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MODELLING

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MECHANICS AND SOILS

The critical state ideas that developed in the 1960's reconciled the behaviour of elementary volumes of soil with the plastic behaviour of elementary volumes of steel. This paper concerns the current development of Pokrovsky's method of modelling composite construction involving larger volumes of soil, and the analysis of such "soil construction" by methods established for steel structures.

The elementary piece of a steel construction (Figure 1) is simply characterised as (a) elastic or (b) plastic, as it flexes and then yields. When a steel portal frame is in use its ability to carry vertical loads V may be affected in some cases (Figure 2) by the presence of a load H at right angles, or in other cases (Figure 3) by an imperfection δ at right angles to the line of action of V. In what follows it will be a help to refer to these cases of the analysis of the performance of steel-construction in order to make an analysis of the observed performance of models of soil-construction.

The words soil-construction are adopted here because the words soil-structure have previously been used to mean the internal fabric within the elementary volume of soil. This paper proceeds from the standpoint of Critical State Soil Mechanics that the elementary volume behaves according to the principle of effective stress; that all time effects relate to steady or transient flow of a pore fluid phase; and that the effectively stressed soil phase is a rigid or elastic or plastic continuum which is time independent. Alternative hypotheses that continue to be explored by others would, if established, invalidate or at least imply some limit to the usefulness of Pokrovsky's method of centrifugal modelling. For example Bjerrum (1973) quotes a curve

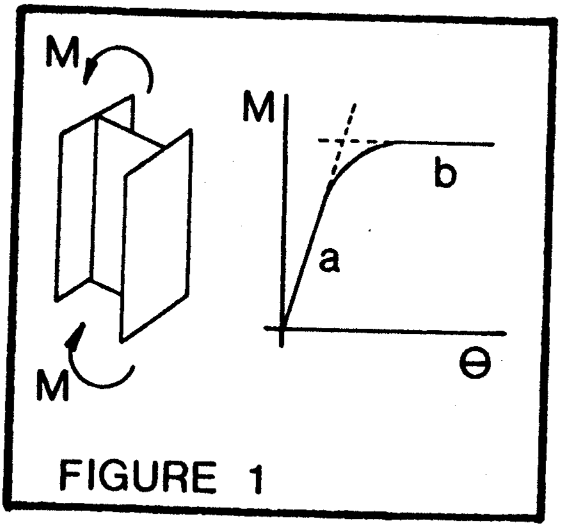

FIGURE 1

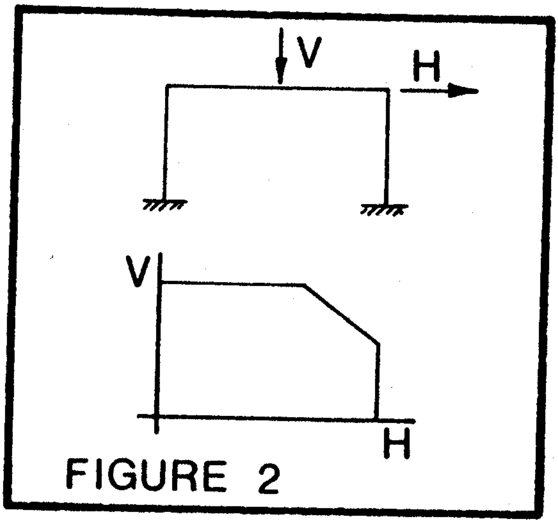

FIGURE 2

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(Figure 4) that shows a strength increase as time-for-deformation decreases. If a model test only takes 10 minutes while the prototype soil-construction really takes 36,000 minutes (25 days) then this creep effect would imply that in a case when Pokrovsky's model appears to have a safety factor of 25 percent the real prototype would fail. A different hypothesis by Palmer and Rice (1973) implies that the absolute size of the region affected by propagation of a progressive failure surface is fixed for a given type of soil and can not be scaled down in models - an effect that they describe as 'catastrophic' for the prospects of centrifugal modelling. There are many different varieties of soil and there may well be certain soil-construction for which creep-effects and progressive-failure effects become central to the understanding of overall behaviour. However this paper is concerned with the analysis of the class of soil for which the behaviour of a reasonably large block of soil - say about a quarter of a tonne - appears in Pokrovsky's model to be in general similarity with prototype behaviour. To date all soils tested have been in this class, but with refinement of experimental observation it is expected that different classes of soil will occur with behaviour that will require refinement of the present admittedly simple analysis.

