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Bored pile design in stiff clay II: mechanisms and uncertainty

1 **Paul J. Vardanega** MEngSc, MIEAust, A.M.ASCE
Graduate Researcher, Department of Engineering, University of Cambridge, UK

2 **Michael G. Williamson** MEng, MA
Graduate Researcher, Department of Engineering, University of Cambridge, UK and Ove Arup and Partners, London, UK

3 **Malcolm D. Bolton** FREng, MA, MSc, PhD, CEng, MICE
Professor of Soil Mechanics, University of Cambridge, UK

The soil mechanics related to pile design in clay has been the subject of substantial engineering research. In a companion paper, various codes of practice were reviewed showing the effect on pile capacity of the different global factors of safety that emerge from the various partial factor combinations for the ultimate limit state. Factors of safety are generally specified based on the opinions of experts. In this paper an assessment will be made of various objective procedures that can be used to reduce uncertainty in the design process, especially regarding the adoption of a pile resistance model and the selection of a soil strength profile as part of a ultimate limit state check, and the estimation of pile head settlement in the context of a serviceability limit state check. It is shown that both total stress and effective stress calculation methods are applicable in London Clay. Estimates of settlement using a non-linear soil stress–strain relationship are made and compared with published data. It is shown that the compression of the concrete dominates the settlement of long piles. Given the low settlements observed, recommendations are made for a reduction in standard factors of safety for bored pile design in stiff clays.

Notation

A_b	area of the base (m^2)
c_u	undrained shear strength (kPa)
D	pile diameter (m)
E_c	elastic modulus of reinforced concrete (kPa)
F	factor of safety
G	permanent load
K_0	coefficient of earth pressure at rest
K_s	earth pressure coefficient for the pile shaft
L_{pile}	total length of a pile (m)
M	mobilisation factor
N_c	bearing capacity factor
N_{60}	SPT blowcount
Q_b	base capacity of the pile
Q_h	pile head load
Q_s	shaft resistance of the pile
$\tan\delta$	effective coefficient of friction between the pile shaft and the clay
u_0	initial pore pressure
V	variable load
w	pile axial displacement
w_h	pile head displacement
z	depth below ground surface (m)
z_c	depth below top of clay layer (m)
z_w	depth of water table below ground surface (m)
α	adhesion factor
$\gamma_{M=2}$	mobilisation strain at 50% of c_u
γ_c	unit weight of concrete
γ_w	unit weight of water
σ_c	wet concrete pressure
$\sigma'_{h,0}$	initial lateral stress (kPa)
σ_m	mud pressure
σ'_s	radial effective stress (kPa)
$\sigma'_{v,0}$	pre-existing effective vertical stress (kPa)
τ	shear strength mobilised
ϕ_{crit}	critical state friction angle

1. Introduction

Terzaghi and Peck (1948) suggested that the global factor of safety for foundations should range from 2 to 3. Meyerhof (1970) reviewed the safety factors commonly used for foundations (Table 1). He described the factor of safety as 'the ratio of the resistance of the structure to applied loads' used 'in order to ensure freedom from danger, loss or risks'. Vardanega *et al.* (2012) used several codes of practice to determine the design capacity of a bored pile in stiff clay. Most modern codes now split the overall safety factor into partial factors reflecting different uncertainties in expected loads and resistances. The various combinations of partial factors in Eurocode 7 (BSI, 2010) lead to an overall factor of safety (FOS) of around 2·5, depending on the particular design method that has been used. The AASHTO (2007) bridge code is written in load and resistance factor design (LRFD) format; it calls for average or expected soil parameters to be used rather than cautious estimates and accordingly imposes a higher global FOS. The Russian SNiP (1985) code for buildings and structures foundations has a significantly lower global factor of safety (around 1·70) but it is based on remoulded soil properties, not on peak soil strengths. These different codes can therefore be seen as specifying an overall safety factor for foundations in the same range as that recommended by Terzaghi and Peck over 60 years ago. Indeed, the drafters of a new code usually ensure that a typical structure designed under its auspices is similar to the structure that would have been designed according to its predecessor, a process known as 'code calibration'. This conservative step effectively guarantees that overall safety factors will, on average, remain the same even though modern codes of practice are superficially more elaborate.

