What are partial factors for?

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SYNOPSIS
Partial factors have emerged within limit state design methodology as a means of adjusting parameters so that appropriate values are derived for design calculations. They have become associated with uncertainty, but their proper use in geotechnical engineering is to safeguard against predictable deformation by factoring down the peak soil strength. Ultimate soil strengths must be used to check for safety against collapse: these parameters are easily and reliably measured and require no partial factor. Considerable effort must be expended to explain to structural engineers the real nature of serviceability and collapse limit states brought about by soil behaviour. A simple, unified approach is suggested which is based on the theory of plasticity.

1. INTRODUCTION
Brinch Hansen (1953) set out a philosophy of geotechnical design in which separate consideration was to be given the selection of design loads and design values of soil strength. Partial coefficients were proposed to factor characteristic loads and soil strength parameters, to derive design values. This approach was immediately influential inside Denmark, and came to be embodied in Danish Code DS 415 for Foundation Engineering, Dansk Ingeniorforening (1978).

Brinch Hansen had perceived a groundswell of dissatisfaction with the previous generation of permissible stress codes, and saw that a single factor of safety must equally fail to provide engineers with objective tests to evaluate performance. He wished to have engineers base their evaluation on the principles of mechanics, and emphasised proper use of plasticity theory in the analysis of collapse, and elasticity theory in the analysis of deformation.

The distinction to be drawn between safety and deformation analyses, and the clarification of performance requirements which could flow from this, has become the hallmark of the modern approach to the so-called limit state design of structures. The “Structural Eurocodes” being drafted through CEN embrace these ideals, and both strong and weak features of their format are discernible in the earlier Danish work.
2. THE DANISH CODE DS 415

2.1 Safety
The characteristic loads were to be taken from structural codes, and organised in three types of load combination – normal, extraordinary and extreme. In each combination, loads were to be increased by certain partial factors. For example, in the case of normal loads, the partial factors \( \gamma_F \) were given as 1.0 for dead loads and 1.5 for live loads. As extra special loads were introduced into the other combinations, the partial factors on the normal load components were reduced.

Everyone agrees that it would be unrealistic to consider both earthquake and hurricane simultaneously. It is, however, less clear that the partial factor on persons, furniture and equipment should drop from its normal 1.5 to 0.5 in the analysis of an extreme accident, as Annex A of DS 415 (1978) proposed. This suggests great volatility. If large office loads are possible, they may well persist until the accident strikes. The use of statistical reasoning to configure realistic load combinations is, however, now widely accepted.

Characteristic values of soil strength were to be based on triaxial tests. Calibration factors were suggested for vane test data on clay, for example. No mention was made of post-peak softening, and it may be assumed that peak strength data were to be used to obtain \( c_u \) or \( (c', \phi') \) envelopes. The degree of conservatism to be used in putting a design line through any scatter was, however, not clear.

In a discussion about safety factors, the first question that must be posed is the objectivity and safety injected into the number to be factored, in this case the characteristic value. Here, it is considered significant that the code permitted an estimate to be made of \( \phi \) for granular materials using an empirical formula which would offer 34° for a rounded, uniform \( (U = 2) \), dense \( (I_D = 70\%) \) sand. More recent evidence (Bolton, 1986) would suggest a value of 40° at moderate stress levels, reducing to 34° only at mean effective stresses of about 2 MPa. If this were so, an extra safety factor of 1.25 on \( \tan \phi \) at normal stress levels was injected (in this particular case) into the characteristic value – assuming engineers used the formula rather than their own test data, of course.

Having established characteristic values of soil strength, the code demanded that partial factors \( \gamma_M \) be applied as follows. Cohesion intercepts were to be factored down by 1.5 generally, but 1.75 in the case of the bearing capacity of foundations. Soil friction was to be reduced by the factor 1.2 on \( \tan \phi \).

