Vertical bearing capacity factors for circular and strip footings on Mohr–Coulomb soil

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The method of characteristics is used to establish consistent factors for the vertical bearing capacity of circular and strip footings on soil which satisfies a linear (c, ϕ) Mohr–Coulomb strength criterion. This method of solution avoids the assumption of arbitrary slip surfaces, and produces zones within which equilibrium and plastic yield are simultaneously satisfied for given boundary stresses. Although similar solutions have previously been published for circular footings, their application has been hindered by errors and confusions over terminology. These are resolved, and the method of solution is explained. It is confirmed that Terzaghi's approach to the superposition of bearing terms containing N_q, N_γ, and N_c is both safe and sufficiently accurate for circular footings, as for strip footings. The values to be adopted are tabulated as functions of ϕ. Differences between the factors applicable to circular and strip footings far exceed the allowances of the empirical shape factors in common use. Some new shape factors are suggested that better represent the relationship between the limiting equilibrium of circular and strip foundations. Some current shape factors attempt to allow simultaneously for the differences in equilibrium solutions and the differences in axisymmetric (triaxial) and plane strain soil parameters. This cannot succeed, since the relationship between strength parameters depends strongly on relative density. The new bearing factors facilitate a more rational approach in which soil parameters appropriate to the geometry can first be determined and then used to find appropriate bearing capacity factors.

Key words: bearing capacity, axisymmetry, method of characteristics, footings, plane strain.

La méthode des caractéristiques est utilisée pour établir des facteurs consistants de capacité portante verticale de semelles circulaires et filantes sur un sol qui satisfait un critère linéaire (c, ϕ) de Mohr–Coulomb. Cette méthode de solution évite l'hypothèse de surfaces de glissement arbitraires, et produit des zones à l'intérieur desquelles l'équilibre et la déformation plastique sont simultanément satisfaites pour des contraintes aux frontières données. Quoique des solutions similaires aient été publiées antérieurement pour des semelles circulaires, leur application a été entravée par des erreurs et des confusions sur la terminologie. Ces problèmes sont résolus et la méthode de solution est expliquée. Il est confirmé que l'approche de Terzaghi à la superposition des termes de portance contenant N_q, N_γ, et N_c est en même temps sécuritaire et suffisamment précise pour les semelles circulaires, comme pour les semelles filantes. Les valeurs à adopter sont mises en tableau en fonction de ϕ. Les différences entre les facteurs applicables aux semelles circulaires et filantes dépassent de beaucoup les tolérances des facteurs de forme empiriques d'usage courant. L'on suggère de nouvaux facteurs de forme qui représentent mieux la relation entre l'équilibre limite des fondations circulaires et filantes. Des facteurs de forme courants tentent de tenir compte en même temps des différences dans les solutions d'équilibre et des différences dans les paramètres de sol axisymétriques (triaxiaux) et en déformation plane. Ceci ne peut pas réussir puisque la relation entre les paramètres de résistance dépendent fortement de la densité relative. Les nouveaux facteurs de portance facilitent une approche plus rationnelle dans laquelle les paramètres de sol convenant à la géométrie peuvent d'abord être déterminés, et utilisés par la suite pour trouver les coefficients de capacité portante appropriés.

Mots clés : capacité portante, axisymétrie, méthode des caractéristiques, semelles, déformation plane.

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Introduction

The derivation of bearing capacity for foundations on frictional soil relies first on the establishment of strength parameters based on effective stresses, and then on the use of bearing capacity factors. This paper uses the established approach of modelling the strength envelope by a simple constant-ϕ relation: this is later expanded into the more general (c, ϕ) envelope. The objective is to derive corresponding estimates of bearing capacity factors in both plane and axisymmetric load cases.

The currently accepted calculation procedure for bearing capacity factors is the method of characteristics (Sokolovskii 1960). This assumes that limiting stresses have been reached at every point, and solves for plastic equilibrium in the vicinity of the applied load. The arbitrary assumption of the shape of a slip surface, made in the limit equilibrium analyses of Terzaghi (1943) and Meyerhof (1951) for example, is avoided. Doubts regarding the method of characteristics in this application include:

(i) the difficulty of assigning boundary conditions against footings, especially where the mobilization of tangential friction should be such as to oppose relative motion and when the kinematics of plastic soil strain is itself uncertain, and

(ii) the difficulty of accepting the assertion that certain zones (e.g., in the far field, or in a wedge "trapped" beneath

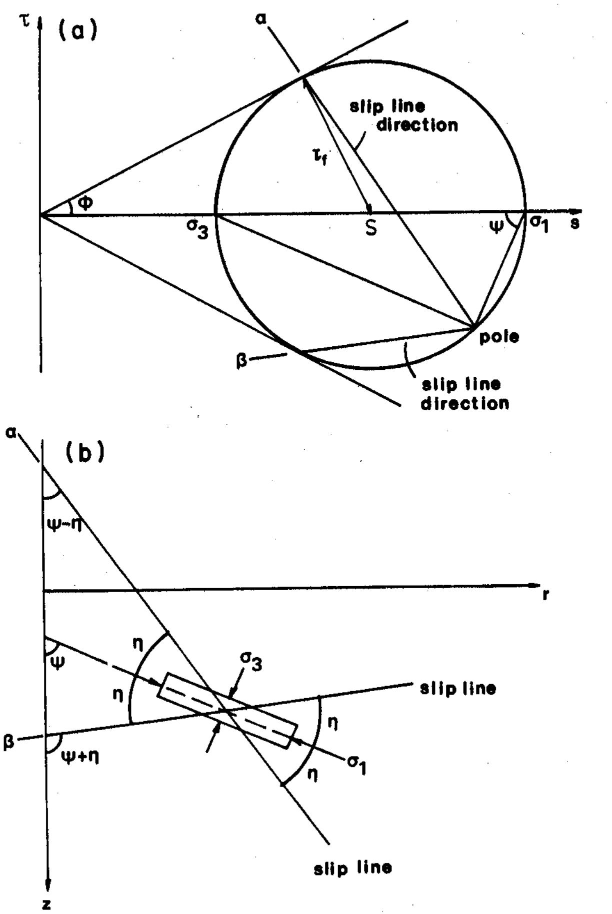

FIG. 1. (*a*) Mohr circle under failure condition. (*b*) Sign convention and notation.

the centre of a footing) are not in a limiting state simply because the characteristics have not been extended that far.

