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LETTER TO THE EDITOR

DISCUSSION ON: A MATHEMATICAL MODEL OF PERMEABILITY ALTERATION AROUND WELLS

(*J. S. Olarewaju, International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 14, 191–207, 1990*)

The paper by Olarewaju[1] concerns the non-steady-state flow of water around a borehole, producing at a constant rate, where the permeability close to the borehole differs from that in the bulk soil. A solution to the transient flow equation for an axisymmetric reservoir with a permeability discontinuity is presented in terms of modified Bessel functions. We are currently examining the installation effects of grouted bodies and, as part of this work, have developed a finite difference solution to the radial transient flow equation, such a method being generally more flexible. In particular, this allowed us to examine the same problem.

The basic task is to solve the radial transient flow equation:

$$\frac{\partial^2 P_D}{\partial r_D{}^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \qquad (1)$$

subject to boundary and continuity constraints, where P_D is the dimensionless pore pressure, r_D the dimensionless radius and t_D the dimensionless time as defined in Olarewaju.[1] Olarewaju presents, graphically, solutions for a range of the ratio between the permeabilities of the inner and outer region, and also for differing values of the radius of the transition from one zone to another. Our results differ from those of Olarewaju by orders of magnitude, and it is necessary to consider how the results may be checked.

Firstly, consider the simpler case of uniform properties, the solution of which corresponds to the initial portion of the solution to the composite case. There is, thus, a single region, of inner radius $r_D = 1$ and outer radius r_1, equal to the transition radius.

The transformation $R = \ln(r_D)$ and $\lambda r_D = t_D$ reduce equation (1) to

$$\frac{\partial^2 P_D}{\partial R^2} = \frac{\partial P_D}{\partial \lambda} \qquad (2)$$

which is one form of the one-dimensional Terzaghi consolidation equation. The conditions are, for $0 \le R \le D \; (= \ln(r_e))$,

$$\frac{\partial P_D}{\partial R} = -1 \quad \text{at } R = 0$$

$$P_D = 0 \quad \text{at } R = \text{D}$$

$$P_D = 0 \quad \text{at } \lambda = 0$$

and the problem is to calculate the variation in the well pressure P_0. An approximate solution to this can be found using the classic method of parabolic isochrones. At a given time λ, the isochrone will have penetrated to a depth d, as shown in curve 1 of Figure 1. In this problem, the slope $\alpha = 1$.

This solution will be valid until the isochrone reaches the outer boundary. From the properties of a parabola and consideration of flow rates and volume changes, the approximate solution expressed in the original variables is

$$P_0 = \sqrt{(\tfrac{4}{3} t_D)}, \quad 0 < t_D < \tfrac{1}{3} r_1 \ln^2(r_1) \qquad (3)$$

With this approximate solution it is now possible to proceed. Figure 2 presents our finite difference prediction of the pressure behaviour in a composite system where the permeability of the outer region is 10 times that of the inner region, compared with equation (3). The approximate solution agrees quite well with the finite difference solution for small time factors, and both agree qualitatively with the solutions presented by Olarewaju but with a difference in the time factor of the order of 100 times.

Figure 3 shows the effect of the transition ratio for a permeability ratio 10, while in Figure 4 the permeability ratio is 0·1 and the time t_D has been divided by the ratio of the transition radius to the well radius. Figures 3 and 4 should be compared with Figures 3 and 5, respectively, of Olarewaju's

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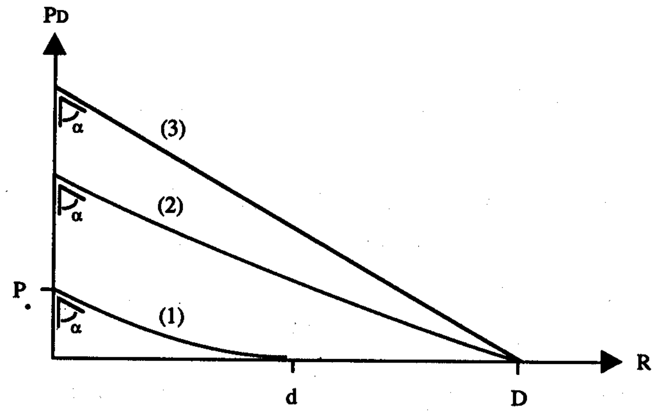