Irrespective of code requirements, however, the designer faces various sources of uncertainty in completing design calculations. This paper considers some fundamental uncertainties that arise in selecting an appropriate soil mechanics calculation procedure for bored piles in stiff clay, and in choosing an associated soil material parameter from a scattered distribution of test values. It does so not only in regard to familiar ultimate limit state (ULS) criteria for the pile plunge load but also in terms of pile head settlements in relation to serviceability limit state (SLS) criteria. For the ULS check, the alternatives of total stress analysis (the α method) and effective stress calculations (the β method) will be considered. A ULS check by the α method will depend on the definition and selection of a design strength profile from scattered estimates of undrained soil strength increasing with depth. A ULS check by the β method involves quite different uncertainties involving the angle of interface friction and, more significantly, the effective lateral stress. And in respect of SLS settlement calculations, the designer has the problem of selecting an appropriate soil stiffness representing non-linear behaviour, and a stiffness modulus for concrete, in order to create reasonable expectations for the performance of piles in load tests. Previously published data for London Clay are used to illustrate these issues in the context of a practical design example.

	Variable load, V	Permanent load, G	Water pressures	Cohesion, c	Friction angle, φ
Loads/soil property	1·2–1·5	1·0	1·0–1·2	2–2·5	1·2–1·3

Table 1. Commonly employed partial factors (after Meyerhof, 1970)

2. Uncertainties in total stress analysis

2.1 The α method for shaft resistance

For a clay deposit with a c_u value that is dependent on depth (z_c), the shaft resistance is calculated using Equation 1

1. $$Q_s = \pi D \alpha \int_0^L c_u \mathrm{d}z_c$$

where D is the pile diameter (m); c_u is the undrained shear strength (kPa) varying with depth; α is an empirical adhesion coefficient (for bored piles in London Clay is taken as 0·5); L is the length of pile in the clay stratum (m); and z_c is the depth below top of the clay stratum (m).

The pile base capacity in clays is generally determined using Equation 2

2. $$Q_b = A_b N_c c_u$$

where A_b is the area of the base (m^2); N_c is the bearing capacity factor, which varies depending on the sensitivity and deformation characteristics of the clay, but generally taken as 9 (e.g. Meyerhof, 1976); and c_u is the undrained shear strength (kPa) at the base.

2.2 Design problem and site data

Simpson *et al.* (1980) set out the details of the soil investigation used for the construction of the British Library on Euston Road in London. This case study will be used in the discussion of the α method of bored pile design. A simplified soil profile and the pile to be analysed are sketched in Figure 1. The water table is conservatively taken to be at ground level. The pile is a cylindrical, 600 mm diameter concrete pile, bored and cast in situ. The design is to be based on original data from six boreholes, the relative locations of which are shown on Figure 2. Undrained soil

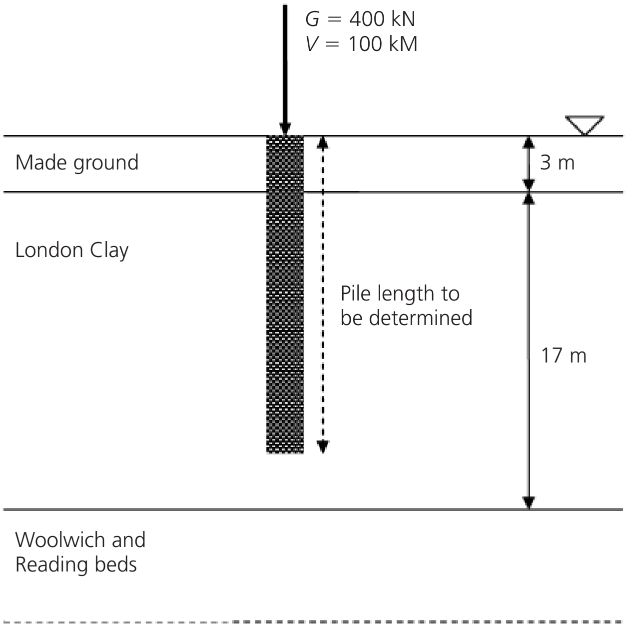

Figure 1. Design problem and idealised soil profile

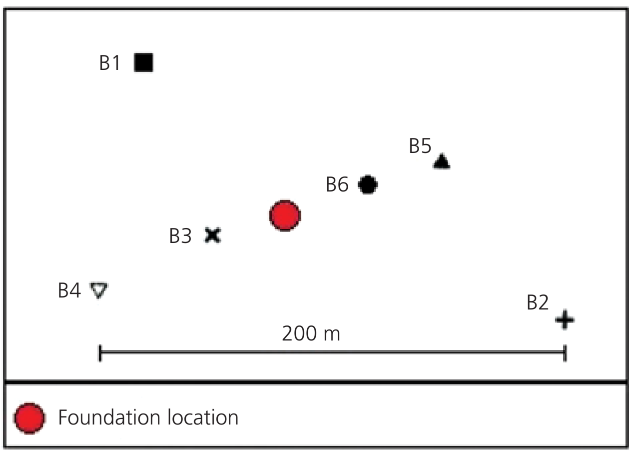

Figure 2. Borehole locations

strength data obtained from triaxial tests on 102 mm diameter samples, and also through a correlation with standard penetration test (SPT) data, is shown in Figure 3. The SPT N_{60} values were converted to c_u using Stroud's (Stroud, 1974) correlations (see Vardanega *et al.*, 2012). Examination of Figure 3 shows that the soil strength appears to increase linearly with depth, and that scatter is a significant design issue in this case.