In retrospect it seems that the principal value of critical state soil mechanics was that its simple set of calculations began to explain quite complicated data of non-linear stress-strain behaviour without appeal to special principles unfamiliar in the wider field of applied mechanics and engineering plasticity. Once we understood this to be the case, many earlier uncertainties about the validity of Pokrovsky's centrifugal model test were removed. Although certain technical problems had to be surmounted in novel developments of centrifugal tests with measurement of the relevant effective stress parameters, the centrifuge group was sustained in its work by the confidence that well-known simple principles of applied mechanics must apply so that useful analogue models of real soil-construction could surely be made. Others who have concentrated on digital models, have valued critical state soil mechanics as the point of departure for the development of new test-equipment and newly refined calculations to introduce non-linear characteristics of real soil into large finite element or other calculations, and have discussed this in many papers. Both developments currently are proper outcomes of the development of critical state soil mechanics in the 1960's.

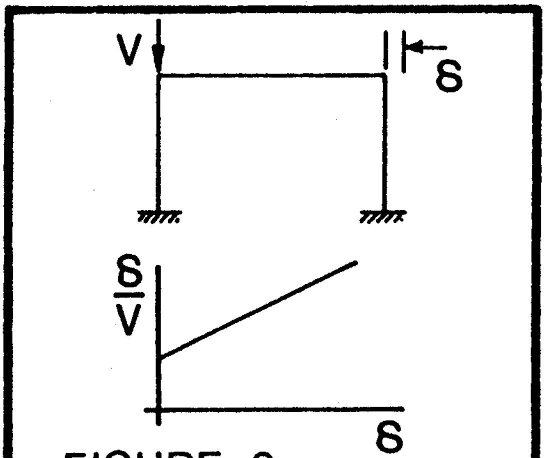

FIGURE 3

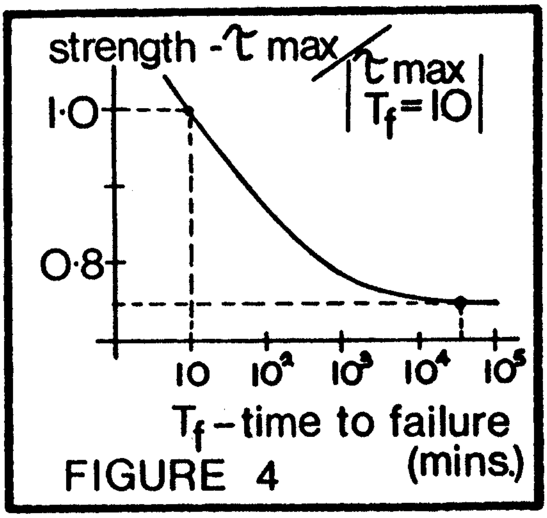

FIGURE 4

A FAILURE ENVELOPE FOR A FLOOD LEVEE

The state of equilibrium of soil-construction may be perturbed by various causes - by extremes of temperature, by explosion, by earthquake vibration, by changes of groundwater levels, by external static loading etc. Pokrovsky has applied his method widely, including the explosion problem, but our work has been limited to the variation of water levels and the application of static loading.

Referring to Figure 2 we see two axes V and H indicating the level of perturbation by two independent causes and the envelope of three straight lines indicating limits at which 'failure' occurs. In this sense 'failure' means a state where a little further perturbation results in an uncontrolled change of state. The safety factor F, F' or F" could be defined in various ways for a state P (VH) in Figure 5: the dotted line shows failure (1) when V increases to FV and H stays constant, the dashed line shows failure (2) when H increases to F'H and V stays constant, the chain dotted line shows failure (3) when V and H both increase by a factor F". An engineer may attach a lower probability to a change of one parameter than the others and may calculate a weighted factor F (F, F', F") in economic appraisal of a steel-construction. Similarly with soil-construction a safe state can be thought of as a point at a safe distance from limiting boundaries in any direction.

It is usual to characterise the stability of a soil embankment slope by the non-dimensional group $C/\gamma H$ relating soil cohesion C to the product of soil density γ and slope height H. For a given slope angle and soil internal friction angle this group falls to a calculable value at slope failure: while it exceeds this value by a factor $F > 1$ the slope is safe by that factor. Engineers normally regard the determination of the value of C as open to more error than γ and H, and usually consider F as applied to soil strength (C/F). However a 1/N scale model tested in a centrifuge can first be brought into equilibrium at acceleration N times gravity which makes it similar to the full scale soil-construction and then the centrifuge speed can be rapidly increased until failure is observed at F N times gravity. In this case we would argue that, in the centrifugal model, undrained shear strength C of such a large block may not be open to the same errors as would be expected