It is interesting to reflect that the higher partial factor on cohesion in foundation capacity accords with common sense, since foundations are highly susceptible to settlements which would go unnoticed in general earthworks, but that this justification is in discord with the prime directive that safety and deformation analyses should be separate. Since the undrained strength of clays increases due to partial drainage under foundations and reduces in cuttings, for example, the justification for selectively increasing \( \gamma_F \) on the grounds of safety of foundations seems poor.
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2.2 Deformation

DS 415 (1978) proposed that the state of deformation of the structure should be checked on the actual state of normal use, based on actual adverse influences, and without any partial factor being applied either to characteristic loads or on the characteristic strength and deformation parameters.

Plate loading tests in the field, and oedometer tests in the laboratory, were recommended where fairly precise determinations of deformation were required. Otherwise, certain empirical values were suggested for soil stiffness; these were clearly intended to be conservative. This again draws attention to the logical difficulty of imposing partial factors on characteristic values which could themselves vary by an order of magnitude depending how they were assessed.

3. THE EMERGING EUROCODES

3.1 Characteristic values

Presumably conscious of the absurdity of applying marginal partial factors to poorly defined characteristic parameters, the drafters of Eurocode 1 (ENV 1991 - 1) attempted to harmonise all such definitions using probability theory.

Characteristic values of actions (loads etc) and material properties should, in the EC1 proposal, be selected as having a prescribed probability (usually 5%) that a more unfavourable value will occur. However, the drafters have felt obliged to add that where there is a lack of information on the statistical distribution of the property, a nominal value may be substituted. EC1 intends that the characteristic value should represent field behaviour. A conversion factor should be applied to sampled data where a regression relationship has been established between test measurements and field behaviour.

The drafters of EC7 Geotechnical Design have attempted to give some life to the “nominal value” to be used in the absence of statistics, requiring that “the characteristic value of a soil parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state”. The clear intention of the EC7 drafters has been to guide engineers to a value which would emerge from a back-analysis of a failure, should one ever occur. The most confusing aspect of this otherwise enlightened approach is the apparent need to apply a partial factor to this most carefully selected value.

3.2 Partial factors in EC1

According to EC1, partial factors should depend on the probability of an unfavourable deviation from the characteristic value, inaccuracy in the model of behaviour used for calculations, and the consequences of breaching a limit state. It is emphasised that partial factors deal with uncertainty and variability.

An informative Annex to EC1 summarises the ideas behind partial factor design. The key concept is the selection of target probabilities for limit state events. The drafters suggest a nominal probability per building lifetime of 1 / 10 000 for ultimate limit states (ULS:
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collapse which compromises safety), and
1/10 for serviceability limit states (SLS: deformation which compromises efficient utilisation). These targets should ideally be met by fixing probability density functions for all the variables, and combining them according to the calculation models employed in the design process.

A shortened form of this method is to use reliability theory in which the shape of the distribution of each parameter is assumed (often log normal) and the probability of outlying values is computed from a reliability index which measures the deviation of that value from the measured population mean, in units of the measured standard deviation. The answer depends entirely on the shape assumed for the distribution, especially for the extreme values demanded at ULS when the partial factors are expected to reflect an extension from the characteristic (5% probability) to the design value (0.01% probability).

Even this more modest agenda fails through lack of data. Where sufficient data does seem to exist, the calculated probabilities are frankly admitted to be nominal (erroneous). In stead, the Eurocode drafters have had to fall back on taking completely arbitrary values for partial factors. This alternative “principle” is defined as “calibration to historical and empirical design methods”. This, then, is the lifeless heart of the partial factor design method in EC1.

The nebulous ideas of “calibration” and “uncertainty” have led to a sort of arithmetic cancer. Brinch Hansen’s separate partial coefficients for loads and resistances have divided and spread in an uncontrolled manner. When any sub-group proposed a different partial factor, the EC1 drafters now had no philosophical position from which to oppose its inclusion. Every material property, every load, every calculation method, was subject to possible error and therefore to the imposition of a partial factor.

In an aside, the EC1 drafters do point out that choosing a different partial factor is not the only way of altering the probability of failure; an alternative approach is to improve the accuracy of the calculation method. This more scientific thought will be reviewed later in the context of soil plasticity calculations.

