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A micro mechanical model for overconsolidated behaviour in soils

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ABSTRACT: This paper presents a simple micro mechanical model for the behaviour of overconsolidated soils. We consider the simplest case of isotropic unloading and reloading of an isotropically normally compressed aggregate. The model is based on the definition of a unit cell which contains a rigid kernel representing a large grain, surrounded by a matrix of finer particles. On initial unloading, the model exhibits elastic behaviour followed by "kinematic yielding" which is the onset of sliding of grains. On reloading, kinematic yielding may not occur, but the inhomogeneous stresses may induce "clastic yielding" which is the onset of particle crushing. The model offers realistic hysteresis loops featuring shakedown on cyclic loading.

1 INTRODUCTION

Figure 1 shows a typical plot of voids ratio e versus the logarithm of mean effective stress p' for a soil which has been isotropically normally compressed, unloaded and reloaded. McDowell, Bolton and Robertson (1996) proposed that the micro mechanical origin of plastic compression in soils lies in the crushing of grains. In particular, they showed that the linear-log compression line C-D-I is consistent with the evolution of a fractal distribution of particle sizes. In this case, particle size disparity must be a hidden feature of all constitutive behaviour.

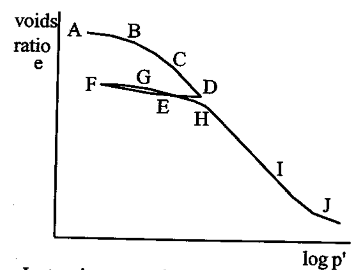

Figure 1. Isotropic normal compression, swelling and recompression.

The unload-reload behaviour D-E-F-G-H which would occur "inside the yield surface" in stress space is assumed (incorrectly) in conventional plasticity models to be entirely elastic. More sophisticated models such as the kinematic hardening yield surface (Hashiguchi, 1993) have proved to be useful in predicting hysteretic behaviour, but give little information about the micro mechanics of deformation. A mean field approach predicts that a monotonically increasing isotropic strain cannot give rise to the sliding of grains, and is of no use here. What is required is a micro mechanical model which accounts for the existence of a fractal geometry and which permits frictional dissipation during global isotropic unloading and reloading.

Consider the arrangement of particles following normal compression to D. The largest particles have the highest co-ordination numbers, since they are in contact with many smaller particles. The smallest particles have the lowest co-ordination and will be the most unstable: in particular, the smallest particles which are in contact with the largest particles will have the lowest co-ordination, so that on initial unloading, they will be the first to move. On further unloading, slightly larger particles will rearrange, and so on. The largest particles are stable and remain relatively undisturbed in the elastic-frictional fractal matrix. It makes sense to define a unit cell which has one of the largest particles at its centre, and which accounts for the percentage by sample volume of such particles: the volume fraction of the largest particles is known to influence the frictional behaviour of the aggregate (Lee, 1992).

2 A 'KERNEL CELL' MODEL

Fig. 2(a) shows the chosen unit cell which is spherically symmetric and consists of one of the largest particles (the "kernel") of radius a, at the centre of a matrix of finer particles. The radius b of the cell is taken as half the mean distance between the centres of the largest particles in the aggregate. The kernel is assumed to be rigid, and the Young's modulus E of the matrix is assumed to be constant. The displacement u at radius r is measured positive radially outwards, and compressive stresses and strains are positive. Incremental strains are related to incremental displacements by the equations:

$$\dot{\varepsilon}_r = -\mathrm{d}\dot{u}/\mathrm{d}r \tag{1}$$
$$\dot{\varepsilon}_\theta = \dot{\varepsilon}_\phi = -\dot{u}/r \tag{2}$$

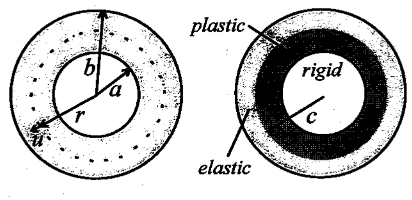

Figure 2. Definition of the kernel cell, showing elastic and plastic zones.