Methods such as the finite element technique can account for kinematics and equilibrium everywhere, and can be used to derive bearing capacities in close agreement with simpler solutions, albeit with more computing effort (Griffiths 1982). Since the demonstration of limiting equilibrium following Sokolovskii (1960) is more strenuous than that following Terzaghi (1943), and computationally easier to handle, it seems sensible to select the method of characteristics as the standard calculation approach, to be checked independently by back-analysis of case studies wherever profitable.

The objective of this paper is not to offer empirical evidence, however, but to discuss the extension of the method of characteristics to axisymmetric (circular or, approximately, square) footings. It is not widely known in engineering practice that characteristic solutions have been found for axisymmetric footings by Cox et al. (1961) and Cox (1962), providing estimates rather different from the application of commonly used shape factors to multiply plane solutions. Solutions by the method of characteristics should logically form the basis of engineering judgement for circular footings such as spuds for offshore rigs, as they do with plane problems. Engineers could then rely on the analysis to represent the difference between equilibrium equations in the two cases and could fall back on empirical judgements, which will certainly prove necessary for the assessment of appropriate soil strength parameters.

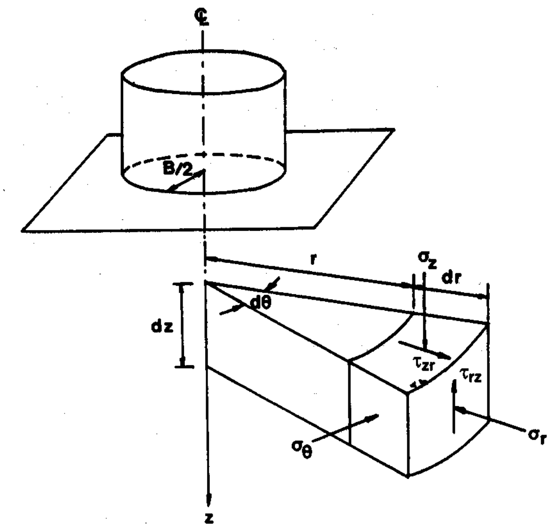

FIG. 2. Cylindrical coordinate system and stress components.

The method of characteristics

Sokolovskii's (1960) solution for strip footing was generalized by Shield (1955) to include the axisymmetric case for Tresca (cohesive) material. Cox et al. (1961) got axisymmetric solutions for a general (c, ϕ) material, albeit weightless, and Cox (1962) included self-weights. Here Cox used a very strange substitution, namely $c^* = c + \sigma_0 \tan \phi$, where σ_0 is an arbitrary surcharge defined as the atmospheric pressure (≈100 kPa), and a dimensionless parameter $G = \gamma B/(2c^*)$. The apparent misunderstanding of the physical principle of effective stress does not alter the mathematical acceptability of Cox's solution; it simply hinders the use by engineers of the original paper because of the likelihood of misinterpretation of the terms. Furthermore, Cox always lumped together the surcharge (N_q) and self-weight (N_γ) effects, which avoids Terzaghi's (1943) superposition assumption but makes the results very hard for the engineers to interpret.

Larkin (1968) took a more meaningful set of nondimensional parameters, normalizing stresses by dividing by $0.5\gamma B$ and distances by dividing by $0.5B$, but producing results for plane strain that differed by a factor of 2 from those of Prandtl (1920), which are known to be exact.

Taken together these extensions of bearing capacity theory are unnecessarily complex and confusing and restricted to far too narrow a range of ϕ compared with what might be needed in practice. Although the mathematical formulation for axisymmetric solutions was correct, engineers in practice have continued to rely on empirical shape factors. The results of applying the method of characteristics to plane or axisymmetric footings will be recast below in a form that should prove useful in practice.

The following assumptions are fundamental to this approach:

(*i*) The soil is rigid–plastic so that change-of-geometry effects such as settlement causing additional surcharge are not included, and imperfections in plasticity because of strain softening and progressive failure are ignored.

(*ii*) The soil obeys the Mohr–Coulomb yield criterion, so that its states of plastic equilibrium are consistent with a straight strength envelope acting as a tangent to the Mohr circle of stress drawn for the principal plane containing the major and minor effective stresses (Fig. 1*a*). The cohesion intercept will be dealt with later. For now, the shear strength τ_f is related to the mean stress s by $\tau_f = s \sin \phi$.

(*iii*) The intermediate principal stress is taken to be irrelevant to the yield criterion, but it enters axisymmetric analyses as the hoop stress σ_θ, which influences radial equilibrium. The Harr and von Karman (1909) hypothesis states that σ_θ should be equal to one of the two principal stresses lying in the axial plane, i.e., either $\sigma_\theta = \sigma_1$ or σ_3. It is assumed here, following Cox et al. (1961), to be equal to the minor principal stress, i.e., as small as possible. Lau (1988) has evidence from finite element analysis of punch indentation that supports this assumption, which must generally be either true or safe, since it will be shown that the effect of σ_θ is to increase the bearing capacity.

The mathematical formulation can now broadly follow Larkin (1968). The equations of equilibrium for a toroidal element (Fig. 2) can be written in cylindrical coordinates r, θ, z as

$$[1] \quad \frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \qquad \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = \gamma$$

The four stress components can be expressed in terms of mean stress s and the inclination ψ subtended to the z axis by the major principal stress (Fig. 1*b*):

$$[2] \quad \sigma_r = s(1 - \sin\phi \cos 2\psi), \qquad \sigma_z = s(1 + \sin\phi \cos 2\psi), \qquad \tau_{rz} = s \sin\phi \sin 2\psi, \qquad \sigma_\theta = \sigma_3 = s(1 - \sin\phi)$$

The characteristic lines that will embody the solution are lines α and β on which ϕ is mobilized. In the literature they are often referred to as slip lines, but their significance is in relation to the equations of equilibrium, not those of displacement. The geometry dictates that their slopes can be written

$$[3] \quad \frac{dr}{dz} = \tan(\psi - m\eta) \qquad \text{where} \qquad \eta = \frac{\pi}{4} - \frac{\phi}{2}$$

and m takes the value $+1$ for an α line and -1 for a β line (shown in Fig. 1*b*).