Figure 1. Sketch of pressure variation in the transformed region

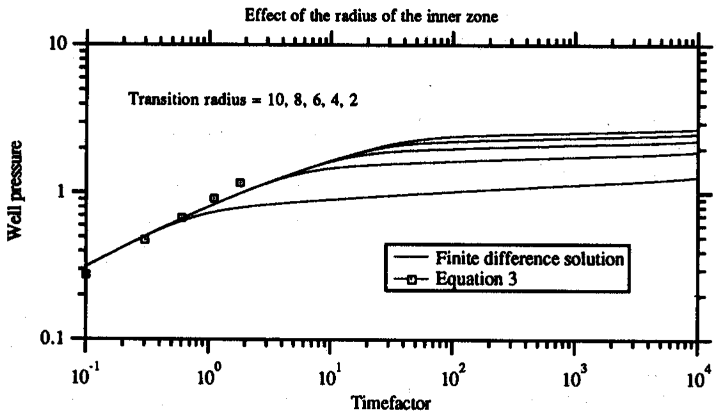

Figure 2. Effect of the transition radius (permeability ratio = 0·1)

paper. It is clear that, although the finite difference solutions appear to be similar in form to Olarewaju's solutions, they differ quantitatively to a very great but variable extent.

In the case of uniform properties, if the outer radius is very large compared to the well radius, then the well could be treated as a line source. This problem was solved exactly by Theis,[2] and by Jacob,[3] the former by analogy with heat flow and the latter directly. These two solutions are identical. Although both Theis and Jacob were concerned with transient flow in a confined aquifer where the strains were vertical, while Olarewaju considers horizontal strains to dominate, the resulting equation is identical in form. Specifically, it is the coefficient of consolidation which differs. If the specific parameters of the above problem are substituted into the solution of Theis and of Jacob, then the pore pressure as a function of radius and time is

$$P_D = \frac{1}{2}\left\{-0{\cdot}577216 - \ln(x) + x - \frac{x^2}{2{\cdot}2!} + \frac{x^3}{3{\cdot}3!} - \frac{x^4}{4{\cdot}4!} + \cdots\right\} \tag{4}$$

where $x = r_D^2/4t_D$.

Although equation (4) might be expected to be valid only at some distance from the borehole, Jacob asserts that it can be used to measure the drawdown at the borehole wall. The specific case of uniform permeability and an external radius equal to 100,000 times the inner radius was solved by both the finite difference method and by this line source method, the results being shown in Figure 5. The finite difference solution and the line

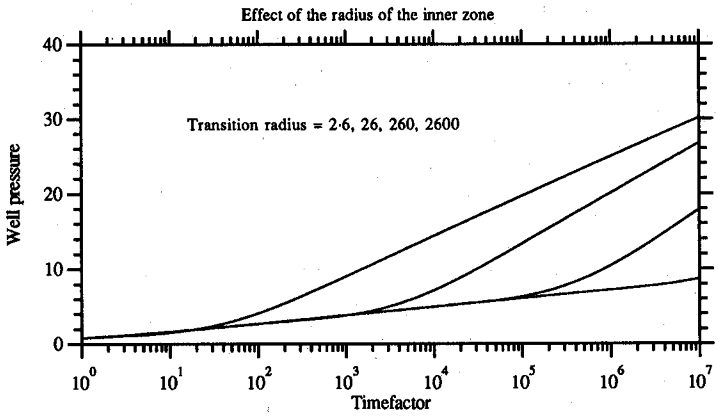

Figure 3. Effect of the transition radius (permeability ratio = 10)