2.3 Dealing with soil shear strength variation

Soil variability is of major concern to foundation engineers. Lack of data to compute reliable means and standard deviations of soil properties generally makes probabilistic methods impossible to implement (e.g. Bolton, 1993). Variability of soil

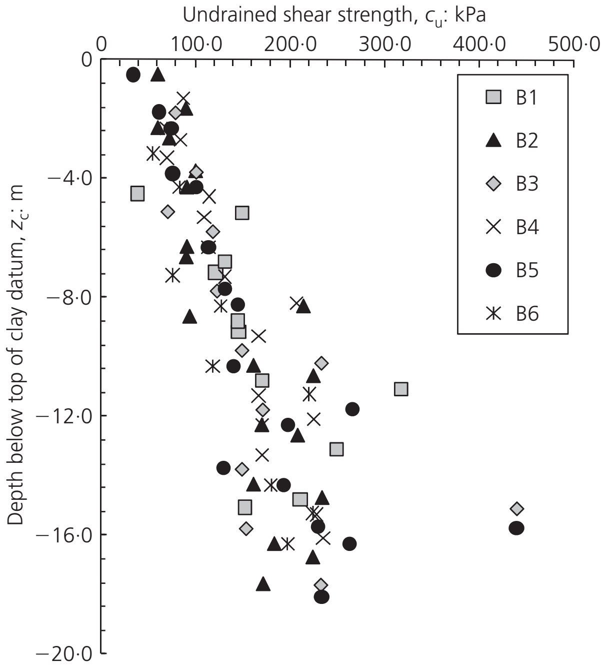

Figure 3. Results from triaxial testing and converted SPT data

properties increases in importance as the size of the foundation decreases; for example a larger raft foundation can better accommodate soft spots and weak layers than a single footing. An isolated pile supporting a column (Meyerhof, 1970) is an intermediate case; shaft capacity is averaged over many layers, but base capacity is susceptible to a single weak stratum. A salient issue is to capture and model the change in soil parameters with depth.

The following procedure should be used to model the soil shear strength variation for pile design purposes (see Figure 4).

(*a*) First, the soil data need to be subdivided into layers based on

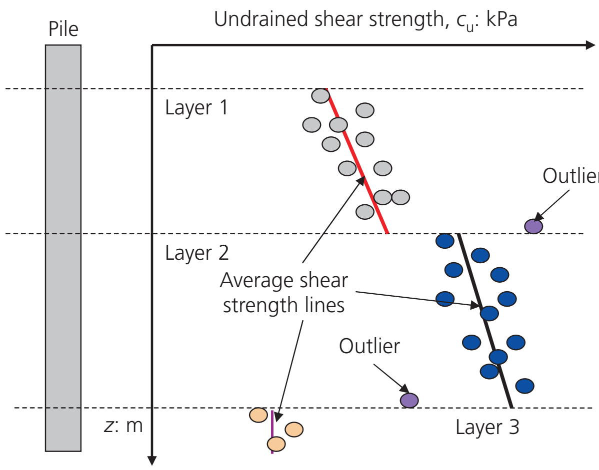

Figure 4. Dealing with soil shear strength variation

geology. This will usually be done from an inspection of the borehole records.

(*b*) Outliers of the shear strength data should be removed based on an inspection of the test data, knowledge of local conditions and from trial regression analyses.

(*c*) The shear strength (c_u) should be averaged down the pile shaft. For calculations of shaft friction a 50th percentile line should be drawn through the test results. If sufficient data are available a linear regression may be appropriate (e.g. Patel, 1992). The engineer may wish to pivot the regression line about the centroid of the data to obtain a more representative fit to the data at all points throughout the stratum, ensuring that 50% of the data points lie above and below the line. If few data are available then a 'cautious estimate' or 'worst credible' average line should be considered (e.g. Simpson *et al.*, 1981).

(*d*) The shear strength (c_u) at the base is more uncertain. If each data point remaining on the scatter plot is deemed valid by the engineer then it must be accepted that the lowest of the observed values is statistically possible to be representative of c_u at the base of any given pile. Therefore a 5th percentile estimate of shear strength should be used for the c_u at the base (e.g. Burland and Cooke, 1974).