2.1 *First unloading: elastic behaviour*

It is assumed that the fractal crushing which occurred during isotropic normal compression, has left the material "soaking" in a uniform stress field, so that the radial stress σ_r and the circumferential stresses σ_θ, σ_φ are each everywhere equal to the preconsolidation pressure p_O. At all subsequent stages of the unload-reload process, it is assumed that the radial pressure $\sigma_r(b)$ on the boundary of the cell at $r=b$, is equal to the current global isotropic pressure p. On first unloading, the behaviour of the unit cell is entirely elastic. Stress increments in the matrix are related to increments in global pressure by Lamés equations for an elastic matrix containing a rigid kernel:

$$\dot{\sigma}_r = \frac{\dot{p}}{1+k(a/b)^3}\left(1+\frac{ka^3}{r^3}\right) \tag{3}$$

$$\dot{\sigma}_\theta = \frac{\dot{p}}{1+k(a/b)^3}\left(1-\frac{ka^3}{2r^3}\right) \tag{4}$$

where

$$k = 2(1-2\nu)/(1+\nu) \tag{5}$$

and ν is the Poisson's ratio for the matrix. The volumetric strain increment for the unit cell is related to the increment in global pressure by

$$\dot{\varepsilon}_v\Big|_{\substack{UNIT\\CELL}} = 3\dot{\varepsilon}_\theta(b) = \frac{3\dot{p}(1+\nu)k/2\left[1-(a/b)^3\right]}{E\left[1+k(a/b)^3\right]} \tag{6}$$

2.2 *Kinematic yielding*

Yield is assumed to occur according to the Mohr-Coulomb criterion:

$$\frac{\sigma_r}{\sigma_\theta} = K = \frac{1-\sin\phi}{1+\sin\phi} \tag{7}$$

where ϕ is the mobilised angle of friction. This marks the onset of sliding of grains in the matrix, leading to a partial loss of shear stiffness while some normal contact stiffness remains. We describe this as "kinematic yielding", which is more informative than the term "kinematic hardening". By combining (1), (2) and (7), it can be shown that kinematic yielding first occurs at $r=a$. Substituting typical soil parameters, for speswhite kaolin $\nu=0.3$ and $K=0.4375$ at the critical state (Al-Tabbaa, 1988), so that for a soil sample in which the largest particles occupy 20% by volume ($a/b=0.6$), the increase in global pressure to cause yield is $\delta p=-0.486\ p_O$. This corresponds to an overconsolidation ratio of $p_O/(1-0.486)p_O=1.944$, which is of the same order of magnitude as predicted by Al-Tabbaa's two surface model.

After first yield, as the global pressure is reduced, the width of the plastic zone increases as the elastic-plastic interface moves outwards to radius c (Fig. 2(b)). For $c<b$, the total circumferential strain at $r=b$ since initial unloading is related to the total changes in $\sigma_r(b)$ and $\sigma_\theta(b)$ via Hooke's Law. The current radial and circumferential stresses in the elastic zone are given by Lamés equations, subject to the boundary conditions:

$$\text{(i)}\quad \sigma_r(b) = p \tag{8}$$
$$\text{(ii)}\quad \sigma_r(c)/\sigma_\theta(c) = K \tag{9}$$

(iii) compatibility at the interface, $u|_{r=c} = u_c$ has the same value in both the elastic and plastic zones.

The displacement u_c at the interface, following unloading, may be calculated from Hooke's Law and from the deformation of the plastic zone. Consider the assumption that the plastic zone deforms with constant rate of dilation, defined in terms of total strain increments (Hughes *et. al.*, 1977):

$$\delta\varepsilon_r + 2\delta\varepsilon_\theta = -\nu^*\left|\delta\varepsilon_r - \delta\varepsilon_\theta\right| \tag{10}$$

where ν^* is a function of the yield stress ratio σ_r/σ_θ.