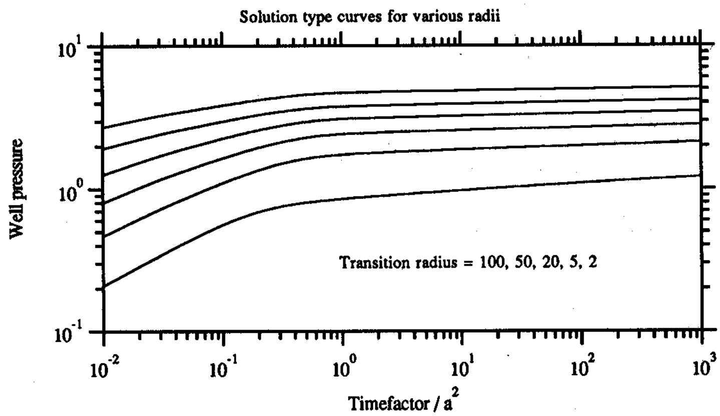

Figure 4. Effect of the transition radius (permeability ratio = 0·1)

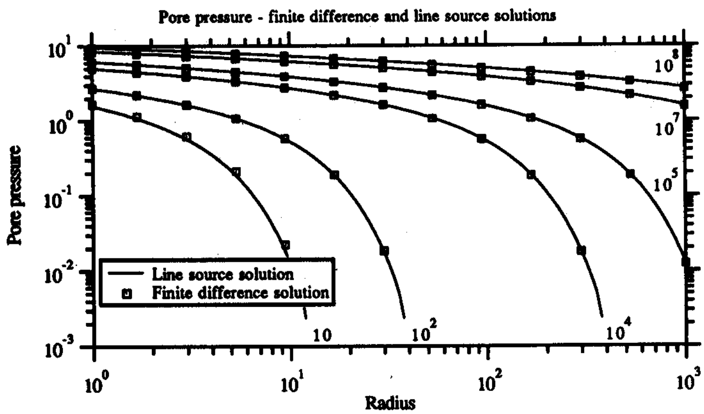

Figure 5. Radial distribution of pore pressure for uniform permeability

source solution are almost indistinguishable, even close to the borehole wall.

The data presented from our finite difference solution appear to be consistent with the approximate solution based on parabolic isochrones, and with the analytical solution for a line source, but not with the data presented by Olarewaju. Moreover, Olarewaju's data do not appear to be internally consistent. A number of examples follow.

For a permeability ratio = 0·1 and transition radius = 10, at $t_D = 10^5$, which is a value of t_D/a^2 of 10^3, Olarewaju's Figure 2 indicates a well pressure of about 3·8, yet his Figure 5 indictes a value of 2·7; at $t_D = 10^3$ these values are 1·6 and 2·4, respectively; and at $t_D = 10^0$ they are 0·11 and 0·26, respectively. For a transition radius = 2, $t_D = 10^3$, the values are 0·73 and 0·98, respectively. A comparison for a permeability ratio 10 is not possible, since no set of data is common to both Olarewaju's Figures 3 and 4, but from the latter the pressure at $t_D = 10^5$ for a transition radius of 10 is 5·0, whereas from the former the pressure should be between 43·1 and 6·9, the pressure at a transition ratio of 2·6 and 26, respectively.

The above discussion concerns the solution of the radial transient flow equation. However, it also appears to us that this equation has not been formulated entirely correctly either by Olarewaju or by Jacob, since equation (1) is valid for the excess pore pressures, not for the total pore pressures. Both authors propose that an increase in the bore pressure causes immediate flow into the soil, generating isochrones of pore pressure of the form shown in Figure 1. However, this is not so, since any change in the well pressure causes a uniform increase of pressure throughout the soil except at other points where the pore pressure is held constant, such as an external boundary. More correctly, a wavefront across which there is a jump in pore pressure equal to the change in the well pressure propagates through the soil at the speed of sound in water.

If, as proposed by Olarewaju, the outer boundary is impermeable, then this would cause a change in the pore pressure which was the same at all radii, causing no additional flow at all. If the outer boundary is taken as being of fixed pore pressure, then it is clear that flow will occur first at this outer boundary, and will only occur at the borehole once the isochrone reaches the inner wall, after a time lag. The steady-state solution is, of course, the same. The question then arises as to what value should be assumed for the outer radius, and it seems possible that this will depend on the speed of sound and on the time lag, as well as on the geometry of the site. It is our opinion that more careful consideration needs to be given to this, before the above solutions are widely applied.

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