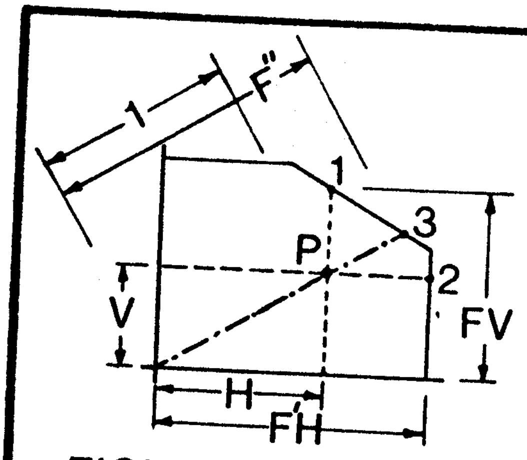

FIGURE 5

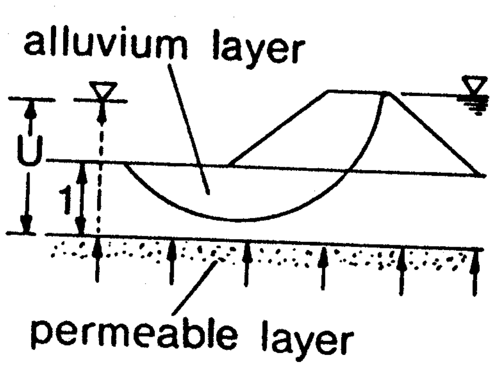

FIGURE 6

in many small test samples, and we must then relate the centrifuge factor F of self weight increase to a non-dimensional group $C/[(F\gamma N)(HN^{-1})]$ with F linked to γ and not to C. It is easy to determine such a factor F by a quick speed change in a centrifuge test and we will plot values as if they lie on a failure surface because this F is more or less the same factor that is normally used by engineers in practice: of course there is no probability of there being a change of gravitational acceleration on a normal slope.

A more realistic perturbation of the model is a change of the ground water level or a change of applied load at constant speed, while the acceleration on the model remains constant at N times gravity and the model remains correctly in similarity with the prototype soil-construction. This may become clearer in an example - the problem of the resistance of flood levees to uplift.

Figure 6 shows a layer of alluvium of unit thickness above a permeable layer with an artesian condition uplifting water pressure to a head height U above the top of the permeable layer. There is a bank to the right of Figure 6 holding back a flood. Will this uplift reduce the bank's safety factor ? When the series of models were tested in our centrifuge each was first brought into equilibrium at the point U = 1, F = 1, in Figure 7. The uplift was then set at a certain value and the centrifuge speed quickly increased until failure occurred at states represented by the points marked by the crosses in Figure 7. Each cross corresponds to a mechanism of failure with the slip circle through the bank and through the alluvium as shown in Figure 6.

The square point in Figure 7 represents a second, different, mechanism - the failure of the alluvium by bursting and blowing up when the uplift head in the permeable layer equals the density of the overlying alluvium, without failure by slipping of the bank. To further explore this second mechanism a different test path was followed (Figure 8.) After bringing the model into equilibrium at U = 1 and F = 1, the centrifuge speed was quickly raised until the bank was very near failure. Then the uplift head was steadily increased, so that the state of the soil-construction traversed a path (Figure 8) closely below the line of crosses. That model failed with a