3.3 Partial factors in EC7

Although EC7 pays lip service to the EC1 “ideal”, the only partial factors to be specified in EC7 are for loads and strengths, and they resemble those used in DS 415.

All actions and all material parameters are generally to have partial factors of unity in deformation checks for serviceability (SLS).

For safety checks (ULS), loads are to be multiplied by partial factors \( \gamma_F = 1.0 \) for permanent loads causing structural collapse, 1.35 for permanent loads causing ground failure, and 1.5 for variable loads causing any ultimate limit state event. Characteristic soil strengths are to be converted to ULS design values by dividing by partial factors \( \gamma_M \) as follows: 1.25 for tangent angle of friction, 1.6 for cohesion intercept in effective stress, and 1.4 for the undrained strength of clays.
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Compared with the Danish code, these factors have marginally affected design values of strength in terms of effective stress, but have quite strikingly reduced the partial factor on undrained strength, particularly in bearing capacity checks. This presumably reflects the judgement that undrained bearing failures are sufficiently unlikely to warrant a higher factor.

While this may correct a theoretical anomaly in DS 415, it exposes engineers to much more risk in their estimation of deformations. Although oedometer tests are satisfactory for the estimation of drained settlement with lateral restraint, the large undrained shear deformations which can accompany loading in the absence of such restraint, and which precede volume changes, are presently poorly understood in practice.

If a student of Brinch Hansen were to compare their DS 415 calculations with those carried out under EC7, the loss of empirical data to fix characteristic values would probably lead to even greater deviations. The practical definition of characteristic strength in EC7 leaves a lot to be desired. As with the earlier Danish code there is no definition of strength, no mention of peak strength softening towards ultimate strength after some small displacement on a rupture surface. Most readers will take it that peak strength is being discussed. This now conflicts with the earlier injunction that characteristic values should reflect field behaviour: when slip surfaces permit failure, the peak strength of sands becomes irrelevant, and the ultimate strength is required.

Engineers will generally use some empirical correlation from CPT or SPT data to estimate $\phi$ for granular soils, and that correlation will probably be based on the peak triaxial strength at the field density (and some possibly unspecified mean effective stress level). Much more variation must be anticipated in the selection of a characteristic value, in the absence of the strong advice given in DS 415. Some will select peak values, some will select ultimate values which they judge relevant to ULS checks, some will correct for stress-level effects, some for plane strain values. The failure to define strength properly makes its factoring a farce, and the subsequent calculations a charade.

4. SOIL STRENGTH

4.1 Total stress analysis

The undrained strength of clay is acknowledged to vary with test type, due principally to the different deformation conditions which tests impose, and to the probably anisotropic nature of material which will have been one-dimensionally consolidated. Kulhawy (1992) draws a comparison between these different test conditions and the soil deformations anticipated in different construction activities: Figure 1. He then establishes a correlation between the different measures of normalised strength, expressed as a ratio of undrained shear strength to initial vertical effective stress. Figure 2 shows Kulhawy's relationships in terms of the ratio between the normalised undrained strength in a test and
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![Diagram of different test scenarios for embankment, loaded wall, and drilled shafts.]

Note: Plane strain tests (PSC/PSE) used for long features.
Triaxial tests (TC/TE) used for near symmetrical features.
Direct shear (DS) normally substituted for DSS to evaluate $\phi$

**Fig 1** Relevance of laboratory strength tests to field conditions (taken from Kulhawy, 1992)

![Graph illustrating normalised undrained strength ratios.]

Note that these are mean lines through data with S.D. on $(s_u / \bar{s}_u) = 0.03$ to 0.05

**Fig 2** Normalised undrained strength ratios (taken from Kulhawy, 1992)
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that developed in a standard isotropically consolidated triaxial compression test. The soils reported in Figure 2 were normally consolidated, but with different effective angles of shearing.

For typical plastic clays with a friction angle $\phi'$ in the region 20° to 25°, the tests with $K_0$ consolidation gave about 90% (compression tests) and 45% (extension tests) of the normalised strength in the standard isotropically consolidated test. Other test types showed normalised strengths falling on or between these values.

