing condition. The strain rate in the tests was about 1%/min. Relationships between compressive stress and axial strain of the improved soils are shown in Fig. 7. The improved soils with relatively small cements content were made because of a limitation of load weight capacity in the test apparatus. However, it is found that these improved soils have a clear peak strength and the effect of cementation appears adequately. In addition, the original soil with a small amount of cement has no clear peak strength and the value of the strength is very small comparing with those of the improved soils.

The finite element mesh for analysis of the model test is shown in Fig. 8. The improved soil and the original soil are assumed both linear elastic perfectly plastic materials. The soil parameters used in the analysis are shown in Table 2. The deformation moduli, E_{s0} , obtained from the unconfined compression tests of the improved soils and the original soil are used as each elastic modulus. Cohesions of the improved soils and the original soil

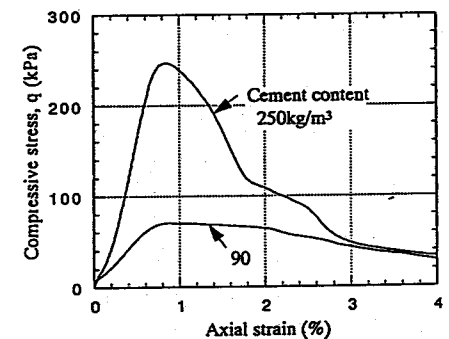


Figure 7. Relationships between compressive stress and axial strain of the improved soils.

are obtained as a half value of the compressive strength. The Poisson's ratios of 0.4 and 0.49 are assumed for the improved soils and the original soil, respectively. Average elastic moduli, Poisson's ratios and yield stresses of the improved ground in each direction are calculated using the proposed homogenisation method (Eqs.(12)–(14) and (18)–(21)). Anisotropic elasticity and perfectly plasticity model with the modified von Mises approach (Eqs.(11) and (23)) are applied in FEM analysis of the improved ground.

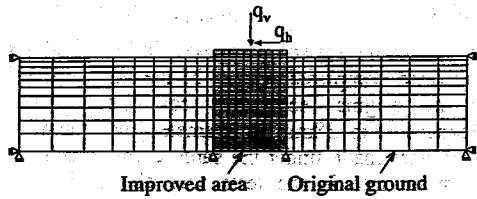
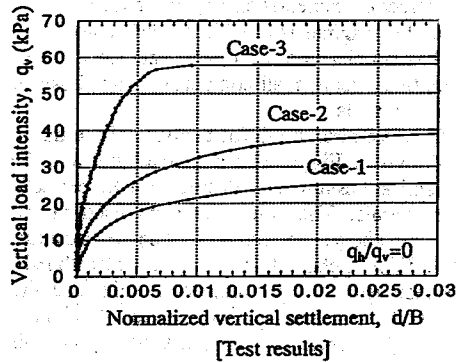
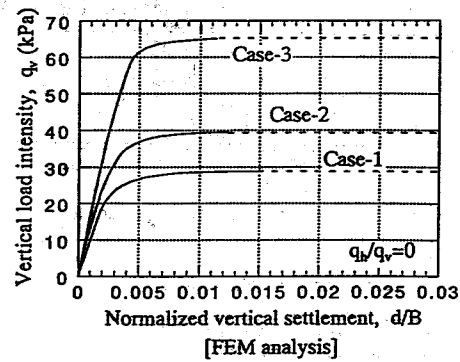


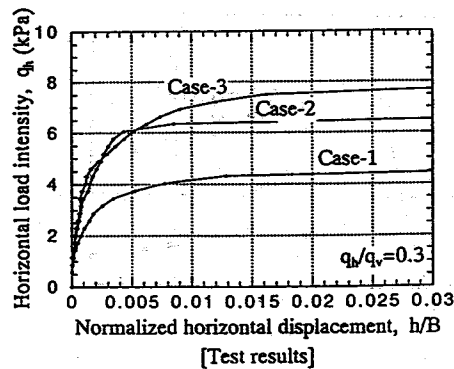
Figure 8. Finite element mesh for the analysis



(a) $q_h/q_v=0$



(b) $q_h/q_v=0.3$



(b) $q_h/q_v=0.3$

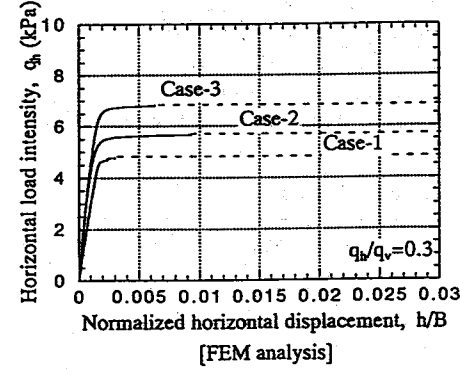


Figure 9. Comparison of FEM analysis results and test results on deformation behaviour of the improved ground.