Substituting (1) and (2) into (10) and integrating, it is evident that the presence of the rigid kernel ($u_a=0$) ensures that there is no radial displacement anywhere within the plastic zone. The dilatancy rate must therefore be defined in terms of plastic and not total strain increments. Following Rowe (1972) we write:

$$\frac{2\sigma_\theta\delta\varepsilon_\theta^p}{\sigma_r\delta\varepsilon_r^p} = -\frac{1}{K_{cv}} = -\frac{1+\sin\phi_{cv}}{1-\sin\phi_{cv}} \tag{11}$$

where ϕ_{cv} is the critical state angle of friction for constant volume shearing. However, by adopting (11), it is not possible to calculate the stress-strain relation for the kernel cell analytically. We first perform an underestimate of the volumetric expansion of the kernel cell by assuming that volume changes are solely elastic everywhere: there is no extra dilation in the plastic zone. This allows the stress-strain relation for the unit cell to be derived in closed form (McDowell, 1997).

When the kernel cell becomes fully plastic, by assuming that only elastic changes in volume occur, the volumetric strain increment for the cell can be related analytically to increments in global pressure (McDowell, 1997):

$$\dot{\varepsilon}_v\Big|_{\substack{UNIT\\CELL}} = -\frac{3\dot{u}_b}{b} = \frac{3(1-2\nu)}{E}\dot{p}\left[1-\left(\frac{a}{b}\right)^{\frac{2+K}{K}}\right] \tag{12}$$

Figure 3 shows the unloading curves predicted by the kernel cell (assuming no plastic volume changes) for a typical normally consolidated sample of speswhite kaolin. The effect of increasing the percentage by sample volume of large particles is shown by increasing the a/b ratio from 0.6 to 0.8.

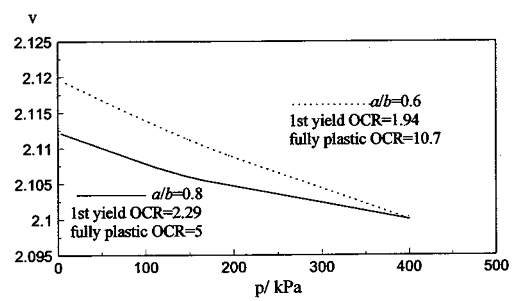

Figure 3. v-p plots showing the effect of a/b ratio.

2.3 *Reloading*

If the aggregate is now isotropically reloaded, the response will be initially elastic, so that the radial and circumferential stress increments will be given by (3), (4). The volumetric strain increment for the unit cell is then given by (6). Comparing (6) and (12), it is evident that for $K<1$ (always true), the stiffness of the non-dilatant fully plastic soil is always less than that of the elastic soil, except for the limiting case of $a/b=0$ (no kernel) when the stiffnesses become equal. The reload line therefore lies above the unload line in v-p space, so that hysteretic behaviour is predicted. Reverse kinematic yielding is assumed to occur when:

$$\sigma_r/\sigma_\theta = 1/K \tag{13}$$

On initial reloading, $\dot{\sigma}_r/\dot{\sigma}_\theta$ takes a maximum value of $(1+k)/(1-k/2)$ at $r=a$. So if reverse kinematic yielding is to occur at all, it can be seen that

$$K > \nu/(1-\nu) \tag{14}$$

For reasonable values of Poisson's ratio $\nu=0.25$ and $K=1/3$, typical of a quartz sand, (14) predicts reverse kinematic yielding after an infinite increase in global pressure. It is evident that whatever values of ν and K are chosen, large increases in global pressure will be required to cause reverse kinematic yielding. However, the inhomogeneous stresses in the kernel cell may induce "clastic yielding", which is the onset of particle fracture (Bolton and McDowell, 1996). We might adopt the criterion that crushing occurs if the major principal stress exceeds p_O. In this case, it can be shown (McDowell, 1997) that clastic yielding occurs just before the major stress reaches the preconsolidation pressure p_O. It is found that crushing first occurs at $r=a$, next to the rigid kernel. This reinforces the tendency for the smallest particles next to the kernel, which have the lowest co-ordination, to have the highest probability of splitting. Furthermore, it is conceivable that when the global pressure reaches the preconsolidation pressure p_O, clastic yielding will have led to an overall reduction in voids ratio after the unload-reload cycle, so that a ratcheting phenomenon is predicted (Fig. 5).