2.4 Determination of the 5th percentile

Figure 5 shows three typical patterns of variation of a soil parameter, as described by Lumb (1966). In case A (Figure 5(a)) there is no reason to suppose that a variation with depth should exist. Perhaps the soil is mapped within one geological unit and the property concerned is an intrinsic material parameter such as the critical state angle of friction φ_{crit}. In case B (Figure 5(b)) there is a good reason for a trend with depth arising from the increase in effective stress. An example could be the undrained shear strength c_u of a clay stratum subject to a simple cycle of overconsolidation, although the trend-line may curve towards zero close to the ground surface if the groundwater table is high. In case C (Figure 5(c)) there is a trend with depth, but the slope may vary at random within limits. The penetration resistance of layered ground, in which there are sequences of soil types (A, B, C, and so on), can indicate different trend-lines with depth for each soil type, but offer random samples of them at increasing depth: A, A, B, A, C, B, C, A, and so on. The engineer must assess the nature of the possible variation before modelling it. Lumb's classification of soil variation shows that the 5th percentile line does not necessarily have to have the same slope as the 50th percentile line. Once outliers on the low side have been removed it is best to trace a lower bound line to the test data (Figure 6) and then shift the line until 5% of the values lie below the line.

2.5 Total stress calculations

Permanent load, $G = 400$ kN
Variable load, $V = 100$ kN
Lumped factor of safety, $F = 2{\cdot}5$
z_c = depth below top of clay datum (m)
Undrained strength trend with depth: $c_u = c_0 + \eta z_c$ where c_0 and η are constants that depend on which line is required (e.g. 5th percentile or 50th percentile).

The rationale of ULS calculations is that all compatible components of resistance are included. The base capacity will be added to shaft capacity to determine the ULS criterion. A separate analysis of settlement will be made later.

The ULS requirement can therefore be written

$$F(G+V) \leqslant Q_b + Q_s \tag{3}$$

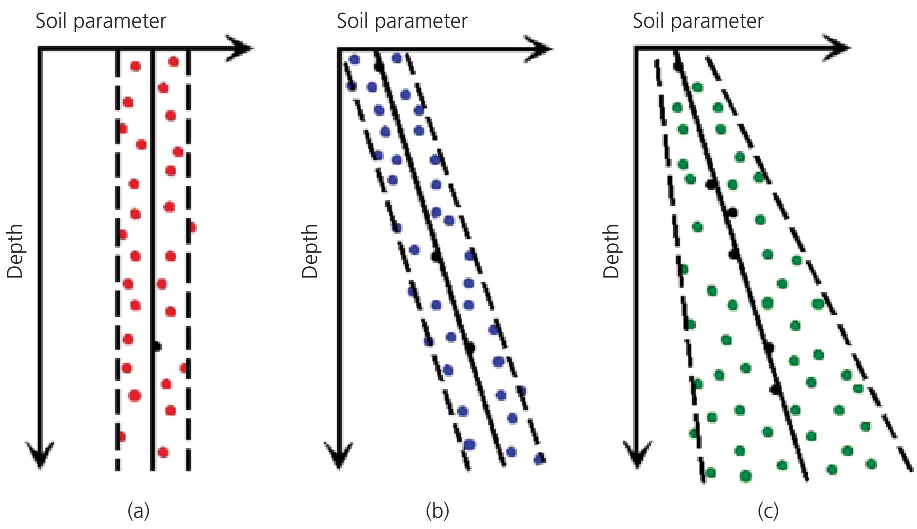

Figure 5. Types of soil variation (after Lumb, 1966 and Cheong and Kaggwa, 2002): (a) case A; (b) case B; (c) case C

