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Limit state design in geotechnical engineering

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1. Development of design procedures

DURING THE PAST DECADE a new fashion has emerged in the presentation of design calculations demanded by codes of practice in the field of structural engineering. The roots of this innovation may perhaps be traced to the widespread rethinking of the whole design process which accompanied the post war boom in construction, demanding as it did the accelerated application of new technology and new materials on a vast scale. In 1951, for example, the Institution of Structural Engineers set up a committee under the chairmanship of Sir Alfred Pugsley to report on safety in structural design. Their 1955 report[1] began by defining some basic requirements:

"The following requirements . . . should be satisfied to a reasonable degree of probability:—

(*a*) That the structure shall retain throughout its life, the characteristics essential for fulfilling adequately the purpose for which it was constructed, without abnormal maintenance cost.

(*b*) That the structure shall retain throughout its life an appearance not disquieting to the user and general public, and shall neither have nor develop characteristics leading to concern as to structural safety.

(*c*) That the structure shall be so designed that adequate warning of damage is given by visible signs; and that none of these signs shall be evident under design working loads".

The report continued by noting that "the main body of evidence regarding the safety of a structure . . . will usually take the form of design calculations", and it proposed that two particular ratios should dominate the discussion:

"(*a*) The ratio of the ultimate load to the appropriate working load, known as the *ultimate load factor*.

(*b*) The ratio of the limiting load to the appropriate working load, known as the *limiting load factor*".

The "ultimate load" was identified as that causing collapse, while the "limiting load" was intended to define the onset of "excessive elastic deflections, limits to which may be set by aesthetic considerations or by some resulting interference with the proper use of the structure, (similar) permanent deflections, (and the) development of local defects, such as cracks . . . ".

The committee went on to quantify, by reference to the relative probability of collapse and the seriousness of its consequences the particular "ultimate load factors" which they thought would be appropriate. This quantification was made in terms of relative excellence in three areas: workmanship (inspection, maintenance and materials), loading (control), and accuracy of analysis.

Each attribute was required to be assessed as "very good, good, fair or poor" and was invested with a correspondingly increasing partial factor. Likewise the consequences of collapse, in terms both of loss of life and loss of property, were categorised as "not serious, serious, or very serious" and made the subject of further factors. The required global "ultimate load factor", calculated by extended multiplication, took values between 1.1 and 6.3. No such quantification was attempted with the "limiting load factor".

In the intervening years, this re-evaluation of the role and style of the structural code-drafter has proved most influential. Following pressure from joint European committees[2], the British adopted CP110 The Structural use of Concrete in 1972: it was coined in the terminology of limit states, which had effectively been laid down in the 1955 report. Any condition which a structure might attain which contravened the basic requirements quoted earlier was designated a limit state. The notion of collapse at ultimate limit states was preserved, as was the concept that partial factors could be assigned on various grounds to reduce the probability of experiencing one. The "limiting load" concept from the 1955 report was relabelled in terms of serviceability limit states in which the structure was supposed no longer to be fit for its purpose, usually on the grounds of deformations or cracks which the owner would find to be intolerable.

The most important innovation in CP110 was the explicit use of probability theory in the selection of 'characteristic' values of strength which would, according to some notional or measured distribution, be exceeded in at least 95% of standardised samples. This foothold for probability theory has by now been converted into a bridgehead. In 1977 CIRIA Report 63 on the rationalisation of safety and serviceability factors in structural codes[3] clearly took the view that reliability analysis held the key to the rational selection of partial factors and implied that redrafted codes such as CP2 Earth Retaining Structures should be drawn into the fold of semi-probabilistic limit state codes which now include water-retaining structures, steel structures and bridges.

In the present year, a European committee is to be established to create a modern limit-state framework for design codes in geotechnical engineering, as a guide to national code-drafting authorities. We are therefore entering a critical period in which a concerted input of effort will be required in order to avoid pivotal philosophical mistakes which could so constrain designers as to affect the degree of security of soil constructions in Europe for many years.

2. Limit states in geotechnical engineering

New technologies often generate a class of events which were hard to preconceive. Localised liquefaction sand boils, for example, have been seen to erupt from beneath the bases of gravity oil production platforms during cyclic storm loading; these events were rather different from the catastrophic liquefaction scenarios which had been envisaged. On a less exalted scale, the possible collapse mechanisms of reinforced earth constructions have severely tested the imaginations of designers, even in the absence of dynamic forces.

In an analogous fashion there is a social pressure on engineers to react more intelligently to the foolishness of man and the power of an untamed nature, by encompassing within their designs an allowance for accidental impact, explosion, earthquake and flood. Even these occasional and violent limit state events, therefore, are to be subject to some prior thought.

These new requirements clearly demand a fresh approach: there is often no empirical track-record to rely on. If limit state concepts are to be applied in geotechnical engineering they should therefore be of maximum use in these development areas. The well-established technologies of foundation and slope engineering are less likely candidates for reshaping, at least in the first instance, since the evolutionary processes of trial, error and competition must already have achieved a degree of optimisation. Limit state design for run-of-the mill constructions is more likely to be acceptable when its record of success in new fields has been established.

Although limit state *events* may firstly be experienced at field scale, they are usually far too expensive to research at full scale. The clarification of the *mode* of behaviour, including our understanding of the *parameters* which were involved, has usually been possible only by modelling some aspects of the limit state event in a laboratory. Only when a large number of limit state events has been modelled, corresponding to a wide variation in the values of all parameters, is it possible to validate some design *envelope* of values within which limit state events should not occur. The word validation in this context, implies only that a formula has been subjected to a trial in which a reasonable attempt has been made to disconfirm its safety, without success.