3 DILATANCY

It would be expected that incorporating a dilatancy rule into the model would increase the predicted volumetric expansion of the kernel cell for the same reduction in global pressure. If the ratio of plastic radial and circumferential strain increments is

$$\dot{\varepsilon}_r^p/\dot{\varepsilon}_\theta^p = -2/\rho \tag{15}$$

then (11) can be written:

$$\sigma_r/\sigma_\theta = K = \rho K_{cv} \tag{16}$$

where K is related to the mobilised angle of friction for kinematic yielding by (7). The stress-strain relation for the unit cell now has to be found numerically. Fig. 4 shows the effect of introducing dilatancy. The dotted line shows the numerical solution using the kaolin parameters and a/b=0.6, for K=0.4375 (no plastic volume change) which corresponds to a critical state angle of friction of 23°. The plot compares exactly with the closed form solution in Fig. 3. Fig. 4 shows an additional plot for ρ=2/3, which corresponds to an increased angle of shearing resistance of 33°.

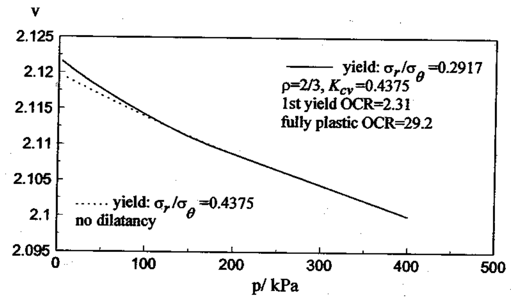

Figure 4. *v-p* plots showing the effect of dilatancy.

The introduction of dilatancy increases the curvature of the unloading curve, and this is an important feature of the kernel cell model. If when a normally consolidated soil is unloaded, no plastic volume changes occur (no dilatancy), then the volumetric strain of the soil must be elastic. The ideally elastic moduli of soils vary with confining pressure p as $p^{1/2}$ (Viggiani and Atkinson, 1995), so it follows that

$$\mathrm{d}v/\mathrm{d}(\ln p) \propto vp^{1/2} \tag{17}$$

In this case the slope of the unloading curve in v-ln$p$ space decreases with decreasing p, and the curvature is "in the wrong sense", comparing with Fig. 1. However, it is likely that kinematic yielding will first occur at a much lower OCR than predicted by the kernel cell due to local deviatoric stresses in the aggregate after normal compression, so that the perfectly elastic portion of the unloading curve may not be not visible in Fig. 1. Fig. 4 shows that dilatancy would have a remedial effect on the curvature predicted by (17). The kernel cell therefore suggests that the process of dilatant shearing during isotropic unloading may be responsible for the form of the unloading curve in Fig. 1.

4 CONCLUSIONS

The kernel cell provides a useful and interesting commentary for the unload-reload behaviour of soils (Fig. 5), such that the unloading curve in v-p space is due to kinematic yielding, and the reload line is due to elasticity followed by clastic yielding, which leads to compaction on cyclic loading if the pores are drained or partial liquefaction if volume is constrained to be constant.

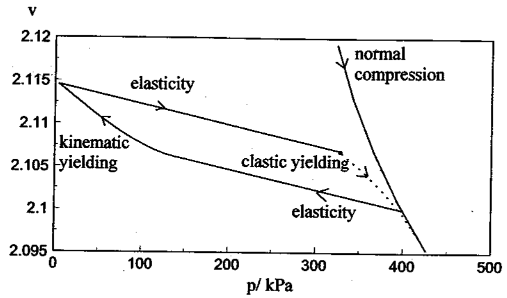

Figure 5. Typical unload-reload cycle produced by the kernel cell model.

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