These equilibrium and yield equations are a set of hyperbolic partial differential equations that reduce to two ordinary differential equations expressing the increase in stress along each characteristic line ($m = \pm 1$) in terms of the changing inclination ψ and position (r, z):

$$[4] \quad ds \cos\phi + m 2s \sin\phi \, d\psi + n\frac{s}{r}[\sin\phi \cos\phi \, dr + m(\sin^2\phi - \sin\phi) dz] = \gamma(-m \sin\phi \, dr + \cos\phi \, dz)$$

The factor n has been introduced for convenience; in axisymmetric analysis for circular footings it takes the value $+1$, whereas it happens that plane solutions for long strip footings are given if n is set to zero.

Method of computation

Following Larkin (1968) the variables will now be normalized according to a scale length of $0.5B$, so we will take

$$[5] \quad \Sigma = \frac{s}{0.5B\gamma}, \qquad R = \frac{r}{0.5B}, \qquad Z = \frac{z}{0.5B}$$

Equation [4] can now be written in finite difference form, suitable to the solution of the intersection of an α line, which passed through a known "point" (R_1, Z_1, Σ_1, ψ_1), and a β line, which passed through another known "point" (R_2, Z_2, Σ_2, ψ_2). Shi (1988) suggested the following substitution:

$$[6] \quad \begin{aligned} A &= \frac{-2\Sigma_1}{(R + R_1)\cos\phi}[\sin\phi\cos\phi(R - R_1) + (\sin^2\phi - \sin\phi)(Z - Z_1)] - (R - R_1)\tan\phi + (Z - Z_1) + \Sigma_1 + 2\Sigma_1 \tan\phi\psi_1 \\ B &= \frac{-2\Sigma_2}{(R + R_2)\cos\phi}[\sin\phi\cos\phi(R - R_2) - (\sin^2\phi - \sin\phi)(Z - Z_2)] + (R - R_2)\tan\phi + (Z - Z_2) + \Sigma_2 - 2\Sigma_2 \tan\phi\psi_2 \end{aligned}$$

The finite difference equations can now be written as

$$[7] \quad (R - R_1) = (Z - Z_1)\tan(\psi_1 - \eta), \qquad (R - R_2) = (Z - Z_2)\tan(\psi_2 + \eta)$$

$$[8] \quad \Sigma = \frac{A\Sigma_2 + B\Sigma_1}{\Sigma_1 + \Sigma_2}, \qquad \psi = \frac{B - A}{-2\tan\phi(\Sigma_1 + \Sigma_2)}$$

which are suitable for iteration.

If in Fig. 3 points P and Q are known, and W is to be determined, R_w and Z_w can be found from [7] and then ψ and Σ can be calculated from [8], by putting $\psi_1 = \psi_P$ and $\psi_2 = \psi_Q$. However, in general, the characteristics are curved and the solution can be improved by updating for ψ, putting $\psi_1 = (\psi + \psi_P)/2$ and $\psi_2 = (\psi + \psi_Q)/2$ and repeating until there is convergence (within some target accuracy) to a stable set of values (Sokolovskii 1960).

Proceeding from the known boundary condition (normal stress σ_0) on the free surface KS (Fig. 4), the entire stress field within KLMO can be determined. Firstly, a value for R_K is assumed. Secondly, the boundary KS is subdivided with a set of equally spaced points. The solution then marches in towards the footing boundary. When the calculation is complete, it should be checked to see if the β characteristic starting from K actually finishes at O. If not, R_K is adjusted

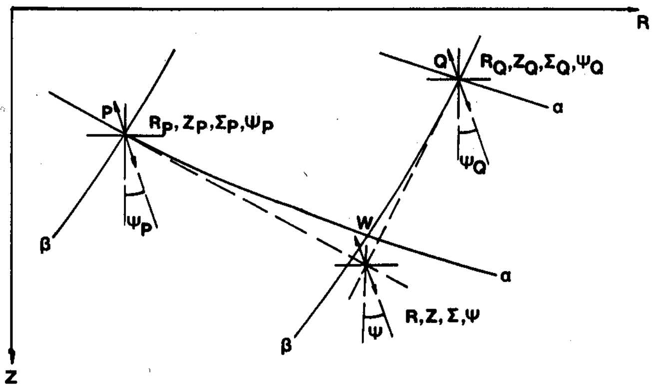

FIG. 3. Computation scheme of new point W from known points P and Q.

according to whether it is too far or too near. The whole calculation iterates until an acceptable closing accuracy is achieved.

A very coarse mesh, applied to a plane strain case with weightless soil in Fig. 4, will be used to illustrate the marching scheme used in the computer program CONPHI.

(1) *Passive zone* — The boundary conditions at S, K1, and K are known. K2 is found using K1 and K; K3 is found from S and K1; L is found from K3 and K2. S, K3, and L have become the known boundary conditions for the fan zone.

(2) *Fan zone* — Node S can be viewed as a degenerate β characteristic with unique R and Z, but varying ψ and Σ. The first two terms of [4] demonstrate that the significant effect in the fan zone is that stress increases exponentially with rotation Δψ of the β line from K3 to L1, for example, L1 being found from S (at $\Delta\psi = \pi/4$) and K3. Similarly, L2 is found from L1 and L, L3 from S (at $\Delta\psi = \pi/2$) and L1, and M from L3 and L2. S, L3, and M have now become the known boundary conditions for the active zone.

(3) *Active zone* — On the footing contact plane, Z is known and the value of ψ, being a function of mobilized friction on the footing, can at least be assumed. The mobilization of friction depends on the detailed kinematics, which are a function of dilatancy and are not available in this equilibrium solution. A simple extreme case is provided by a frictionless interface, as here, for which $\psi = 0$. The solution for Σ and R at M2 can therefore be found from the condition at L3; then M1 can be found from M2 and M; finally, Σ and R can be found at O using M1 together with the known values of Z and ψ at O. O is intended here to close on the centreline.

At this stage, all the variables on the footing contact plane are known. The bearing pressure σ_z can now be found by multiplying Σ with $0.5B\gamma$ to give s, substituting this, together with ψ, into [2]. The mean bearing pressure under the footing can now be found by numerical integration.