4.3 Comparison between numerical analysis and test results

Comparison between the FEM analysis and the test results on the deformation behaviour of the

Table 2. Material parameters used in analyses.

(a) Improved soil and original soil						
	Elastic modulus (kPa)		Poisson's ratio		Shear strength ($\phi=0$)	
	E_s	E_r	ν_s	ν_r	c_{us} (kPa)	c_u (kPa)
Case-1	17262	4171	0.40	0.49	29.96	2.66
Case-2	17262	4171	0.40	0.49	29.96	2.66
Case-3	58238	4171	0.40	0.49	113.29	2.66

(b) Improved ground								
	Elastic modulus (kPa)			Poisson's ratio		Yield stress (kPa)		
	E_v	E_h	\bar{G}_{hv}	ν_{vh}	ν_{hh}	σ_{yv}	σ_{yh}	σ_{vhv}
Case-1	7051	5331	2063	0.47	0.48	17.32	5.80	4.11
Case-2	9669	6786	2798	0.45	0.47	28.24	6.26	5.03
Case-3	16066	6203	3204	0.47	0.49	53.98	6.40	6.04

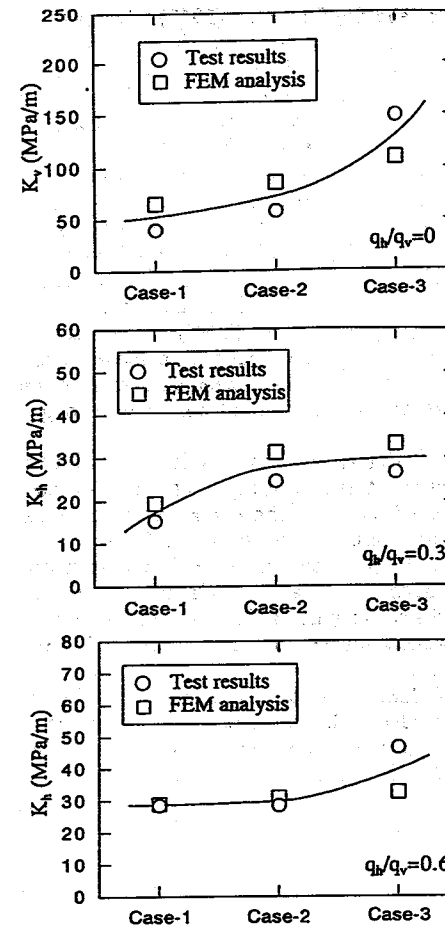


Figure 10. Comparison between FEM analysis and test results on the coefficients of vertical and horizontal subgrade reaction.

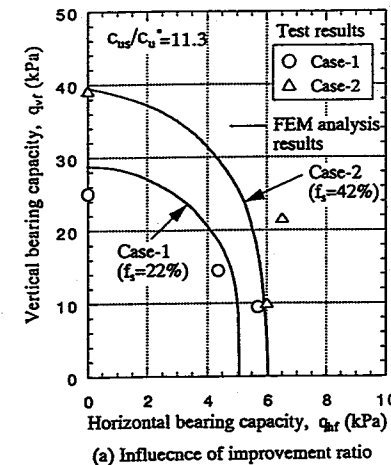


Figure 11. Influences of improvement ratio and strength ratio on bearing capacity of improved ground.

Table 3. Test results of model tests.

Inclined load ratio q_h/q_v	Case-1		Case-2		Case-3	
	q_{rv} (kPa)	q_{rh} (kPa)	q_{rv} (kPa)	q_{rh} (kPa)	q_{rv} (kPa)	q_{rh} (kPa)
0	25.00	0	39.20	0	57.90	0
0.3	14.50	4.35	21.67	6.50	26.00	7.80
0.6	9.50	5.70	10.00	6.00	11.53	6.92