3. Uncertainty

The research of a safe envelope is usually an entirely deterministic affair, in which the greatest possible care is taken to control and measure every parameter of a carefully engineered system. The application of these hard-won concepts to a real field-scale problem therefore suffers from two sorts of uncertainty:

(*i*) System uncertainty, that not all limit state modes are well understood even if the materials on site were perfectly mapped, and that even a high-grade site investigation may leave undetected some crucial stratum or unusual feature which may participate in a limit-state event, and

(*ii*) Parameter uncertainty, recognising that the attempt to define the value of a property by the expedient of recording spot values from a number of samples will result in a distribution, necessitating some choice.

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It is vital to appreciate that the classical use of probability theory in estimating the reliability of processes is restricted to parameter uncertainty, and that the general attitude of many structural engineers since the war has been to suppose that structural systems have been completely determined. The danger of complacency on this score is brought out by a paragraph in CIRIA Report 63, defining its scope:

"A majority of structural failures occur because of causes other than inadequacies in the safety margins, whereas structural costs are related directly to such margins. The notional risk assessments provide no direct indication of total risk, which, from examination of case histories, is shown to be of the order of ten times the probability of failure calculable by reliability theory. It is therefore advisable to consider separately the influence on safety of control procedures to be associated with any new code. This aspect of reliability assessment has not been a subject of this study".

The implication here is that 90% of failures occur because the design calculations were irrelevant to the structure which was actually created. No doubt the authors had in mind errors such as the accidental transposition of top and bottom steel in a concrete slab. We may, perhaps, be forgiven for considering also the progressive collapse of the Ronan Point flats following a gas explosion, the collapses during construction of the Milford Haven and Yarra box-girder bridges, or the saga of high alumina cement. If 90% of structural failures are incapable of prevention by classical reliability theory, to what percentage of geotechnical failures will the theory be useful?

J. M. Keynes wrote on page 343 of his Treatise on Probability[4], "If we are given a penny of which we have no reason to doubt the regularity, the probability of heads at the first toss is $\frac{1}{2}$; but if heads fall at every one of the first 999 tosses; it becomes reasonable to estimate the probability of heads at the 1 000th toss at much more than $\frac{1}{2}$. For the *a priori* probability of its being a conjuror's penny, or otherwise biassed so as to fall heads almost invariably, is not usually so infinitesimally small as $(\frac{1}{2})^{1000}$".

D. I. Blockley, on page 172 of his more recent book[5] on structural design, put the problem even more succinctly,

"It is clear therefore that if we wish to measure system uncertainty we should not do so using probability theory".

System uncertainty in geotechnical engineering refers to doubt concerning the hypothesis which the engineer develops concerning the disposition of earth, rock and water and their projected behaviour.

There are always an infinite number of tenable hypotheses. Popper[6] discusses his uncomfortable finding that it is only the vaguest hypotheses which have a high probability of being absolutely true. The most useful hypotheses are rarely the most probable: they usually possess a high information content, however, which makes them easily testable.

An analogy would be the comparison between the usefulness of a tool-box containing only a single committee-compromised all-purpose tool which had a certainty of being used albeit to questionable effect, to that of a conventional tool-box containing a number of small well-defined instruments each of which had a low probability of being used on a particular job but a certainty of being useful on those specific tasks for which they had been designed.

It is demonstrated below that the overwhelming majority of uncertainties in geotechnical design are of the geotechnical system, rather than of its parameters, and that the automatic application of statistical methods of any sort is fraught with danger and paradox.

4. Interpreting geotechnical data

4.1 Permeability k

Suppose that a number of 100mm diameter rock cores have been obtained from a succession of boreholes along the line of a tunnel which is to pass beneath a tidal estuary, and that an estimate of inflow is required. Should some mean value of the laboratory permeability of the cores be used in conjunction with an appropriate flow regime? If the engineer feels that the contribution of fissure flow in the rock could be significant, he would be aware that any treatment of intact specimens would lead to an underestimate of the flow rate.

If his experience or research tells him that his rock has been identified in previous excavations as a relatively porous permeable acquifer with insignificant joints and possessing strong stratification then he may follow the classical approach of estimating the horizontal permeability of the ground at each borehole by the weighted *arithmetic mean* of the samples $k_a = \sum_{r=1}^{n}(k_r d_r)/\sum_{r=1}^{n} d_r$ and the vertical permeability by the *harmonic mean* of the samples $k_h = \sum_{r=1}^{n} d_r/\sum_{r=1}^{n}(d_r/k_r)$.

If there was evidence that the stratification was not horizontal, these inferences would of course have to be altered slightly. If, on the other hand there was evidence that the ground simply comprised erratic and variable patches, like a plum pudding, it has been suggested that it could be treated as isotropic with a permeability given by the *geometric mean* $k_g = (k_1\ k_2\ k_3\ k_r\ k_n)^{1/n}$ of the random samples, though this is by no means conclusive.

Each of these statistical strategies relies on there being an adequate density of sampling, and in these circumstances adequacy should be defined in terms of an interval between samples much less than the diameter of the tunnel. For if the sampling was less frequent than this, a single extreme value of large permeability could, in association with an assumption of stratification, indicate that the tunnel could pass at that section through a substantial acquifer with that extreme value of permeability, which ought not to be averaged with smaller values in any way whatsoever. If the engineer was concerned that his decision on this matter was too badly informed, he would presumably demand in-situ pumping tests at some locations. He would then, it should be noted, be testing his hypothesis of the ground system, rather than blindly accumulating more data.

Of course, in some circumstances, an engineer will be faced with a risky decision which cannot be underpinned by further testing. De Mello[7] cites the example of the possible inclusion, within the supposedly impermeable clay core of an earth dam, of a thin layer of permeable and erodable silt which could be placed at a time when inspection was lax. Only a strongly designed measure such as a steep blanket filter-drain passing behind the core can counter this quite predictable limit state condition. De Mello speaks of the need for physical intuition in facing and avoiding extreme-value problems of this sort and observes "how fallacious to engineering conclusions are the frequent probabilistic formulations wherein the author is forced to begin by assuming a certain extreme-value probability density function".