Principle of superposition

The method of characteristics derived above makes it possible to solve for the bearing capacity of footings taking surcharge and soil self-weight simultaneously into account. Terzaghi (1943) assumed that if the bearing capacity of a foundation on weightless frictional soil due to surcharge σ_0 could be calculated and expressed as $\sigma_0 N_q$, and if the bearing capacity of the same foundation due to self-weight γ alone could be written $0.5\gamma B N_\gamma$, then these com-

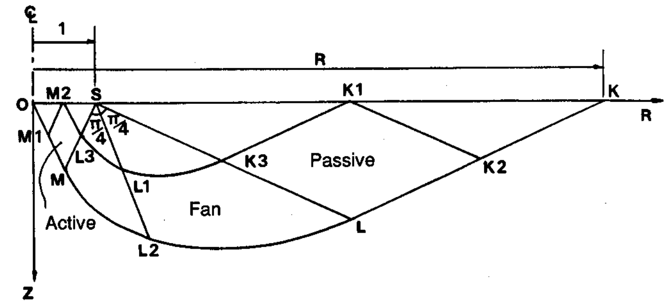

FIG. 4. A typical stress characteristic mesh.

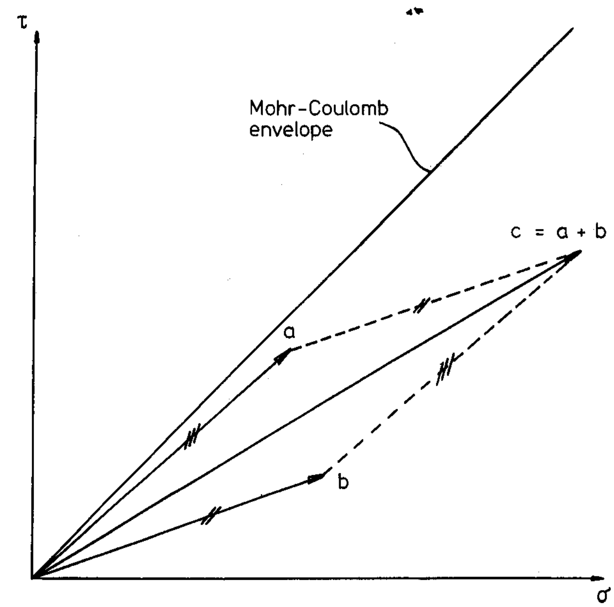

FIG. 5. Effects of superposition.

ponents could safely be superposed. Decomposition of the total bearing capacity into components has, following Terzaghi, been adopted in practice. The validity and utility of this practice, and its possible extension for circular footings, are now examined.

It can easily be demonstrated that superposition must be conservative for materials that obey a linear Mohr–Coulomb envelope with constant ϕ. Suppose, in Fig. 5, that vector ***a*** represents the stress on any plane at a point inside a soil body in limiting equilibrium subject to load case A (e.g., surcharge plus bearing capacity), while ***b*** represents the stresses on the same plane due to limiting equilibrium in some other load case B (e.g., self-weight plus bearing capacity). If loads A were separately in equilibrium, as were loads B, then the combination $C = A + B$ would also be in equilibrium. Furthermore, the stress ***c*** would be the vector addition ***a*** + ***b***, so if ***a*** and ***b*** separately satisfied Mohr–Coulomb so must ***c***. In general, the critical planes mobilizing ϕ at the point in question will not coincide in the two load cases, and the maximum angle mobilized on any plane in case C will be less than ϕ: hence the conservatism of superposition.

Davis and Booker (1971) performed rigorous checks on the superposition assumption for the plane strain case and found that it was indeed conservative. The error was no more than 20% for ϕ in the range 20°–40°. Cox (1962) made no superposition assumption for circular footings and used a dimensionless parameter $G = 0.5\gamma B/(\sigma_0 \tan \phi)$ to show that solu-

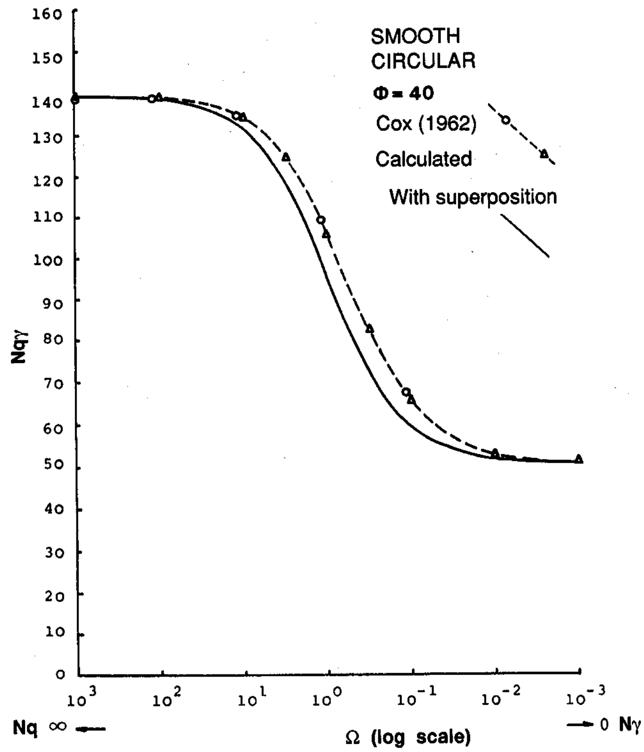

FIG. 6. Effects of Ω on $N_{q\gamma}$.

tions depended in a nonlinear fashion on the relative magnitude of the self-weight term and the surcharge term. It is now proposed to treat σ_0 not as atmospheric pressure following Cox, but as an effective surcharge applied to the plane surface of the soil around the foundation. A new dimensionless parameter, the superposition factor.

$$\Omega = \frac{\sigma_0}{0.5B\gamma}$$

will be used to display the relative importance of surcharge and self-weight effects, and the total bearing capacity σ_f will be related to a combined bearing capacity factor

$$N_{q\gamma} = \frac{\sigma_f}{0.5B\gamma + \sigma_0}$$

Figure 6 depicts the effect of Ω on $N_{q\gamma}$ for smooth circular footings on soil with $\phi = 40°$. By converting Cox's data into the new format, it can be seen that the new calculation produces results that are practically identical, although the numerical solution techniques are different. It can also be seen that $N_{q\gamma}$ remains constant for $\Omega > 10^3$; this corresponds to the N_q limit. Likewise, $N_{q\gamma}$ remains constant at N_γ for $\Omega < 10^{-3}$. These limits can be used to derive the alternative superposition estimate which errs on the safe side, and never by more than the 20% error seen in the vicinity of $\Omega = 1$. Since this result is typical of all those cases that have been checked, and the complexity of bearing capacity tables is much reduced if only N_q and N_γ are required in each case, the general practicability of Terzaghi's (1943) superposition is confirmed.