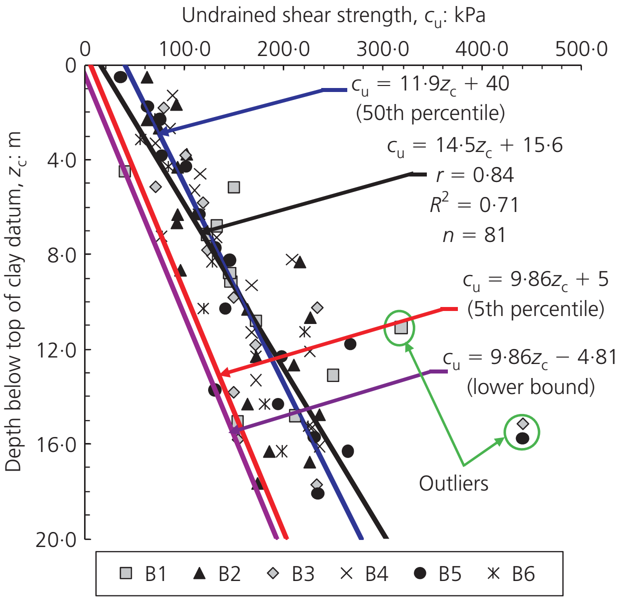

Figure 6. Shear strength relationship with depth characterisation for site

$$2{\cdot}5 \times (400 + 100) \leqslant Q_b + Q_s \tag{4}$$

The base capacity is given by

$$Q_b = A_b N_c c_u = A_b N_c (c_0 + \eta z_c) \tag{5}$$

The 5th percentile strength profile line has the following formula

$$c_u = 9{\cdot}86 z_c + 5 \tag{6}$$

Taking L as the length of pile in London Clay (so that $L_{pile} = L + 3$), and using diameter $D = 0{\cdot}6$ m

$$\begin{aligned} Q_b &= [\pi(0{\cdot}6)^2/4] \times 9 \times (9{\cdot}86L + 5) \\ &= 25{\cdot}1L + 12{\cdot}7 \end{aligned} \tag{7}$$

The shaft capacity is calculated as

$$Q_s = \pi D \alpha \int_0^L c_u \mathrm{d}z_c \tag{8}$$

$$Q_s = \pi D \alpha \int_0^L (c_0 + \eta z_c) \mathrm{d}z_c \tag{9}$$

$$Q_s = \pi D \alpha \left[\frac{\eta z_c^2}{2} + c_0 z_c \right]_0^L \tag{10}$$

The 50th percentile strength profile line has the following formula

$$c_u = 40 + 11{\cdot}9 z_c \tag{11}$$

Substituting Equation 11 into Equation 10

$$Q_s = \pi(0{\cdot}6) \times (0{\cdot}5) \left[\frac{11{\cdot}9\,L^2}{2} + 40\,L \right]$$

$$Q_s = 5{\cdot}6L^2 + 37{\cdot}7L \tag{12}$$

Substituting Equations 7 and 12 into Equation 4, and imposing the equality for the minimum permissible pile length L

$$\begin{aligned} 2{\cdot}5(400 + 100) &= 5{\cdot}6L^2 + 37{\cdot}7L + 25{\cdot}1L + 12{\cdot}7 \\ 0 &= 5{\cdot}6L^2 + 62{\cdot}8L – 1237{\cdot}3 \end{aligned} \tag{13}$$

$L = 10{\cdot}3$ m (taking the positive root).

Therefore, a 13·3 m pile would be specified from the result of this design calculation, and in that case the base would be contributing 271 kN and the shaft 982 kN of the total resistance.

3. Uncertainties in effective stress analysis

3.1 The β method for shaft resistance

The β method makes use of effective stress for determination of skin friction (Equation 14). Meyerhof (1976), Burland (1973), Burland and Twine (1988) and Bond and Jardine (1995) all give similar formulations for the β method

$$Q_s = \pi D \int_3^{L+3} \beta \sigma'_{v,0} \mathrm{d}z \tag{14}$$

where D is pile diameter (m); $\beta = K_s \tan\delta$; K_s is the ratio of radial effective stress σ'_s acting on the shaft and the pre-existing $\sigma'_{v,0}$; $\tan\delta$ is the effective coefficient of friction between the pile shaft and the clay; $\sigma'_{v,0}$ is the pre-existing effective vertical stress (kPa); L is the length of pile in the clay stratum (m); and z is the depth of a point on the pile, from the ground surface (m).

Meyerhof (1976) collected load test data for London Clay and stated that K_s ranges from $0{\cdot}7K_0$ to $1{\cdot}2K_0$, which leaves the appropriate value in considerable doubt since K_0 itself is quite variable. Further elaboration is warranted.

3.2 Uncertainties regarding groundwater pressures

Figure 7 shows the results of piezometric testing on the site under discussion. The water pressures are around 60% of hydrostatic levels, due to base drainage. Higher effective stress levels may accordingly be used for design purposes on this site, but only if the base drainage can be assured in the long term.

3.3 Uncertainties due to construction sequence

The β method relies on estimating effective contact stresses and is not often used in London Clay (Simpson, personal communication, 2009). Burland (1973) and Burland and Twine (1988) preferred the β method for the following reasons.

(*a*) Major shear distortion is confined to a thin zone around the pile shaft and any drainage following loading must be rapid, so fully drained soil strength is more relevant.
(*b*) Installation of a pile by driving will remould the soil adjacent to the pile, while installation by boring will permit swelling and softening, in both cases altering its original undrained strength.
(*c*) There is no clear relationship between undrained strength and drained strength at the pile–soil interface.