No doubt the relatively high value in the standard isotropic test can partly be attributed to having too great a mean effective stress during consolidation. No doubt the spread of values for normally consolidated soils is higher than would be found with for over-consolidated soils, because of their sensitivity to the exact shape of the yield surface.

Whatever else Figure 2 shows, however, it certainly expresses the fact that undrained shear strength is not an intrinsic soil parameter. The strength depends on the excess pore pressures created when the soil is sheared at constant volume, and these pore pressures are a function of the initial shear stress, and the mode of deformation. Kulhawy notes that the variation of strength in terms of effective stresses is much less sensitive to test type.

In principle, correlations such as those in Figure 2 should be used as calibrations, made as part of the process of selecting characteristic values, and should result in strength values appropriate to the modes of deformation of different soil zones, as shown in Figure 1. If there is a perception that extension tests are relevant to the definition of strength in the passive soil zone adjacent to a retaining wall, those tests should be carried out, or published data should be used to convert compression strengths into projected extension strengths. EC1 makes clear that this process of more exact calibration should not be related to the selection of a partial factor, unless it reduces the overall level of uncertainty for example.

The EC7 partial factor on undrained strength of 1.4 should therefore be related, according to EC1, to the typical degree of scatter observed in tests, of whatever sort. Variability of undrained strength in space and time can be real, and due to grain size variations for example. Alternatively, variations may be artificially induced by poor investigation and testing procedures. Variations may be large, or negligible. Although the engineer on the job may well be able to sort all this out, the code drafters do not have the advantage of seeing the ground, and have had to fix a single partial factor of 1.4 against a characteristic value set at, or close to, the lower 5th percentile of the data.

4.2 Effective stress analysis

Figure 3 shows the Mohr-Coulomb strength envelopes in terms of effective stress, which are broadly similar for all soils. The critical state angle of shearing $\phi_{crit}$, which can be treated as a material constant, can be found as the ultimate strength of a soil which is tested
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Fig 3  Strength in terms of effective stress

either loose, or at high effective stress levels
so that the soil volume tends to reduce under shear. At lower stresses, soils which are
dense tend to dilate as they shear giving a
temporary peak strength until the capacity for
volume expansion is exhausted and the strength falls to critical.

Soils comprising more than 50% clay
particles by mass have the extraordinary
capacity to fall in strength below the critical
angle. Shear rupture surfaces develop
polished surfaces with a smaller, residual,
angle of shearing.

The peak dilatant strength can either be
expressed as a secant angle of shearing \( \phi_{\text{max}} \),
or by use of a local tangent \( c' + \sigma' \tan \phi' \). It
has been common in practice to fit tangents,
but this is unwarranted since both \( c' \) and \( \phi' \)
depend on the range of stress being fitted, so
one of the two parameters is redundant.

Bolton (1986) demonstrated that the secant
angle could be written:
Figure 4 is typical of the good fit which can achieved with triaxial test data. $\phi_{\text{dil}}$ is an extrinsic parameter which is solely dependent on the capacity of the soil to dilate on shearing. It is at its largest (circa 20°) when the grains are densely packed, and the stress level is not high enough to permit any grain breakage. $\phi_{\text{crit}}$ is an intrinsic parameter which is a function of the mineralogy, uniformity, and angularity of the grains. For quartz sand it lies in the range 30° (round, uniform grains) to 40° (angular, well-graded) and can be estimated within a degree or so by excavating a loose, dry heap on the desk-top and measuring the angle of repose.

The causes of scatter in angles of shearing resistance determined from different samples of a given soil deposit is now very well understood. Density varies from point to point, so $\phi_{\text{dil}}$ also varies, but $\phi_{\text{crit}}$ remains closely constant within a given formation. The effect of density variation on tangent parameters is to produce a confusing sequence of points which should actually lie on different peak envelopes, but which can be combined statistically to produce probability density functions of $c^\prime$ and $\phi^\prime$ which would necessarily have rather large coefficients of variation, and which would also be statistically dependent on each other for the obvious reason that only one real variable ($\phi_{\text{dil}}$) exists. The case for using secant rather than tangent parameters is therefore overwhelming whether one takes a physical or statistical view-point.
4.3 The brittleness problem

In structural design, the upper yield point of low-carbon steels is ignored, and plastic design is based on the lower yield point. Our structural colleagues also decide not to utilise the reliable strain-hardening which then causes the yield stress to increase up to the ultimate tensile strength: Figure 5. They are concerned to avoid problems with progressive failure and the lack of continuity which would follow complete tensile rupture at any location.