An extension of the theory to cover the more general Mohr–Coulomb envelope, $\tau = c + \sigma \tan \phi$, can now be made. Here, c and ϕ can be taken as the envelope parameters on an effective stress diagram that offer a best fit to strength data over the appropriate range of stress. Prandtl's (1920) substitution can be used to find an expression for N_c, leading to

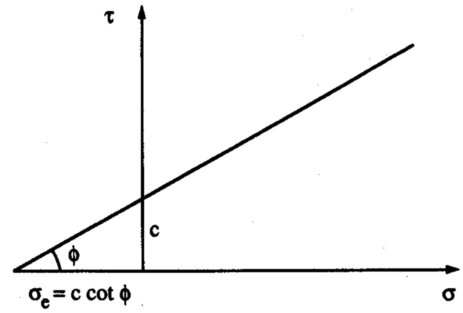

FIG. 7. General Mohr–Coulomb envelope.

the third component of Terzaghi's (1943) bearing capacity, $c\,N_c$. Define an equivalent surcharge $\sigma_e = c \cot \phi$ (see Fig. 7) acting over the whole soil surface including that part that will carry the foundation, causing a hydrostatic stress increase everywhere. The problem may then be seen as a simple constant-ϕ case with the origin shifted. The total bearing capacity due to cohesion would then be $N_q\sigma_e$, of which σ_e was already acting. The *extra* contribution is therefore $(N_q - 1)\sigma_e$. So using superposition we can follow Terzaghi to write

$$[9] \quad \sigma_f = c\,N_c + \sigma_0 N_q + 0.5B\gamma\,N_\gamma$$

finding N_q and N_γ directly for plane or circular footings from the earlier calculations with Ω set to 10^3 and 10^{-3}, respectively, and using $N_c = (N_q - 1) \cot \phi$.

Calculation of values for N_q and N_γ for smooth and rough bases

Computations of N_q and N_γ for smooth, frictionless footings are listed in Table 1 as are solutions for N_γ, which are based on an optimistic treatment of friction opposing the spreading of soil beneath the footing. All the analyses were carried out with a mesh consisting of 96 β characteristics and 220 α characteristics. The rotation step in the fan zone was 3°. Figures 8–13 show the typical stress characteristics and footing pressure distributions for plane strain and axisymmetric cases for both N_q and N_γ at $\phi = 40°$. In each case two solutions are shown, a shallow mechanism (after Hill 1950) consistent with a smooth footing, and a deep mechanism (after Prandtl 1920) in which it is assumed that friction against the footing stabilizes a trapped wedge or cone. Following Meyerhof (1951), it is assumed that the trapped wedge or cone has a base angle $(\pi/4 + \phi/2)$ and mobilizes ϕ on its inclined surface so that it acts as the final characteristic in the marching solution (see the shaded areas in Figs. 9, 11, and 13). Each approach satisfies the condition of symmetry, that the principal stress direction is vertical on the centreline of the footing. Indeed, the trapped wedge offers a vertical major principal stress at every point on its surface.

It is well known (Chen 1975) that both shallow and deep mechanisms offer the same solution for N_q in plane strain, so friction on the footing can have no effect, and this is confirmed in Figs. 8 and 9. Although the same has not yet been proved for circular footings, it will be assumed hereafter that the same identity always holds for N_q. On the other hand, it must be anticipated that the self-weight of the larger plastic zone will lead to larger estimates of N_γ for the deep mechanism. This is indeed found to be the case (see Figs. 10 and 11, for example).

TABLE 1. Bearing capacity factors

φ (deg)	N_q Smooth or rough Strip	N_q Smooth or rough Circle	N_γ Smooth Strip	N_γ Smooth Circle	N_γ Rough Strip	N_γ Rough Circle
5	1.57	1.65	0.09	0.06	0.62	0.68
10	2.47	2.80	0.29	0.21	1.71	1.37
15	3.94	4.70	0.71	0.60	3.17	2.83
20	6.40	8.30	1.60	1.30	5.97	6.04
25	10.7	15.2	3.51	3.00	11.6	13.5
30	18.4	29.5	7.74	7.10	23.6	31.9
31	20.6	34.0	9.1	8.6	27.4	38.3
32	23.2	39.0	10.7	10.3	31.8	46.1
33	26.1	45.0	12.7	12.4	37.1	55.7
34	29.4	52.2	15.0	15.2	43.5	67.6
35	33.3	61.0	17.8	18.2	51.0	82.4
36	38	71	21	22	60	101
37	43	83	25	27	71	124
38	49	99	30	33	85	153
39	56	116	36	40	101	190
40	64	140	44	51	121	238
41	74	166	53	62	145	299
42	85	200	65	78	176	379
43	99	241	79	99	214	480
44	115	295	97	125	262	619
45	135	359	120	160	324	803
46	159	444	150	210	402	1052
47	187	550	188	272	505	1384
48	222	686	237	353	638	1847
49	265	864	302	476	815	2491
50	319	1103	389	621	1052	3403
51	386	1427	505	876	1373	4710
52	470	1854	663	1207	1812	6628