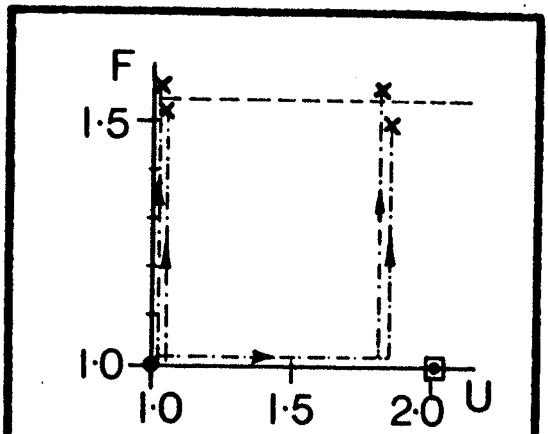

FIGURE 7

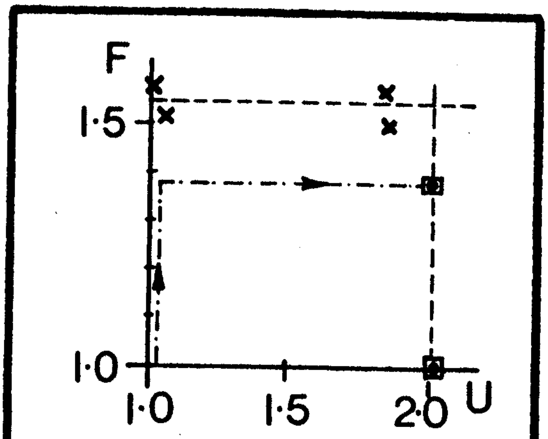

FIGURE 8

blow hole in the alluvium. So it appeared to be a reasonable approximation to put a corner on the failure surface with no intermediate mechanism involving interaction between uplift and bank slip.

Such model test results require confirmation with a variety of natural alluvium and embankment soil samples before they can be relied upon in design decisions. However the analysis of the test results and the planning of critical tests is one illustration of the power of the concepts of mechanics to clarify problems of soil-construction.

A SOUTHWELL PLOT FOR A BURIED PIPE

Soil pressure investigation on sewers by means of models was first undertaken by Pokrovsky (1934) on the instructions of the Academy of Municipal Economics of R.S.F.S.R.: Pokrovsky and Federov (1969) show continued interest in pressures on tunnels thirty five years later. In this general topic of such inexhaustible interest we will consider only some small problems of the buckling of a large thin pipe buried in compact gravel. Specifically such construction was used in the cooling water system of a thermal power station where a welded steel pipe of about 4 m diameter and 20 mm wall thickness and one kilometer length transfers hot water under about 100 m head to the top of a cooling tower. The return-flow culvert is unpressurised and pressure-pipe and culvert are constructed side by side in a wide trench backfilled with gravel. In general the pipe is full of water under pressure but we must consider the problem of a rise of ground water level outside the pipe during the construction period while it is empty.

In the buckling of a long straight beam of stiffness EI (Figure 9) attached to a subgrade of stiffness k it is calculated that the buckling wavelength L depends on the relative stiffness of beam and subgrade $L \propto \sqrt[4]{EI/k}$. A curved beam of radius R can carry an axial thrust T if it is bedded on a subgrade with a pressure p which can be calculated simply from statics as $p = T/R$ (Figure 10). Because the soil is stiff and the structure is flexible in the case we are considering, the cross section of the pipe can be regarded as almost a straight beam in plane strain and the first buckling waves will have length $L \propto \sqrt[4]{EI/k\ (1 - \nu^2)}$ if soil pressures cause failure.

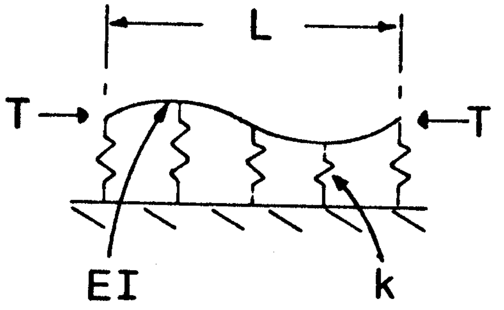

FIGURE 9

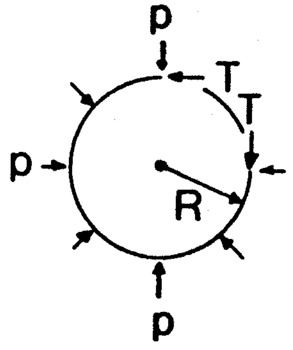

FIGURE 10

Centrifugal model tests of thin pipes buried in sand showed failure to be initiated with a little longitudinal crease of short wavelength in the lower half of the pipe. The crown (the pipe top) moves down when the pipe wall slips towards the little crease and the soil that moves down above the crown is virtually a fluid, without the stiffness of the bed that supported the pipe invert. So the pipe crown buckles with a long wave after the crease has formed (Figure 11). These problems have been well analysed by Meyerhof among others. Less is known about our problem of the effect of rising ground water which tends to lift the pipe invert up from the soil bedding, and centrifugal model tests of this problem were the subject of a special study.

The model shown in Figure 12 had electric resistance strain gauges which measured the change of bending moment and hence the change of curvature of the pipe invert. As the ground water level rose the bending moment changed steadily as H increased in Figure 12. The plot of M/H against M has the familiar form of Southwell's plot of central deflection δ of a strut loaded with a thrust V against δ/V. From the slope of the line it is normally possible to predict the thrust that will buckle a strut, and similarly in this experiment the slope of the line in Figure 13 gave a good prediction of the ground water level at which the pipe invert would buckle in a long wave. The invert moved away from the bedding but it appeared by back calculation of the failure condition in this experiment that the crown and the soil above it did not move (Figure 14).