In conventional geotechnical practice we have taken the opposite view, and neglected the reliable ultimate strength in favour of the temporary brittle peak which is much less reliable than the u.t.s. of steel. We then have to employ significant partial factors to guard against progressive failure. This bad practice must stop. All available evidence shows that when a slip failure is back-analysed, whether in the field or in centrifuge models, in slopes or retaining walls, in sands or clays, the soil strength parameter which emerges is the critical state strength.

Ultimate soil strengths must be used in ultimate limit state checks. The attempted mobilisation of a higher value simply results in mass-accelerations, and the probability of injuries, should accidental loads temporarily exceed peak resistances. ULS design to critical states guarantees stable plastic behaviour, at least in the absence of excess pore pressures.

The failure in EC7 to address this issue, which is pivotal to a real assessment of safety in geotechnical design, is deeply disappointing. The pain is all the more hurtful when one reflects that more money is being spent to identify an unreliable parameter $\phi_{\text{max}}$ when much less money can be spent on disturbed samples to identify a reliable parameter $\phi_{\text{crit}}$ which requires no partial factor and which is the appropriate ultimate strength parameter in any event.

![Brittleness](image)

Fig 5 Britteness

5. SOIL DEFORMATION

5.1 Stress-strain curves

Soils require an order of magnitude more strain to mobilise peak strength than do steel or concrete. Even "good" soils usually require 5% shear strains on first loading, though peak strength can be reached within 1% shear strain on unloading or re-loading. It therefore behoves us to take deformation problems rather more seriously.
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Figure 6 shows a close curve-fitting to the undrained shear stress-strain data of overconsolidated kaolin which had already been taken through some large strain reversals; Sun (1990). Very large stiffness at the origin follows the latest reversal of loading, but after mobilising about one quarter of the peak undrained strength the rate of mobilisation of strength with strain reduces markedly. An unload-reload loop is shown to be relatively narrow. If over-consolidated soil in the field had been through the same loading-unloading history, its mobilisation of strength with subsequent strain would appear to be unusually rapid. Only full-scale field tests will reveal where, in the midst of its inevitable nest of hysteresis loops in stress-strain space, the true strain origin of soil lies. There are certain irreducible uncertainties in the prediction of soil deformation.

Figure 7 shows the drained triaxial compression data of a typical soil fill tested in 70mm diameter samples with 60kPa effective confining pressure: Lee (1993). Axial strains were measured externally, but corrections were made for the compression of the rubber discs used to reduce end friction. Various relative densities between 38% and 81% were achieved by pluviation and vibration, but avoiding compaction. Peak strength required between 2.5% axial strain for dense fill and

**Fig 6 Mobilisation of undrained strength**

![Graph showing mobilisation of undrained strength](image)

**Fig 7 Triaxial tests on a well-graded fill**

(from Lee, 1993)
7.5% axial strain for loose fill, corresponding roughly to shear strains in the vertical plane of 4% and 12% respectively.

Once again, the unload-reload loops remind us that the strain to failure in the field will actually be a function of the strain history. Tests in which fill was physically compacted into larger triaxial moulds prior to testing showed a strong similarity to the reload behaviour of the uncompacted material. Fill which has been made dense by gentle vibration is compliant, and may give trouble with deformations. Fill which has been compacted or pre-loaded may require very small strains to reach peak strength and could give trouble with brittleness.

Obviously, the selection of any single stress-strain expression which fails to embody the hysteresis loops, so familiar to soil testers, will be incapable of explaining why the relative density of granular soils is an insufficient indicator of the deformations to be anticipated in the field.