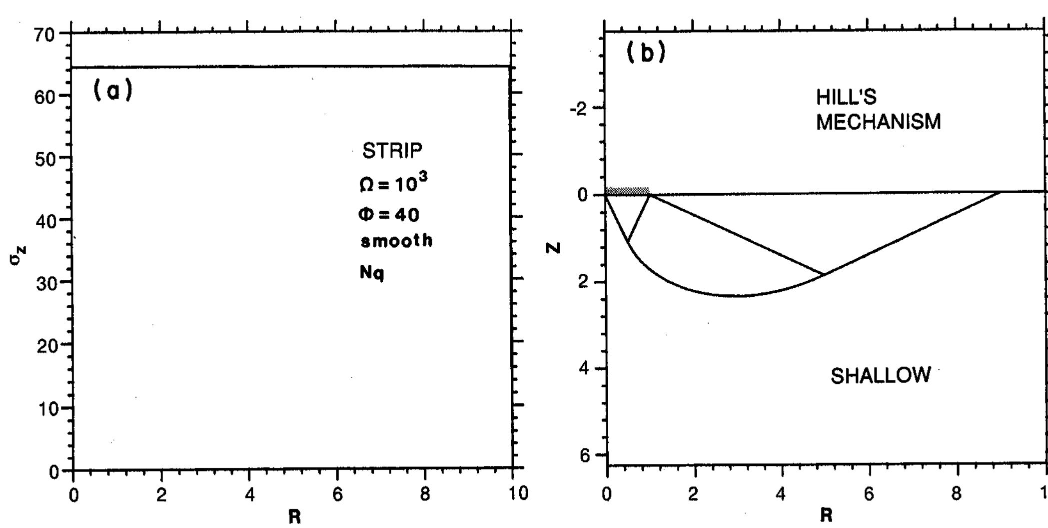

FIG. 8. Footing pressure distribution (*a*) and stress characteristics (*b*) for the plane strain case for N_q for a smooth base.

The assumption of a trapped zone is tantamount to solidifying the soil beneath the footing: presumably, friction mobilized on the base can do no more, but might do less. Although there is no theoretical justification for the assumption, there is experimental evidence in its favour (for example, Ko and Davidson 1973). It will be seen in Table 1 that the effect of allowing for footing roughness in this way increases the value of N_γ by about a factor 3 for strip footings (equivalent to an increase in φ of about 5° overall) and by a factor 4 for circular footings (equivalent to an increase in φ of about 7°). The values in columns 6 and 2 appropriate to rough strip footings satisfy

$$[10] \quad N_\gamma \approx (N_q - 1)\tan(1.5\,\phi)$$

within an equivalent discrepancy on φ of ±2° for φ from 30° to 50°. For example, the value for N_γ indicated by [10] for φ = 35° would be 42.1 compared with the tabulated value of 51.0, and 42.1 would be the tabulated result for

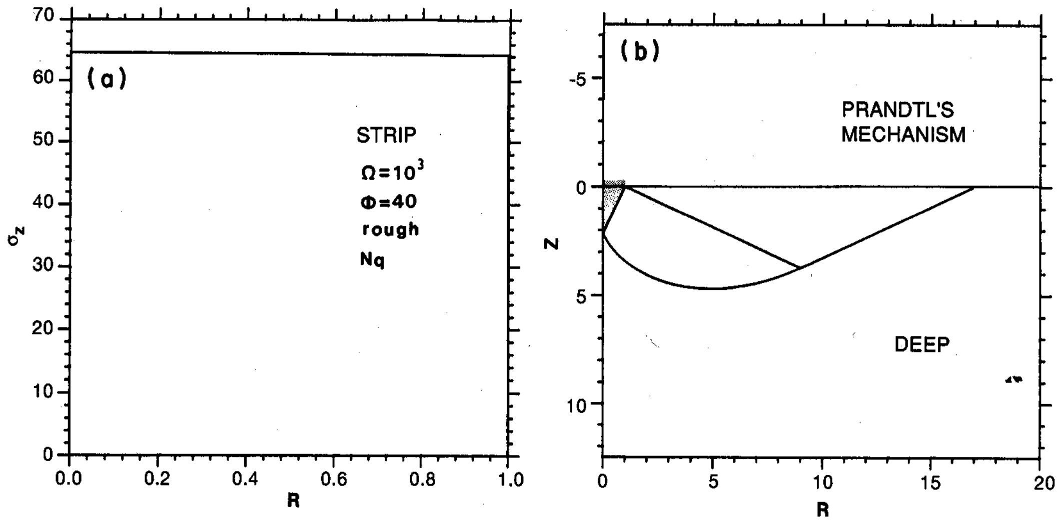

FIG. 9. Footing pressure distribution (*a*) and stress characteristics (*b*) for the plane strain case for N_q for a rough base.

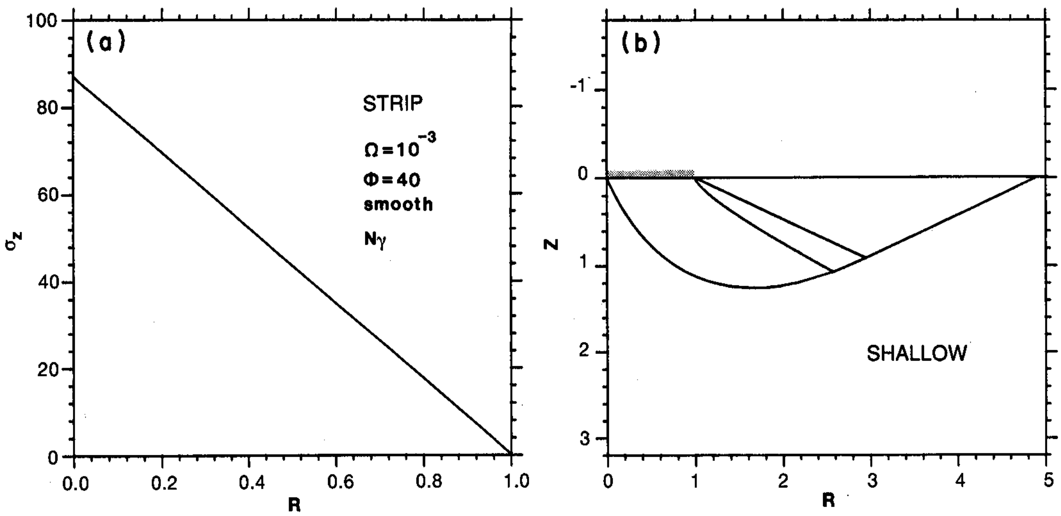

FIG. 10. Footing pressure distribution (*a*) and stress characteristics (*b*) for the plane strain case for N_γ for a smooth base.