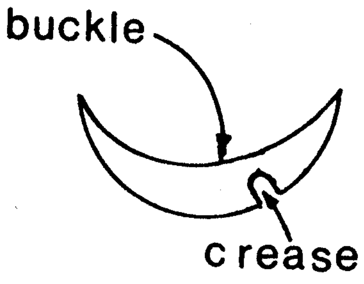

FIGURE 11

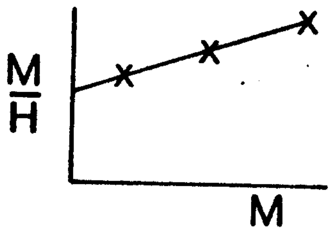

FIGURE 13

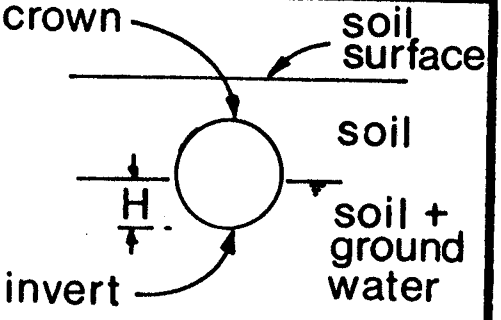

FIGURE 12

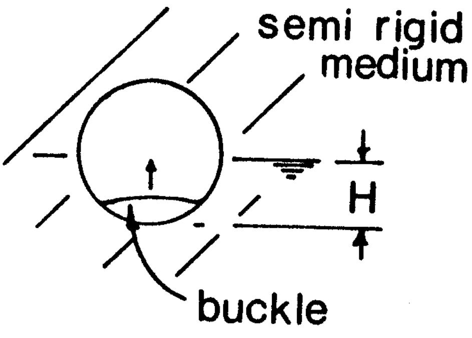

FIGURE 14

Much more can be done on the buried pipe problem in the centrifuge. The technique of observing the growth of imperfections has application in the inspection of prototype pipes after construction. It will be possible to inspect completed construction by taking observations while a heavy vehicle is moved across the soil surface above the buried pipe. Full scale tests on large diameter thin buried pipes are much more costly and time-consuming than centrifugal model tests and the work described above is another example of the usefulness of laboratory tests to clarify the mechanics of soil-construction.

MODELLING SOIL-CONSTRUCTION

We have introduced no new principal in this paper. Three of us were unborn and the fourth barely three years old when Pokrovsky was first applying his principles of centrifugal modelling to a problem of 'municipal economics': we were all taught the principals of engineering plasticity and of structural stability in our engineering courses. One object of this paper is to demonstrate that engineering research workers can engage in useful new explorations and acquire expectations they would not otherwise have had even if they use previously established principals in modelling the realities with which they deal. Only one new idea was introduced in critical state soil mechanics - that soil when sheared with large distortions comes into critical states in which its density is uniquely determined by its effective pressure - and with this notion it proved possible to explain the data of triaxial tests on principles familiar in plasticity. It has not even been necessary to introduce that notion in this paper's discussion of models of soil-construction, where earlier notions of applied mechanics have proved sufficient for the creation of useful predictive models.

It can be confusing to introduce new notions. Pokrovsky first correctly predicted the scale factor for times of consolidation in models of scale n on the basis of "pore water filtration", but then in 1935 produced an alternative calculation of time scale factors on the basis "that the velocity of plastic deformation of the ground is proportional to the pressure gradient". Both calculations gave the same n^2 factor, but their bases are different. There may be ground conditions that are correctly analysed in terms of total stress with a linearly viscous model, but they are quite different conditions from the diffusion-controlled quasi-static deformations of ground considered in critical state soil mechanics.

Perhaps our concern to avoid appeal to new principles may appear limiting, and our other object in this paper is to demonstrate the new scope that we see in the exploration of the mechanics of soil-construction. In the first phase of development of plastic design of steel frame structures there was emphasis on the elementary piece of construction, on the effect of axial load on limiting bending moment etc. In a later phase there was interest in the whole construction, in the mechanisms of failure of steel portal frames etc. Now that we know that many soils are not strange materials we can enter a new phase of application of plasticity (and other systems of applied mechanics) to problems of soil mechanics, to study of the mechanisms of failure of construction etc.

ACKNOWLEDGEMENT

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