4.2 Compressibility m_v

Suppose that a number of oedometer samples had been extracted from a few trial pits in clay at the green-field site of a proposed trade warehouse requiring many columns, and that three boreholes had also been taken to sound rock, affording the possibility of conducting further tests on the samples which had been recovered. Once again, the statistical strategy that should be adopted to deal with the presumably scattered data must reflect the engineer's hypothesis on whether the ground is to be treated as stratified, or heterogeneous.

An arithmetic mean compressibility for the whole soil profile, weighted according to the sampling interval after the fashion of the previous permeability example, might be appropriate to the consideration of average settlement following groundwater lowering after general drainage, but only if the ground were stratified rather than lumpy. An arithmetic mean weighted according to sampling interval and inversely according to some power of the ratio of its depth to the width of the footing might similarly be appropriate to an estimate of the average settlement of a footing.

Of course, the critical design problem is concerned with the possibility that some particularly soft or hard location could be a seat for differential settlements and distortions. If the engineer has omitted to site the trial pits such that some were where the grass looked at its greenest, and some were where it was at its driest, it is unlikely that any statistical process on the data from other locations will offer much of value. One rational course would therefore be to attempt to find the wettest and driest locations and use whatever *extreme values* emerge from the associated tests.

An alternative statistical method would be to fit some analytical probability distribution around whatever data had been achieved, and to allow the automatic process of extrapolation to fix an extreme value, although it should be noted that this strategem contains the hidden hypothesis that compressibility of a sample is statistically independent of the compressibilities of neighbouring samples, which seems unreasonable. The next task would be to decide whether the population of compressibilities should be normally distributed, or whether the logarithm of the compressibility should be normally distributed, or whether some other distribution function should be chosen having special regard to extreme events. This decision, which crucially affects the outcome, can hardly be taken scientifically since it is almost by definition untestable. The final step would be to use estimates of the mean m and standard deviation s of the variate v in order to declare an extreme value that would only be exceeded rarely. If there was a normal distribution of v, the following probabilities would apply:

$v > m + 2.33\ s\ :\ p = 10^{-2}$
$v > m + 3.09\ s\ :\ p = 10^{-3}$
$v > m + 3.72\ s\ :\ p = 10^{-4}$
$v > m + 4.26\ s\ :\ p = 10^{-5}$
$v > m + 4.75\ s\ :\ p = 10^{-6}$
$v > m + 6.0\ s\ :\ p = 10^{-9}$
$v > m + 7.0\ s\ :\ p = 10^{-12}$

Although it is tempting to select one of these values, it is worth recollecting that the geotechnical processes which can lead to soft spots in clay are well-ordered and reliable, and are unrelated to the mathematical decay of probability listed above. And is the probability greater or less than 10^{-9} that a site investigation and construction process with average supervision will result in a footing being built over a backfilled trial pit? Will such an event be more likely if the engineer thinks probabilistically? Clearly, these attempts to use probability theory to *supplant* judgement must result in a diminution of good sense, and an increase in hazard.

4.3 Undrained strength c_u

Consider the data of a sequence of triaxial tests conducted on a stiff fissured clay for the purpose of establishing its short-term stability in a cutting. The distribution of strengths is likely to reflect the distribution of fissures. One rational hypothesis would be to assume that the weakest sample betrayed the weakest fissure orientation, and that the full-scale cutting might provide an opportunity for the ground to find a pattern of fissures at some location which possessed a similar weakness on a large scale. Unless this extreme value hypothesis can be *proved* to be in error, it must be preferred to any statistical averaging process. Critical state soil mechanics[8] might offer an equally powerful group of hypotheses. The clay in the fissures must surely be at least as strong as if it were a remoulded virgin material at the same depth, tested undrained. Furthermore, the fully drained strength of a dilatant overconsolidated soil subject to stress-relief must be an underestimate of its short-term strength. Once again it is the choice of a reasonable hypothesis, using all available knowledge, which is at the heart of the problem.

It has been suggested by Attewell & Farmer[9] that Bayesian statistics may be applicable to the problem of hypothesis selection. They quote the case of a projected tunnel which is likely to be driven through 'good' ground, although there is some 'bad' ground in the vicinity. The engineer supposes that the probability of encountering 'bad' ground is 10%. He then notices that a house over the line of the tunnel exhibits unusually bad cracking. He estimates the probability of cracking supposing bad ground at that location to be 95%, but only 25% on the assumption of good ground. A Bayesian calculation then revises his initial estimate of the probability of encountering bad ground in the tunnel heading, increasing it to 30%. This is wonderfully irrational! It supposes that the weight of prior evidence leading to the 10% estimate (which in practice is presumably derived from one borehole some way off the route) is equivalent to that of observing a collapsing building right above the route. Any engineer who failed to send out a drilling rig to explore the soil beneath such a building, or who delayed his decision until he had done some probability estimates, should be considered seriously unbalanced. It is very difficult to replicate symbolically the subjective judgement of an experienced man who has an unconscious system of belief, making use of a variety of both weak and strong correlations with empirical observations. The first step is to ask whether his thought process can be written down. Failing this, probability calculations are superfluous and irrational.

4.4 Angle of shearing resistance ϕ'

In A Guide to Soil Mechanics[10] the author demonstrated that a wide spectrum of strength data from both sands and clays could be construed in the following manner:

	densest	loosest
peak strength	$\phi'_{crit} + 20°$	ϕ'_{crit}
ultimate strength*	ϕ'_{crit}	ϕ'_{crit}

(*The effective angle of shearing resistance of a true clay can, of course, be further reduced below its critical state value ϕ_{crit} to its residual value ϕ_r', by the agency of relative sliding on a thin rupture surface, analogous to a mechanical polishing process.)

The increment of about 20° (depending on the soil type) up to the peak angle of a 'dense' soil in plane strain is due to dilatancy, and reduces to zero if the potential dilatancy is reduced either by reducing the initial density or increasing the confining pressure so that a critical state is approached, or by cycling the stresses: it is also reduced by a factor of about two-thirds in a triaxial test.