Bond and Jardine (1995) presented results of load tests on displacement piles ranging in length from 3·2 m to 4·16 m, in London Clay at Cannon's Park. The insertion process resulting in observed values of α ranging from 0·49 to 0·93 and values of β ranging from 0·99 to 1·90 respectively. The authors also showed that the residual friction angle ranges from 8° to 13° and confirmed Burland and Twine's (Burland and Twine, 1988) observation that residual angles of friction should be used instead of the critical state friction angle when using the β method for displacement pile design in London Clay.

The sequence of stresses during the construction and loading of a bored pile is quite different, and can be idealised as follows.

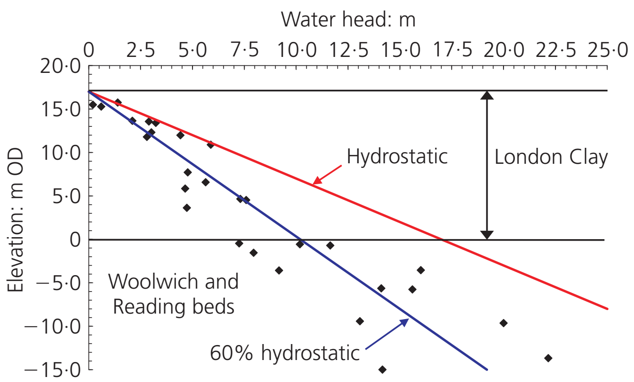

Figure 7. Piezometric data from the site

(*a*) Initial stresses at depth z below the ground surface, where the groundwater table is at depth $z = z_w$, and where base drainage creates a reduction factor f_u on hydrostatic values beneath the water table

$$\sigma_{v,0} = \gamma z \tag{15}$$

$$u_0 = f_u \gamma_w (z - z_w) \tag{16}$$

$$\sigma'_{v,0} = \gamma z - f_u \gamma_w (z - z_w) \tag{17}$$

$$\sigma'_{h,0} = K_0[\gamma z - f_u \gamma_w (z - z_w)] \tag{18}$$

$$\sigma_{h,0} = K_0 \gamma z + (1 - K_0) f_u \gamma_w (z - z_w) \tag{19}$$

(*b*) Undrained cavity contraction from initial lateral stress $\sigma_{h,0}$ to the mud pressure σ_m in the bore (if drilled with bentonite fluid).
(*c*) Local swelling and softening around the bore due to the reduction in radial effective stress.
(*d*) Undrained cavity expansion from mud pressure σ_m to the wet concrete pressure $\sigma_c = \gamma_c z$.
(*e*) Radial transient flow to regain the initial pore pressure u_0 in the clay, together with shrinkage of the set concrete, will cause small changes in the radial stress on the shaft $\sigma_s \approx \gamma_c z$.
(*f*) Drained axial shearing may cause dilation of the clay at the pile interface, leading to a further small change in the radial stress σ_s on the shaft as peak shear stress develops

$$\tau_{max} = (\sigma_s - u_0) \tan \delta_{max} \quad \text{where} \quad \delta_{max} \approx \varphi_{crit} \tag{20}$$

(*g*) Pile axial displacement w initially occurs by dragging down the adjacent soil, but at some limiting value $w = w_{slip}$ it begins to cause slip at the peak shaft shear resistance $\tau = \tau_{max}$.
(*h*) As axial displacement continues to increase, with $w > w_{slip}$, the shear stress falls, $\tau \rightarrow \tau_{res}$, $\delta_{res} \approx \varphi_{res}$.

It is presumed here that the best estimate of the peak value of the fully drained shaft resistance derives from Equation 14 with $\sigma_s = \gamma_c z$, so that the effective earth pressure coefficient for the shaft can be taken as

$$K_s = \frac{\gamma_c z - u_0}{\gamma z - u_0} = \frac{\gamma_c z - f_u \gamma_w (z - z_w)}{\gamma z - f_u \gamma_w (z - z_w)} \tag{21}$$

In the example being pursued here, the unit weight of concrete will be taken as $\gamma_c = 23{\cdot}5\,\text{kN/m}^3$ and the unit weight of London Clay will be taken as $\gamma = 20\,\text{kN/m}^3$, so taking $\gamma_w = 10\,\text{kN/m}^3$ and $f_u = 0{\cdot}6$ for example, a typical range of values for K_s may be calculated from Equation 21. Near the top of the pile, where $z = z_w$, $K_s = 23{\cdot}5/20 = 1{\cdot}17$ is obtained; near the base of a long pile where $z >> z_w$, $K_s = 17{\cdot}5/14 = 1{\cdot}25$ is obtained. The calculated range is quite narrow even when the drainage factor f_u is allowed to vary. Therefore the value $K_s \approx 1{\cdot}2$ is taken.