5.2 Plastic deformation mechanisms
On the left hand side of Figure 8 appear the familiar plastic equilibrium solutions for cohesive material around retaining walls, beneath footings, and surrounding piles. On the right hand side of the figure are shown corresponding plastic deformation solutions. Their key feature is the elimination of slip surfaces.

The solid lines marked on the deformation cartoons are zero-extension lines. The 45° wedges around the cantilever wall contain soil all of which shears to the extent $\gamma = 2\theta$ when the wall rotates $\theta$. The soil in the far-field is treated as rigid. This mechanism is adequate for the prediction of near-field strains, conforming well to the results both of centrifuge tests and non-linear finite element simulations: Bolton and Powrie (1988), Bolton et al (1989). The consistency of the equilibrium and deformation diagrams is adequate: "active" and "passive" zones correspond, and the mobilisation of a uniform plastic strength $c_{mob}$ is consistent with the development of a uniform plastic shear strain $\gamma_{mob}$.

In the case of the footing, Prandtl’s near field plastic zone mobilising $c_{mob} = q / N_c$ is also associated with the mobilisation of an average shear strain $\gamma_{mob} = 2w/B$. Here, the constancy of shear strain within the near field has been sacrificed, but an allowance has been made for additional far-field strains using cavity expansion theory. Correspondence between “active”, “fan” and “passive” zones is qualitatively correct. The justification for the mechanism lies in its close relationship to physical and numerical simulations: Sun (1990).

The kinematic solution to the friction pile relies entirely on an integral of strain reducing from a value of $2w_o/d_o$ at the pile interface, to zero at large radius: a sort of power curve used in Figure 6 was chosen to relate mobilised shear stress to mobilised shear strain: Bolton (1992).

It is interesting to note that these “geotechnical” idealisations of familiar soil continuum problems each predict that the
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STATICS

\[ \sigma_p = \sigma_v + 2c_{mob} \]
\[ \sigma_a = \sigma_v - 2c_{mob} \]

KINEMATICS

\[ \gamma_{mob} = \frac{2\theta}{r} \]

\[ c_{mob} = \frac{q}{5.14} \]

\[ \gamma_{mob} \approx \frac{2w}{B} \]

\[ c = \frac{Q}{2\pi r L} \]

pile adhesion \[ c_{mob} = \frac{Q}{\pi d_0 L} \]

\[ \gamma_{mob} = \frac{2w_0}{d_0} \]

Fig 8 Plastic equilibrium and compatibility

DGF

BOLTON
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Shear strain mobilised in soil by the displacement of the adjacent structure is twice the proportional displacement of the structure, defined as lateral displacement over wall height or vertical displacement over foundation width. In order to limit the proportional displacement to 1/200 as a typical serviceability criterion, the soil shear strain would have to be limited to 1%.

5.3 Design for serviceability
This suggests a general approach to the initial design of serviceable geo-structures based on this new application of the theory of plasticity. Select a proportional displacement as a serviceability limit. Double it and obtain the plastic increment of strain permitted in the adjacent soil. Read off the permissible strength mobilised at this strain on a stress-strain curve reflecting the best understanding of the appropriate soil state, stress history, and future load path. Figure 9 demonstrates that the mobilised angle of shearing is just as easily derived in this way as is the mobilised undrained strength. The absolute change of volume of sands in drained tests is negligible up to peak strength, and the constant volume kinematics of Figure 8 will prove adequate.

Once the appropriate values of permissible plastic strength have been obtained, they can be used in conventional plastic equilibrium solutions such as the bearing capacity equation, or earth pressure coefficients. Instead of viewing plastic solutions as restricted to large deformations at constant plastic strength they can now be seen to be useful at intermediate strains on the plastic hardening curve. Elastic assumptions are no longer necessary.

The final step necessary with clays is to open the drainage and observe transient flow and the dissipation of excess pore pressures. Long-term consolidation settlements can then be calculated and added to the immediate undrained displacements, following the technique of Skempton and Bjerrum.