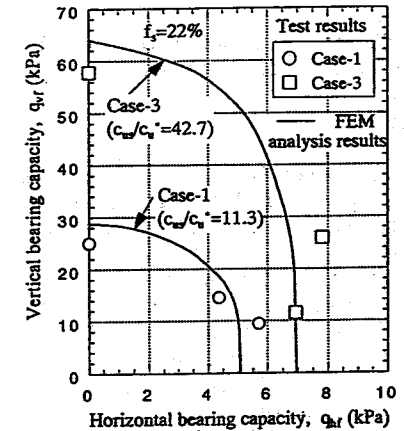
improved ground are shown in Fig.9 (a) and (b). Fig.9 (a) shows the relationship between vertical load intensity, q_v , and normalized vertical settlement, d/B , in the condition of inclined load ratio, $q_h/q_v=0$, where d is vertical settlement and B is width of the rigid plate. Fig.9 (b) shows the relationships between horizontal load intensity, q_h , and normalized horizontal displacement, h/B , in the condition of inclined load ratio, $q_h/q_v=0.3$, where h is horizontal displacement. The results of FEM analysis represent well the difference of the test cases in the deformation behaviour of the improved ground obtained from the model tests. Although the deformation obtained from the analysis at small load density overestimates as compared with the test results, this reason is that the secant modulus, E_{50} , defined at a half point of compressive strength is used as Young's modulus. Hence, the following coefficients of vertical and horizontal subgrade reaction at the half value of bearing capacity are defined as

Coefficient of vertical subgrade reaction ;

$$K_{v50} = \frac{q_{fv}/2}{d_{50}}$$

Coefficient of horizontal subgrade reaction ;

$$K_{h50} = \frac{q_{hv}/2}{h_{50}}$$



where q_v and q_h are vertical and horizontal bearing capacities and d_{50} and h_{50} are vertical settlement and horizontal displacement at 50% of bearing capacity, respectively. Figure 10 shows the comparison between FEM analysis and test results on the coefficients of vertical and horizontal subgrade reactions. The value of K_v or K_h depends on the test cases. The results of FEM analysis give relatively good agreement with the test results.

The results of the bearing capacities obtained by the model tests are shown in Table 3. Figure 11 shows the relationships between the vertical and horizontal bearing capacities of the improved ground. The influences of improvement ratio and strength ratio are shown in Fig. 11 (a) and (b), respectively. It is found that the bearing capacity increases with increase in improvement ratio and strength ratio, and the horizontal bearing capacity is very small as compared with the vertical bearing capacity. The results of numerical analysis agree well with such a tendency. It is therefore confirmed that the proposed method is effective for numerical analysis of the improved ground.

5 CONCLUSIONS

In order to evaluate strength-deformation properties of the improved ground with cement-treated soil columns, a homogenization method is proposed in this study. The following conclusions are obtained:

- 1) The generalised two-phase mixture model is proposed in the consideration of relation to Eshelby's tensor.
- 2) Average elastic moduli and the yield criterion of improved ground are obtained from the two-phase mixture model.
- 4) Bearing capacities and coefficients of subgrade reactions in vertical and horizontal directions of the improved ground are predicted from the numerical analysis based on the proposed homogenisation method.
- 5) The validity of the method is confirmed by 1g-model tests of the improved ground.

REFERENCES

- Adams, D.F. and Doner, D.R. (1967) : Transverse normal loading of a unidirectional composite, *Journal of Composite Materials*, Vol.1, pp.152-164.
- Eshelby, J.D. (1957) : The determination of the elastic field of an ellipsoidal inclusion, and related problems, *Proceedings of the Royal Society, London*, Vol.A, No.241, pp.376-396.
- Hashin, Z. (1962) : The elastic moduli of heterogeneous materials, *ASME Journal of Applied Mechanics*, Vol.29, pp.143-150.

Hashin, Z. and Shtrikman, S. (1962) : A variational approach to the theory of the elastic behavior of polycrystals, *Journal of the Mechanics and Physics of Solids*, Vol.10, pp.343-352.

Mura, T. (1982) : *Micromechanics of defects in solids*, Martinus Nijhoff Publishers.

Omine, K. and Ochiai, H. (1992) : Stress-strain relationship of mixtures with two different materials and its application to one-dimensional compression property of sand-clay mixed soils, *Proceedings of Japan Society of Civil Engineers*, No.448/III-19, pp.121-130 (in Japanese).

Omine, K. and Ohno, S. (1997) : Deformation analysis of composite ground by homogenization method, *Proc. of 14th International Conference on Soil Mechanics & Foundation Engineering*, pp.719-722.

Omine, K., Ochiai, H. and Yoshida, N. (1998) : Estimation of in-situ strength of cement-treated soils based on a two-phase mixture model, *Soils & Foundations*, Vol.38, No.4, pp.17-29.

Reuss, A. (1929) : Berechnung der Fließgrenze von Mischkristallen auf Grund der Plastizitätsbedingung für Einkristalle, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol.9, pp.49-58.

Voigt, W. (1889) : Ueber die Beziehung zwischen den beiden Elasticitätsconstanten isotroper Körper, *Annalen der Physik und Chemie*, vol.38, pp.573-587.