However, it must not be forgotten that additional uncertainties arise from the variations in radial stress referred to above, especially regarding the time permitted for softening during stress relief under drilling mud in relation to the subsequent time available for hardening under the pressure of wet concrete. So the reliability of the β method based on the use of Equations 14, 20 and 21 should ideally be based on long-term fully drained loading trials to failure. Such tests would be very time consuming and they are highly unlikely ever to be carried out in practice. Slow maintained load (ML) tests on piles in the field are generally carried out at a design load of twice the nominal working load for a period of a few days, so that a settlement criterion can be checked (Fellenius, 1980). Extrapolation techniques based on hyperbolic load–settlement relations have proved reasonably accurate and offer the opportunity to make settlement predictions after only 1 day of testing (Fleming, 1992). Faster constant rate of penetration (CRP) tests and quasi-dynamic tests rely on rate effect corrections, and generally require prior correlations with ML tests on the same soil (Brown *et al.*, 2006). The increasing use of monitoring systems during construction, and in service, will provide better justification of settlement predictions in future, although obviously not of failure conditions. Discussion of settlement predictions based on ground investigation findings will be conducted in the next section.

For the β calculations that follow, K_s from Equation 21 is taken as 1·2 all the way down the pile shaft. Figure 8 shows this in comparison to the K_0 database for London Clay collected by Hight *et al.* (2003). K_s of 1·2 is seen to correspond to a lower bound to the K_0 data for London Clay, although the preceding discussion suggests that this is a coincidence. However, Fleming *et al.* (2009: chapter 4) do suggest the use of $K_s = (K_c + K_0)/2$. K_c is the earth pressure coefficient in the soil owing to the placement of wet concrete.

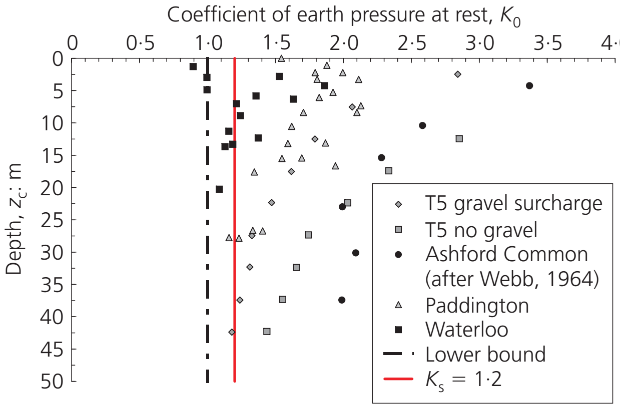

Figure 8. K_0 data from various sites in London (replotted from Hight *et al.*, 2003)

3.4 Effective stress calculations

As before, the base capacity will be added to shaft capacity to determine the ULS criterion. The conservative assumption is made that the water table is at ground level, and that the made ground has weight but no reliable friction. Calculations leading to the drained shaft resistance are set out below.

$\varphi_{\text{crit}} = 22°$
$\gamma_{\text{soil}} = 20\,\text{kN/m}^3$
$\gamma_w = 10\,\text{kN/m}^3$

$$Q_s = \pi D K_s \tan\varphi_{\text{crit}} \int_3^{L+3} (\gamma - \gamma_w) z\,dz$$

$$Q_s = \pi(0{\cdot}6) \times (1{\cdot}2) \tan 22° \gamma' \int_3^{L+3} z\,dz$$

$$Q_s = \pi(0{\cdot}6) \times (1{\cdot}2) \tan 22°(10) \left[\frac{z^2}{2}\right]_3^{L+3}$$

22. $$Q_s = 4{\cdot}57L^2 + 27{\cdot}4L$$

The fully drained ultimate base resistance should conservatively be calculated using the critical state angle of shearing resistance. Berezantsev *et al.* (1961) offer the following bearing capacity equation

23. $$q_f = A_k\gamma' D + B_k\alpha_T\gamma' L_{\text{pile}}$$

where A_k and B_k are functions of φ, α_T is a function of both φ and the pile depth to diameter ratio, and γ' is the effective unit weight of the clay. The calculation for the proposed 0·6 m diameter pile in London Clay finds, for submerged hydrostatic groundwater conditions

$$q_f = 30 + 41L_{\text{pile}} = 30 + 41(L + 3)$$

so that the drained ultimate base resistance becomes

24. $$Q_b = 43 + 11{\cdot}5L$$

Putting Equations 22 and 24 into Equation 4, it is possible to solve to find

$$2{\cdot}5(400+100) = 11{\cdot}5L + 43 + 4{\cdot}57L^2 + 27{\cdot}4L$$

25. $$0 = 4{\cdot}57L^2 + 38{\cdot}9L\text{–}1207$$

so that the length L of pile within the clay is given as $L = 12{\cdot}5$ m.