Although this approach is apparently new in geotechnical engineering, it would be familiar to structural and mechanical engineers who have to deal with annealed non-ferrous metals for which the degree of "cold working" (i.e. strain hardening) is the key to predicting the current yield stress. In aluminium air-frames it was long the practice to set as a "proof stress" the tensile stress at which the degree of irrecoverable axial strain would be acceptable; a 0.2% proof stress indicated that 0.2% plastic extension could be expected if the members had not been pre-loaded. In this spirit, what is proposed here is the use of a 1% proof shear stress for soils (or such other strain limit as may be desirable in a particular case).

5.4 The mobilisation factor
The suggested deformation limit of 1% shear strain would be mobilised in a triaxial test at about 0.67% axial strain. Conventional testing techniques with external strain measurement are perfectly adequate at this strain level, if
Fig 9 The 1% proof shear strength

Fig 10 The mobilisable strength

Fig 11 Soil-structure interaction
care is taken with sample trimming and the elimination of bedding errors by taking the sample through a cycle representative of its recent stress history.

Nevertheless, it may well be helpful to suggest certain strength reduction factors which might on average safeguard against excessive deformations. A mobilisation factor $M$ is defined:

$$M = \frac{T_{\text{peak}}}{T_{\text{mob}}}$$

which for undrained strength becomes

$$M = \frac{c_u}{c_{\text{mob}}}$$

and for effective stress parameters

$$M = \frac{\tan \phi_{\text{max}}}{\tan \phi_{\text{mob}}} \text{ or } \frac{(c' + \sigma' \tan \phi)_{\text{max}}}{(c' + \sigma' \tan \phi)_{\text{mob}}}$$

Sands which have been pre-cycled by compaction, or which are to be subjected to lateral unloading behind a retaining wall, are well known to require very much less strain than 1% to mobilise peak strength.

Quartz sands subject to virgin loading at moderate stress levels ($\sigma'_r < 100\text{kPa}$) may require $M = 1.5$ if dense ($I_D > 75\%$) and $M = 1.75$ if medium dense ($75\% > I_D > 50\%$), the higher factor also applying to dense sands at higher stress levels ($100\text{kPa} < \sigma'_r < 400\text{kPa}$). Sands which are loose, or which are more heavily stressed in relation to the crushing strength of their grains, require larger factors and are also subject to significant volume reduction which must separately be accounted for. These suggested mobilisation factors could be reduced by about 20% if the serviceability criterion were relaxed to 2\% shear strain, and similarly increased by 20\% if the deformation limits were tightened by a factor of two.

Clays must be tested wherever possible, but a factor $M$ in the range 1.5 to 1.75 would generally seem to restrict over-consolidated clays to within 1\% shear strain. Normally consolidated clays require larger factors and are also subject to significant volume reductions which must be assessed.

6. **Mobilisable Strength**

Safety demands that no attempt be made to mobilise more than the ultimate strength of soil, which can usually be identified as a critical state. Serviceability demands that a mobilisation factor in the range 1.5 to 1.75 be used to reduce design strength below its peak value, except where the soil is to be unloaded or where it has been pre-cycled. The mobilisable strength of soil, accounting for both large and small deformations, could be taken to be the smaller of the two alternatives, as shown in Figure 10.

In case (a) the soil design value is deformation limited because the permissible strength for serviceability is below the ultimate strength. In case (b) the soil design value is strength limited, because the ultimate strength falls below the strength at which permissible
strains are first exceeded prior to peak. Case (a) exhibits the compliance problem; case (b) exhibits the brittleness problem. Neither case is explicitly recognised in EC7, though the consequences for the rational design of associated structures is profound.

It is satisfying to realise that the strengths which would be arrived at by applying these new rules are very similar to the strengths to be derived by applying the more conservative advice for characteristic strengths together with the recommended partial coefficients which appeared in the old Danish code. This feeling of satisfaction must be tempered when it is recalled that the proposals in the new Eurocodes reduce the Danish partial factors, and relax the conservative Danish advice on the estimation of characteristic strengths.

7. THE STRUCTURAL INTERFACE
The mobilisable strength may not in fact be mobilised if structural constraints prevent soil shear strains reaching 1%, or if other influences occur (swelling of clays, cyclic loading of sands etc). Consider the example of a propped cantilever wall: Figure 11.