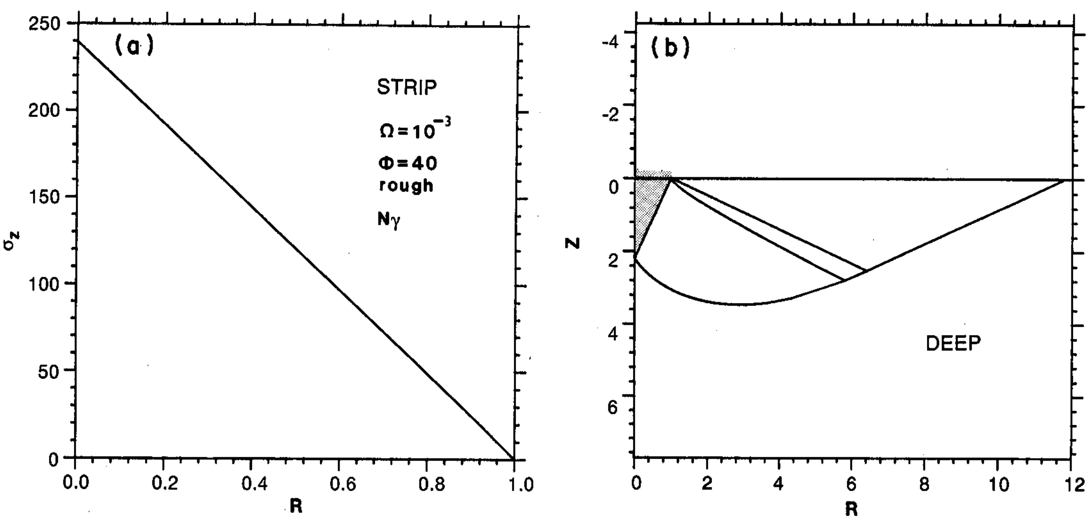

FIG. 11. Footing pressure distribution (*a*) and stress characteristics (*b*) for the plane strain case for N_γ for a rough base.

ϕ = 33.8°, which is 1.2° less than requested. The same approximation [10] also applies to the relative values in Table 1 for rough circular footings. Meherhof (1961) first suggested an expression such as [10], with a factor for strip footings of 1.4 rather than 1.5.

The shallow mechanism clearly offers a safe solution to the

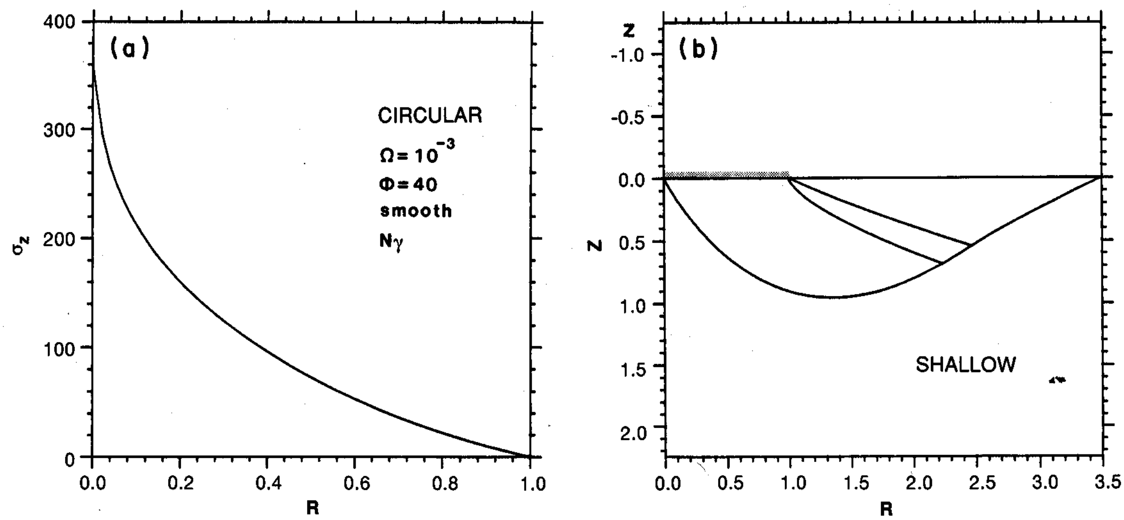

FIG. 12. Footing pressure distribution (*a*) and stress characteristics (*b*) for the axisymmetric case for N_γ for a smooth base.

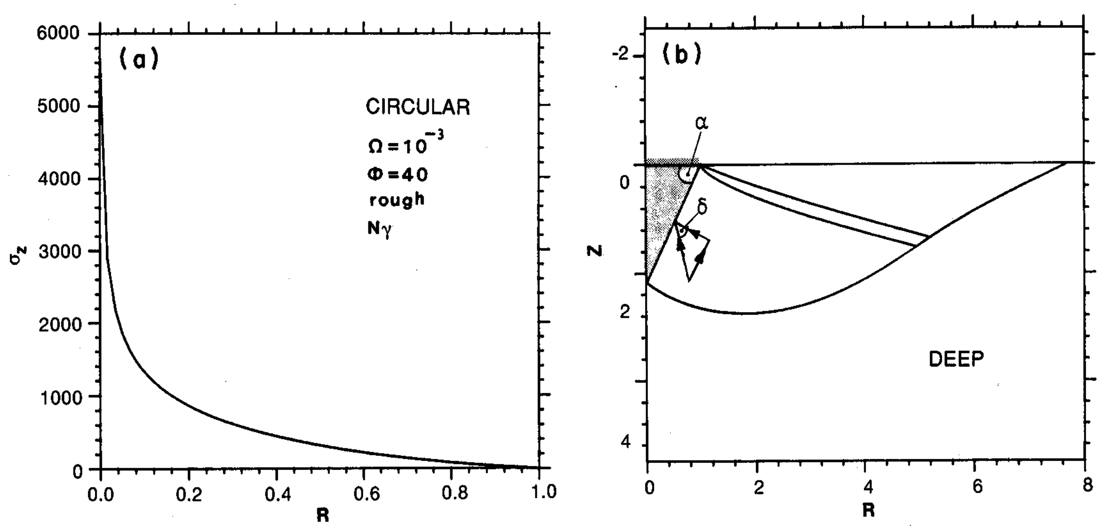

FIG. 13. Footing pressure distribution (*a*) and stress characteristics (*b*) for the axisymmetric case for N_γ for a rough base.

problem of vertical bearing capacity, but it will err too far on the safe side in many circumstances. The only collapse mechanisms that can operate in a given situation are those that offer the soil–footing system a kinematically admissible displacement field. The possible displacement fields should take soil dilatancy into account and relate principal directions of compressive stress and strain. Although soil strains are outside the scope of the current work, Fig. 14 illustrates some circumstances in which the shallow Hill (1950) mechanism might actually operate for foundations that could appear rough. In these cases the footing can stretch and bend, or split, so that the soil just beneath the footing, and close to the centreline on either side, can acquire relatively large lateral displacements as the footing penetrates the soil.