Now consider a widespread stratum of sand which in location *A* is uniformly loose and in location *B* is mainly dense but with occasional loose pockets. From the standpoint of a routine probability analysis the following sort of decision making may emerge as typified in CIRIA Report 63[3] para. 7.5.11:

peak ϕ'	location *A*	location *B*
mean m	34°	44°
standard deviation s	1°	5°
design ($m - 4.75\ s$)	29.35°	20.25°

This obviously silly result can be corrected only by recognising that the critical state angle, which could be roughly 30° in this example, is a properly pessimistic estimate of the shearing resistance at either location, always assuming that the sand really was present. The critical states of a soil should dominate the discussion of the prevention of collapse of constructions in soil with an uncertain potential for dilation. The angle ϕ_{crit} for a wide variety of sands and silts lies in the narrow range 29° – 34°, while the angle for many clay minerals is about 10° lower.

4.5 Water pressures

The deformation of a soil skeleton is held to be solely due to changes of effective stress. The pore pressure in a soil is therefore central to any discussion of its behaviour. The problem will be to select a hypothetical 'worst' distribution of pore pressures appropriate to the various stages of construction and utilisation which the ground profile must tolerate. *Accurate* hypotheses will stem only from an intimate knowledge of local hydrology, seepage conditions, pore pressures due to shear, rates of loading or load cycling and rates of dissipation and consolidation. *Pessimistic* hypotheses may suffice, however: for example, the groundwater may often be taken at ground level in low-lying sandy soils subject only to static loads. Whatever mechanisms are involved, it is fortunately very much less likely on most sites that the free water surface is 1m above ground level, than 1m below ground level. It is, however, not always easy to arrive even at a pessimistic bound in cases where pore pressures are subject to dynamic loads: fundamental mechanisms remain to be clarified.

5. Selecting limit state modes

The degree of confidence that may rationally be felt in the accuracy of limit state modes used in soil mechanics cannot be very high. Far too few, having been observed, have been made the subject of a scientific enquiry. Many designers still rely in their decision-making on small laboratory model tests conducted by prominent engineers of the last generation. It is now recognised that sand under small confining pressures dilates very strongly, enhancing its peak angle of shearing resistance and especially modifying its collapse mechanism.

Our methods of measuring the soil strength of small elements have improved dramatically since the war, so that it is unlikely that a modern soil laboratory would even reach an agreement on the value of ϕ' invoked in many venerable experiments. Neither has a very wide spectrum of limit state events been investigated. In the past many research workers have contented themselves by conducting repeatable experiments irrespective of whether limit state events were being observed. Even where model collapses have been promoted it has been usual to avoid problems of soil-structure interaction, and dynamic or three-dimensional loading, and to use especially simple soils in the absence of pore fluids. The continuing development of centrifugal model testing now affords a significant improvement in the realism of limit state events depicted in models.

6. Limit state envelopes

The theoretical envelopes which are used to guard against collapse modes have been developed within the framework of the theory of plasticity. Two simple styles have developed, the kinematical method popularised by Coulomb and the statical method advocated by Rankine. When dealing with perfectly plastic materials these alternative approaches can be proved to provide upper-bound and lower-bound estimates of the collapse load, respectively.

Although this provides an extremely useful starting point, no geotechnical designer can hide behind the theorems for long since his materials are always less than perfect. It is not often pointed out that the use of any angle of shearing ϕ' greater than ϕ_{crit} in a collapse calculation can hardly be supported on *theoretical* grounds. If such an angle *were* to be used its safety would therefore have to rest solely on experimental evidence good enough to convince the designer that no foreseeable events would cause sufficient strain-softening on a thin rupture surface to reduce the soil to its critical density.

Clearly Rankine's statical method is to be preferred if it can be said to be inherently pessimistic, and if kinematical methods are available to check that its solutions are not grossly conservative. It is particularly valuable when such pessimistic steps as effacing any supposed friction on various surfaces leaves a soil construction in an easily determined state of stress.

Consider, for example, Fig. 1 which depicts a reinforced concrete L-wall. The pessimistic system of Fig. 2 is sufficiently simple to be easily determined. The passive zone of contact in front of the wall has been removed on the assumption that a narrow service trench may be required

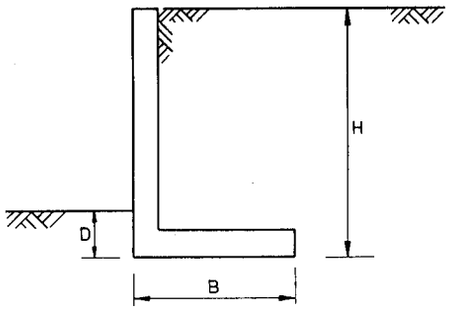

Fig. 1. Idealised L-wall

there at some time. The assumption of frictionless wounds along *QN* and the top of the base allows the stresses under the foundations and on the cantilever stem to be derived. A uniform base stress has been inferred, inclined at δ. If any surcharge were to be carried, it would be imposed as an equivalent blanket over *QR* so that it caused the maximum destabilising effect.

Three families of collapse modes must be considered: a deep seated slip through remote soft zones, the structural failure of the reinforced concrete, and a monolithic failure with limiting soil stresses along *QNL*. The stresses being completely determined means that the structural collapse calculations for bending and shear in the stem, and bending with shear and tension in the base, are relatively straightforward. The proper end product of the monolithic collapse calculations should be the ratio *H/B* required to promote failure.