The wall embedment can be selected by considering the mobilisable soil strength, deducing earth pressure coefficients, and taking moments for equilibrium about the prop. The soil in every region will have $M = M_{\text{target}}$.

The wall section can be selected using the same earth pressure distributions, but more unfavourable cases might exist. If the passive pressure in region P is fully mobilised (i.e. $M_P = 1$) by clay swelling, larger active pressures $A$ will be induced for equilibrium ($M_A = M_{\text{target}}^2$). A similar situation will arise if the active pressures in region A are not mobilised due to cyclic loading (e.g. compaction of granular fill). The pressures in region P will then be forced to rise to hold the wall in equilibrium. These are examples of pressure distributions which would give the most onerous load effects in structural serviceability checks. These cases are similar to the problem of locked-in stresses in steel frames, and are not to be considered at ULS since the structure would be capable of deforming so as to relieve the extra pressure. The maximum possible pressures are derived logically by soil mechanics, so the structural bending moments etc are conservatively derived and need no further partial factor.

8. CONCLUSIONS
8.1 Partial factors
1) Partial factors are properly applied when engineers have been unable to select the correct design value directly on the basis of their own tests. Code writers step in with a large and relevant data base which modifies the perceptions of engineers who had only seen their own data.

2) In production engineering, sampling and quality control offer an example of the proper use of partial factors based on statistics. In geotechnical engineering it is always better for the person with the ground investigation report to select appropriate values.
3) The agenda of the reliability caucus in the Eurocode process has failed through inevitable lack of data. In calibrating against existing codes, the drafters have found no scientific grounds to put factors in one place rather than another, so they have put them everywhere. This is the opposite of the ideal of limit state design in which nominal (unscientific, unmeasurable) influences were to be replaced by objective determinations. No partial factor should be accepted in the absence of a scientific justification.

4) Evidence suggests that serviceability checks will be more critical than the collapse checks arbitrarily imposed in the current draft of EC 7. Whereas collapse is an extreme value problem, deformation of a strain-hardening plastic material is a problem for averages. A soft spot tends to carry less than its fair share of load, so that the average displacement is controlled by the average stiffness in the zone of deformation. If a statistical treatment of serviceability were to be devised, it would be on an entirely different basis than the poorly constituted problem of uncertainty connected with the tails of probability density functions.

8.2 Soil mechanics

1) Ultimate limit states of soil bodies must be checked using ultimate soil strengths, consistent with large deformations.

2) Serviceability limit states of soil bodies must be checked using the strength-strain mobilisation curve which leads to peak strength. This is highly non-linear.

3) The theory of plasticity includes kinematics as well as more familiar equilibrium concepts. Deformation mechanisms with distributed plastic strains, and no slip surfaces, are available to describe plastic strain-hardening up to peak strength. "Geostuctural mechanisms" which approximately satisfy both equilibrium and compatibility have been devised for many problems.

4) A good rule of thumb is that a properly defined proportional displacement of a structure will mobilise soil shear strains of double that amount. This indicates how much strain-hardening can be allowed in the soil up to the limit of tolerable displacements.

5) A mobilisation factor $M$ reducing peak strength needs to be in the region 1.5 to 1.75 even in "good" soils such as overconsolidated clays and quartz sands, unless the soil is unloading or has been pre-cycled. It is suggested that the partial factors listed in EC7 under ULS checks be replaced by the newly proposed factors, which should appear under SLS checks. These new factors satisfy the scientific test imposed above in section 8.1, conclusion (3). In the medium term, engineers should be encouraged to perform more reliable stress-strain tests on representative samples.

6) A mobilisable strength is one which exceeds neither the ultimate or serviceability limits. It can be used freely in plastic equilibrium analyses.

7) If structures can not tolerate soil displacements of any significant extent, they become vulnerable to self-stressing effects in the soil, such as swelling in clays or compaction effects in sands. Upper bounds can be placed on these effects using soil mechanics principles.
9. REFERENCES