Shape effects

Many semiempirical shape factors have been suggested for the conversion of values for N_q and N_γ from plane strain to axisymmetry. Terzaghi (1943) reduced the N_γ term by a factor of 0.6, but left N_q unaltered. Meyerhof (1963) used a factor $(1 + 0.1K_p)$, where K_p is the coefficient of passive earth pressure on both, which enhances the plane factors by 1.3 at $\phi = 30°$ and by 1.76 at $\phi = 50°$. Other authors suggest intermediate values.

As Meyerhof (1963) points out, the final estimate of bearing capacity must take account of two effects, the greater capacity of circular footings on soil with a given ϕ, and the reduced ϕ of soil in axisymmetry (triaxial tests) compared with plane strain. It is logical to follow Meyerhof's procedure, selecting axisymmetric values directly from Table 1 in this case instead of applying indirect shape factors, and attempt to ensure that an appropriate ϕ value is used. Careful back-analysis of the punching of circular footings such as spud foundations will reveal whether triaxial strengths are appropriate. The final outcome might be that the ultimate bearing pressure of a circular footing is found to be less than that of a strip footing on the same soil, as Terzaghi (1943) would have predicted.

In detail, it will be seen from Table 1 that the shape factors for rough foundations creating deep mechanisms vary from about 1 at low ϕ values to about 3.5 at high ϕ values, a range exceeding even the recommendation of Meyerhof (1963). If it is desired to base axisymmetric values on plane values and use some simple factor, it will be found for deep mechanisms that axisymmetric values for N_q or N_γ can best

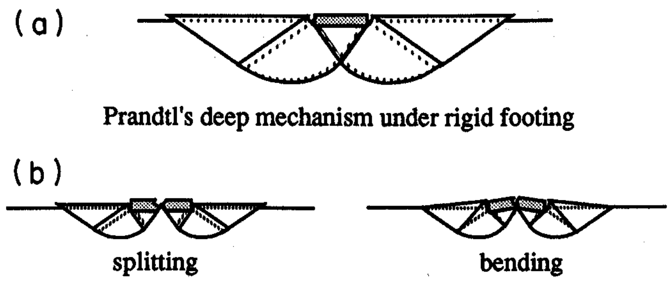

FIG. 14. The influence of footing deformability on the soil mechanism. (*a*) Prandtl's (1920) deep mechanism under rigid footing. (*b*) Hill's (1950) shallow mechanism under deformable footing.

be found by first increasing ϕ by a factor of 1.1, and then finding plane strain values for this enhanced ϕ value. The error expressed as an equivalent discrepancy on ϕ is less than 1° in the range 30–50°. It must be recognized that the enhancement factor of 1.1 is purely to account for load spreading in axisymmetry; it is not determining what ϕ value to enter for the soil. In the case of shallow mechanisms, the plane strain value can first be enhanced by scaling up ϕ by 10%, but the resulting N_γ value must then be divided by 2 to get an acceptable estimate of the axisymmetric capacity according to the values in Table 1.

Coincidentally, a quite different argument is often used to enhance triaxial ϕ values by a factor 1.1 to estimate plane strain values under similar conditions of density and stress. If this were also applied, it would transpire that bearing capacity factors for rough, rigid footings could be selected from plane strain bearing factors using a ϕ value 1.1 times the triaxial value, and irrespective of whether the footing was a circular pad or a strip. Footing shape would not then influence the ultimate bearing pressure.

The first mention of a 10% increase to obtain plane strain strength from triaxial strength was that by Bishop (1961) based on some tests on compacted granular soils. For loose sands the increment is very small, or negligible, as was also shown. Bolton (1986) showed that for dense sands under low confining pressures the increment to secant ϕ values could approach 20%, but it was also demonstrated that Mohr–Coulomb envelope was curved when a large range of stress was to be considered. Changes of both c and ϕ would therefore be necessary to capture the difference in envelope between plane and triaxial strengths relevant to footings. The selection of appropriate strength parameters is outside the present scope of investigation. Nevertheless, the fact that the plane strain factor on triaxial ϕ could lie between 1.0 and 1.2 means that a careless choice could easily lead to an error in bearing capacities of a factor of 3.

Conclusions

Two issues are involved in the estimation of the drained bearing capacities of shallow circular footings. The first task is the establishment of a calculation procedure, based on soil strength parameters, which is capable of predicting bearing capacity. The second task is the establishment, through back-analysis of prototypes or models, of a means of defining representative values of soil strength to use in given circumstances. This paper addresses the first task so that the second can subsequently be attempted.

A great deal of previous effort has gone into the comparison between strip and circular footings. Unfortunately, empirical investigations conflate the two issues referred to above: the use of different triaxial or plane soil strength parameters is confused with the possible existence of different bearing capacity factors. Other problems such as the nonlinearity of soil strength envelopes will inevitably require empirical treatment. In these circumstances it is essential to remove empiricism from the calculation procedure wherever possible.

A coherent analysis of the collapse of plane and axisymmetric footings has been undertaken. Although it cannot be claimed that the assumptions which have been made are inevitable, they are at least consistent between the two cases. The method of characteristics has been used to confirm the utility of separating N_q, N_γ, and N_c components of bearing. A table of bearing capacity factors has been produced for a wide range of ϕ values. Quite distinct values for plane and axisymmetric cases are listed; deep mechanisms beneath rough, rigid footings provide much larger self-weight components of bearing capacity than shallow mechanisms beneath smooth or compliant footings. Bearing capacity factors for a given angle ϕ are shown to vary over a factor 4 from case to case. An error of 5° in ϕ produces an error of a factor of 2 to 3 in each factor.

It is suggested that engineers find bearing capacity factors directly, rather than relying on shape factors, and should attempt to define strength envelopes appropriate to the soil density, stress range, and strain conditions. Further work on back-analysis is called for to validate the empirical judgements that will be required in the selection of representative strength parameters.

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