If the present example is restricted to the case of the gross movement of a monolith founded on, and backfilled with, a dry granular material with a unique value of ϕ', the requirements of plastic equilibrium are:

horizontal $\gamma BH \tan\delta = \frac{1}{2} k_a \gamma H^2$ ... (1)

vertical $\frac{1}{2} N_{\gamma\delta} (\alpha B) \gamma_{sub} + N_{q\delta} \gamma_{fill} D = \frac{\gamma H}{\alpha}$ ...(2)

moment $\gamma BH \frac{(1-\alpha) B}{2} = \frac{1}{6} k_a \gamma H^3$ ... (3)

Although equations *1*, *2* and *3* would normally be turned into "factors of safety" by dividing their left-hand sides by their right-hand sides, such a step forms no part of a logical analysis of limit states. We have three equations and three unknowns (B, δ and α) in terms of two soil parameters γ and ϕ' (or derivatives of ϕ' such as $k_a = (1-\sin\phi')/(1+\sin\phi')$, N_γ and N_q) both of which must be treated as predictable, and geometrical parameters *H* and *D* which can be treated as fixed.

Of course, the equations are transcendental, but a programmable calculator can be used to explore the ratios *H/B*, *H/D* which lead to collapse; an iterative method can provide all the sensitivity analysis that is needed in a matter of minutes. Eqn. *2* offers the greatest obstacle to the rational analysis of collapse scenarios, since it must be recognised that the bearing capacity factors N_q and N_γ are functions of the inclination δ of the base stress. Meyerhof for example proposed that the factor N_γ be reduced by $(1-\delta/\phi')^2$ whereas N_q should be reduced less significantly by $(1-\delta/90°)^2$. In addition, it may be difficult to prevent the groundwater table from rising up to foundation level on some future occasion, thereby effectively reducing γ_{sub} to $(\gamma-\gamma_w)$: a relatively cheap but robust drainage system can be relied upon above that level, however.

For walls which are less than oversquare ($H \not< B$), it transpires that it is reasonable to neglect the N_γ term in eqn. *2*, for the purpose of calculating limit states. This is convenient, because a pessimistic plastic analysis[10] exists for N_q under inclined loads, offering:

$$N_q = \frac{(1+\sin\phi'\cos 2\psi_o)}{(1-\sin\phi')} \exp\left[2\tan\phi'\left(\frac{\pi}{2}-\psi_o\right)\right] \quad \ldots (4)$$

where $2\psi_o = \delta + \sin^{-1}(\sin\delta/\sin\phi')$

A set of solutions to these eqns. *1*, *2*, *3*, and *4* were obtained with a Texas Programmable 58 calculator, and are presented in Fig. 3 in terms of envelopes of ratios *H/D*, *H/B* within which collapse limit states should not exist. For comparison purposes, the spot values which would be created by a conventional method are also shown. The assumptions were

$\tan\delta \not> \tan\phi/1.6$ ... (5)

(σ_v) max $\not>$ ultimate bearing capacity/2 ... (6)

middle third rule, $\frac{e}{B} \not> 1/6$ ... (7)

The conventional method evidently achieves a fairly uniform degree of safety, although the practitioner may be surprised to see that the margin expressed as a reduction in ϕ' to promote collapse may be as small as 6°. If the designer forgot to apply the reduction factor on vertical bearing capacity which is the consequence of the shear component across the base of the wall, this margin of safety would effectively disappear. It would be useful at this point for a designer to check kinematic-

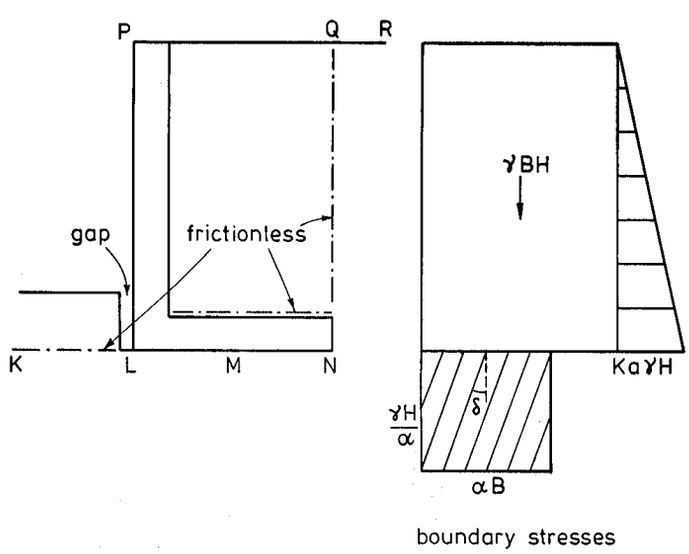

Fig. 2. Pessimistic representation of L-wall

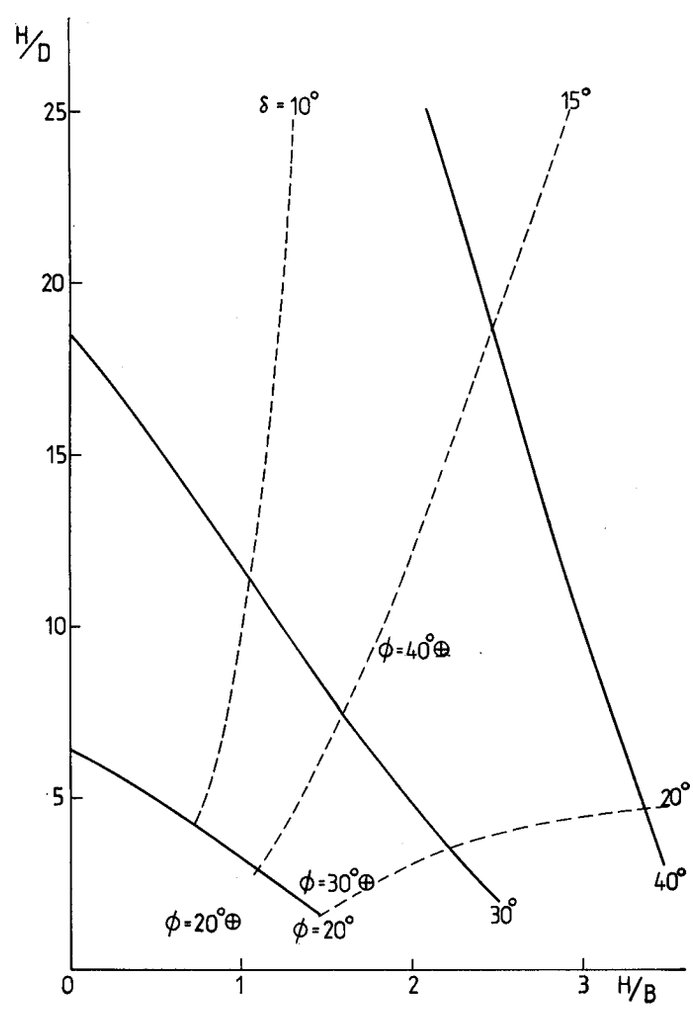

Fig. 3. Collapse envelopes for L-walls compared with conventional designs

ally admissible slip surfaces to ascertain that the gap between pessimistic and optimistic forecasts was not too large. It would then be obvious that the most critical uncertainty in the application of the envelopes of Fig. 3 lies in the selection of an appropriate value for ϕ'.

The mode of thinking of a limit state devotee might be thus:

(*i*) Whatever degree of compaction is achieved behind the wall, and however compact may be the subgrade, it would be wrong to design a wall which would collapse if extraneous ground movements promoted softening in rupture bands. Therefore take $\phi' = 30°$ if this corresponds to an estimate of its critical state angle of shearing resistance.

(*ii*) The geometry $H/D = 5$, $H/B = 1.97$ would not fail monolithically if $\phi' = 30°$ all around the monolith. This point on the envelope accords roughly with the minimum perimeter (proportional to $H + B$) required to provide a given grade separation $(H-D)$; in other words, the accumulated length of the substructure (D + B) is then at its minimum value of $0.7H$.

(*iii*) If the condition $H/D = 5$ is to represent a future limit state event, a decision must be taken on the value to be selected for D in the ideal condition after construction. If, for example, the surface on the lower level is to be paved, it would be sensible to notionally remove the pavement before assigning depth $D = 0.2H$.

The limit state event which has been forestalled is then typified as the removal for repair of a lower level road construction, and the construction of a further narrow service trench down to the base of the foundation, during reinstatement following an episode of long-wall mining subsidence which had coincidentally promoted the settlement of an active wedge of backfill, with localised softening after dilation.

The conventional scheme might, by contrast, proceed thus

(*i*) Insist on at least 92% of the dry density achieved after optimum compaction of a selected granular material. Guess that an average peak value for ϕ' might be 38° in a triaxial compression test, and use 35° for design.

(*ii*) Use the conventional safety factor described in eqns. *5, 6* and 7 above to achieve proportions of $H/D = 4.6$, $H/B = 1.92$.

(*iii*) Put in hand sufficient site investigation and testing to ensure that average values for ϕ' should indeed exceed the value of 38° which was presumed.

Although the wall profile produced by the different approaches is almost identical, the conventional logic is deficient in so far as it fails to define the conditions which would promote collapse. The most obvious practical consequence is that the detail required in the site investigation for the foundation, and in the control of the density of the backfill, is amplified at great cost for no good reason. A limit state designer who had inferred critical state soil strengths in a future limit state event would be indifferent, regarding overall stability, of the degree of compaction which had been achieved at the end of construction. He would concern himself with serviceability limit states in a later, and quite distinct, calculation which might relate the degree of soil strain to the detailing of the wall face, the estimation of maintenance requirements, and the desirability of restricting surface settlements.

He would then be in the position to specify any degree of soil compaction appropriate to the relative economic significance of the possible malfunctions, without prejudicing the *safety* of the construction. Limit state envelopes, after the fashion of Fig. 3, should be used to enquire into the combination of the soil variables (ϕ', u etc.) and system parameters (H, B, D etc.) which promote each of the conceivable limit state modes. The likelihood that the state of a soil construction will pass beyond one of the limiting envelopes is associated principally with the likelihood that one of the formative assumptions is grossly in error — for example that the wall drainage system will not fail. This is a matter for judgement.

7. Conclusions

7.1 Deterministic calculations based on observable mechanisms offer a more reliable route to decision-making in geotechnical design than do the processes of statistical inference. There is an analogy here with the alternative methods of designing the capacity of reservoir spillways based either on the manipulation of a storm mechanism or the extrapolation of stream-flow data. The determinist view in flood hydrology was detailed by Kelway[12]: "(*a*) Selection of the maximum recorded storm in the region

(*b*) Optimisation of the storm mechanism

(*c*) Maximisation of the processes involved in the mechanism

(*d*) Transposition of the storm to the catchment location

(*e*) Orientation of the storm to give . . . the maximum areal fall, over the catchment".

This is to be contrasted with the stochastic method of extrapolating stream flow records taken over a few years or decades to predict floods with longer return periods which are more severe than any on record.

Advocates of this stochastic approach, following a discredited period of over-enthusiasm in which frequency distributions were pulled out of the air and in which outrageous extrapolations were often made, now recognise that it is unsafe to extrapolate a set of stream flow data taken over N years to the problem of floods which are supposed to recur less frequently than $2N$ years. By the regional pooling of all available data, and by the use of carefully researched historical records, it is thought that estimates of the 500 year or perhaps the 1 000 year flood may empirically be made. The collection of data is universally recognised to be paramount.

Similar schools of thought may develop to deal with the estimation of soil parameters. It must be recognised

(*i*) That the *mechanisms* involved in the behaviour of soil bodies are more easily researched, and comparatively well-understood, than is the case in climatology or hydrology.

(*ii*) *The variability* of soil conditions in Great Britain exceeds by many orders of magnitude the variation in rainfall, this variability being still evident within almost any given 10km grid square on a geological map. The use of probability analysis would therefore demand many orders of magnitude more data than is available to the hydrologist: the state of affairs is, however, that soil data is somewhat sparser than hydrological data.

Only in regions which possess an extremely simple geology should design authorities consider basing the values of some parameters on the prior evidence of a pool of data. They would still be vulnerable to erratic strata and misunderstood mechanisms: the problem of geotechnical design *is* the extreme value problem. It is to be expected, therefore, that the stochastic school of thought will be much less influential in geotechnology than has been the case in hydrology.

7.2 The avoidance of well-identified limit state events offers a methodology which ought to be as attractive to designers as it is to research workers and teachers. Fundamental thinking often results in a design similar to that wrought simply by a process of natural selection of those forms which were fittest to their task. Perhaps it would have been more surprising if this were not so. This evidence can obviously not be used to fault either fundamental or empirical design methods: rather it suggests a means whereby practical methods and technologies may evolve by a process less dependent on trial and error than might otherwise have been the case.

7.3 The principal requirements of a limit-state designer are five-fold:

(*i*) Attention must be paid to the imaginative selection of systems and parameters relevent to a given construction. The omission of a limit-state mode, such as the potential cracking of a rolled silt core in an earth embankment, will not in general be rectified by the application of large safety factors against those limit-modes that *have* been recognised.

(*ii*) The possible variation of each parameter must be considered. Facile concepts, such as the supposed stochastic independence of parameters in a limit state calculation, which is necessary to a simplistic reliability analysis based on probability theory, should be regarded with suspicion. As soil approaches collapse it strain-softens on shear surfaces while cracks open up in zones of tension. Experienced designers allow for strengths below peak in an imagined progressive failure, and are free to invoke water pressures in joints and cracks. It is clear that the mass permeability of a soil construction and the strength of its soil constituents, are functions of the remoteness of various limit states and are therefore not independent of the other variables.

A stochastic thinker might suppose that if the probability of a high water table behind a gravity wall was 10^{-2} per year, and if the proportion of backfills which had been observed to have softened to a critical state was 10^{-1}, then the probability of encountering both together was 10^{-3}. A limit-state thinker would be aware that zones of dilation and softening could be an adjunct to groundwater pressures large enough to destabilise the wall, and would therefore invoke both conditions simultaneously in his design calculations. Extreme values of ground parameters characteristically occur simultaneously. Even limit-state *modes* are not necessarily independent and can trigger each other, as happened at Aberfan.

The increasingly popular practice of applying pre-judged partial factors to parameters such as the soil strength are based on stochastic reasoning and are therefore likely to be invalid. Fortunately there exist critical state parameters which discount the strength component due to dilatancy, which is the variable element in the shearing resistance of elements of a given soil.

In many situations the designer will be able to discover a physically meaningful

limitation to the variation of an uncertain parameter. Where this is not possible he will be forced to rely on frequent sampling and a hypothesis regarding the spatial or temporal distribution of the property concerned.

(*iii*) Site investigation techniques must be used not simply to generate accurate spot values of parameters but also to test hypotheses. Geophysical methods are increasingly useful in the confirmation of the existence of defects or erratic conditions, such as peat lenses in fluvio-glacial sands, solution cavities in chalk, or clay gouge on bedding planes in limestone. A paradoxical danger of the probabilistic view of sampling is that the increased cost of data gathering may squeeze out geophysical methods which offer a more fuzzy appreciation of the whole site but which also offer the engineer his best chance of validating the hypotheses which underpin his probabilistic analysis.

(*iv*) Collapse calculations based on stress analysis which is known to be pessimistic should be used in preference to kinematic techniques such as wedge or slip-circle analysis which is recognised to be inherently optimistic. Kinematically admissible mechanisms are useful in determining the possible degree of conservatism in their statical counterparts.

Where good statically admissible stress fields are not available, it is necessary to optimise mechanism geometries in order to achieve a reasonably low upper bound solution: it would then be wise to test out the theory by conducting centrifugal model tests. The techniques of achieving tight bounds to plastic collapse problems, and of calibrating approximate methods against centrifugal model data have recently been discussed[11,13] in relation to soft-ground tunnelling.

(*v*) Serviceability calculations ought increasingly to be based on deformation parameters, and in particular on soil-structure stiffness ratios[10]. There is a need for approximate small-strain mechanisms which feature zones of soil straining in simple patterns, which are broadly compatible with the deformation of associated steel or concrete constructions. Finite element analyses are presently too cumbersome and costly for general purposes other than the calibration of back-of-the-envelope approximations, or the generation of tabulated solutions.

Conventional design methods often contain large factors against collapse which are said to ensure that yielding does not take place: elastic soil strains are frequently considered to be negligibly damaging. Design methods would be considerably clarified if these factors were called 'serviceability factors' rather than 'factors of safety', and if the prediction of damaging deformations was kept separate from the more serious concern of avoiding collapse.

Finally, more attention ought to be paid to ductility and continuity in soil constructions, following Pugsley's basic requirement (*c*) quoted at the outset. This might now be rephrased: every conceivable collapse limit state should be preceded by a serviceability limit state which would offer sufficient time for an appropriate evacuation to take place.

7.4 The consequential role of a code of practice might be

(*i*) To list the limit state events which are known to apply to given forms of construction, ideally marrying each one to a photograph or sketch, and providing an acceptable mechanistic description of its antecedents.

(*ii*) To offer a guide to the selection of pessimistic soil properties based on their critical states, and including advice on deteriorations such as the cracking of clay and the blocking of filter drains, with further guidance on the criteria to be used in the selection of limiting groundwater pressures.

(*iii*) To set out the role of the site investigation in terms of attempts to refute successively more refined prior hypotheses which the engineer has made, using in sequence: geological maps and records, site surveillance with occasional trial pits, continuous sampling or probing, and the recovery of specific samples or in-situ testing at specific locations. The more significant the construction, the more worthwhile it will be to conduct a careful design based on expected soil properties elicited in a high-quality site investigation. Inexpensive constructions whose occasional premature failure could be tolerated could go ahead if the first two phases of the investigation uncovered no surprises, and if the designer is prepared to use pessimistic soil parameters.

(*iv*) To offer, for constructions of various types to fulfill a given function, a series of design charts in the form of envelopes of variables within which given modes of collapse should not occur, so that the time spent on the arithmetic of a particular geometry and a particular collapse mode can be minimised and the time spent on selecting the correct form of construction and its most critical collapse mode can be maximised.

(*v*) To list separately whatever design concepts, details or calculations are considered wise in avoiding serviceability limit states under normal circumstances, but ensuring that observable deformations *will* occur such as to provide sufficient warning of any impending collapse.

(*vi*) To list the observations which may be made during and after construction which would enable an engineer to confirm or refute his principal hypotheses regarding the materials and their behaviour, should this be felt necessary.

References

1. Institution of Structural Engineers (1955): Report on Structural Safety, The Structural Engineer, **33**, May, 141-149.
2. Comite du Beton — Federation internationale de la Precontrainte (1970): International recommendations for the design and construction of concrete structures, Cement and Concrete Association.
3. CIRIA (1977): Rationalisation of safety and serviceability factors in structural codes, Report 63.
4. *Keynes, J. M.* (1921): A treatise on probability. Macmillan.
5. *Blockley, D. I.* (1980): The nature of structural design and safety. Ellis Horwood Ltd.
6. *Popper K. R.* (1977): The logic of scientific discovery. Hutchinson.
7. *De Mello, V. F. B.* (1977): "Reflections on design decisions of practical significance to embankment dams". Geotechnique, **27**, No. 3, 279-355.
8. *Schofield, A. N. & Wroth, C. P.* (1968): Critical State Soil Mechanics. McGraw Hill.
9. *Attewell, P. B. & Farmer, I. W.*: Principles of Engineering Geology, Chapman and Hall.
10. *Bolton M. D.* (1979): A Guide to Soil Mechanics. Macmillan.
11. *Schofield, A. N.* (1980): "Cambridge Geotechnical Centrifuge Operations". Geotechnique, **30**, No. 3, 227-268.
12. *Kelway, P. S.* (1975): Discussion on Reservoir Flood Standards, ICE Flood Studies Conference, p. 90.
13. *Davis, E. H., Gunn, M. J., Mair, R. J. & Seniviratne, H. N.* (1980): "The stability of shallow tunnels and underground openings in cohesive material". Geotechnique **30**, No. 4, 397-416.

Dynamic point resistance

(continued from page 37)

A static point resistance value, to be used in combination with the damping values, has been found from in-situ data derived from a cone penetration test, a pile during driving, and a gravity platform skirt penetrating the soil during installation. This static pile point resistance value is expressed as a percentage of the locally measured cone resistance.

The validity of the proposed point resistance value needs confirmation from more in-situ data, and the question of the shape effect of penetrometer cone/pile annulus needs to be resolved to increase accuracy.

The proposed dynamic point resistance relationship for unplugged open-ended piles for prediction of pile driveability is represented by eqn. 2.

Clay

Data from laboratory tests performed by several investigators prove to correlate well when re-arranged to a scale of the fifth root of velocity, yielding point damping values which decrease with increasing shear strength and decreasing liquidity index. Only for stiff/hard clay do the damping values still need investigation.

In-situ cone penetration test data then serve to provide static resistance data during driving; the complete dynamic point resistance relationship for unplugged open-ended piles for the prediction of pile driveability is represented by eqn. 3.

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References

1. *Heerema, E. P.* (1979): "Relationships between wall friction, displacement velocity, and horizontal stress in clay and in sand, for pile driveability analysis". **Ground Engineering**, January, Vol. **12**, No. 1.
2. *Smith, E. A. L.* (1960): "Pile driving analysis by the wave equation" Transactions of ASCE, Paper No. 3306, Vol. **127**, Part I.
3. *Gibson, G. C. & Coyle, H. M.* (1968): "Soil damping constants related to common soil properties in sands and clays". Research Report No. 125-1, Texas Transportation Institute, Texas A. & M. University, September.
4. *Litkouhi, S. & Poskitt, T. J.* (1980): "Damping constants for pile driveability calculations". Geotechnique **30**, No. 1, pp. 77-86.
5. *Dayal, U. & Allen, J. H.* (1975): "The effect of penetration rate on the strength of remoulded clay and sand samples", Can. Geotech. J. **12**, pp. 336-348.
6. *Terzaghi, K. & Peck, R. B.* (1948, 1966): "Soil mechanics in engineering practice". John Wiley & Sons, New York.
7. *Brinch Hansen, J.* (1968): "A revised and extended formula for bearing capacity". Lecture in Japan, October.
8. *De Beer, E.* (1970): "Experimental determination of the shape factors and the bearing capacity factors of sand". Geotechnique **20**, No. 4, pp. 387-411.
9. *Lunne, T. & St. John, H. D.* (1979): "The use of cone penetrometer tests to compute penetration resistance of steel skirts underneath North Sea gravity platforms". 7th European Conference on Soil Mechanics and Foundation Engineering, Brighton.
10. *De Beer, E.* (1971, 72): "Methodes de deduction de la capacité portante d'un pieu a partir des resultats des essais de penetration". Annales des Travaux Publics de Belgique, *4 5 6*.
11. *Heerema, E. P.* (1978): "Predicting pile driveability: Heather as an illustration of the "friction fatigue" theory". European Offshore Petroleum Conference, London, Paper No. 50, 1978; reprinted in **Ground Engineering**, April 1980, Vol. **13**, No. 3.
12. *Heerema, E. P. & De Jong, A.* (1979): "An advanced wave equation computer program which simulates pile plugging through a coupled mass-spring system". Conference on Numerical Methods in Offshore Piling, Institution of Civil Engineers, London, May.