Therefore a 15·5 m long pile would be specified to achieve a drained safety factor of 2·5 with hydrostatic groundwater and a water table at ground level.

The evidence of Figure 7 suggested that the site water pressures are 60% of hydrostatic and if the correspondingly higher effective stress levels are used, the design calculation changes to

$$Q_s = \pi(0{\cdot}6)\times(1{\cdot}2)\tan 22°(14)\left[\frac{z^2}{2}\right]_3^{L+3}$$

26. $$Q_s = 6{\cdot}4L^2 + 38{\cdot}4L$$

Similarly, Equation 24 becomes

27. $$Q_b = 60 + 16{\cdot}1L$$

The revised ULS criterion then demands

$$2{\cdot}5(400+100) = 16{\cdot}1L + 60 + 6{\cdot}4L^2 + 38{\cdot}4L$$

$$0 = 6{\cdot}4L^2 + 54{\cdot}5L - 1190$$

28. $$L = 10{\cdot}0 \text{ m}$$

which would result in the specification of a 13·0 m long pile. In this case the base is contributing 221 kN and the shaft 1024 kN. This is very similar to the undrained strength calculation.

4. Uncertainty in SLS calculations of pile settlement

4.1 Non-linear settlement analysis

Many common pile settlement calculations assume either a linear elastic soil or simple elasto-plastic model (e.g. Guo and Randolph, 1997; Mattes and Poulos, 1969; Randolph, 1977; Randolph *et al.*, 1979). However, soil stress–strain behaviour is not linear-elastic. Fleming (1992) described a technique of interpreting pile data using hyperbolic functions for the shaft and base capacities. Vardanega and Bolton (2011a) proposed the following non-linear model for shear strength mobilisation of clays and silts based on a large database

29. $$\frac{\tau}{c_u} = 0{\cdot}5\left(\frac{\gamma}{\gamma_{M=2}}\right)^{0{\cdot}6}$$

As defined by BSI (1994), the quantity c_u/τ_{mob} is the mobilisation factor, M, which is equivalent to a factor of safety on shear strength. The definition of $\gamma_{M=2}$ is the shear strain when half the undrained shear strength c_u has been mobilised, that is at a mobilisation factor $M = 2$. The determination of $\gamma_{M=2}$ is most reliably based on site-specific information, such as from high-quality cores reconsolidated in the laboratory and tested with local strain measurement. Vardanega and Bolton (2011b) processed such data from Jardine *et al.* (1984), Yimsiri (2002) and Gasparre (2005), which are used to compile Figure 9 showing mobilisation strain $\gamma_{M=2}$ reducing from about 1% at 5 m depth to about 0·5% at 40 m depth in overconsolidated London Clay with the trend-line

30. $$1000\gamma_{M=2} = -2{\cdot}84\ln z + 15{\cdot}42$$

In the settlement analysis which follows, a rigid pile is assumed, with no slip at the soil–pile interface, and concentric circles of influence around the pile shaft are considered, ignoring resistance at the toe: see Figure 10. Superficial deposits are also ignored in the following analysis.

Randolph's equation of vertical equilibrium at any given radius (e.g. Fleming *et al.*, 2009) gives

31. $$\tau = \frac{\tau_o r_o}{r}$$

Substituting Equation 31 into Equation 29 results in

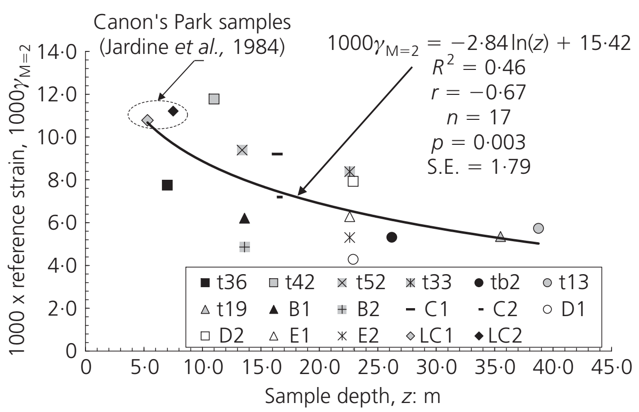

Figure 9. Mobilisation strain plotted against sample depth for three sites in London Clay (after Vardanega and Bolton, 2011b)

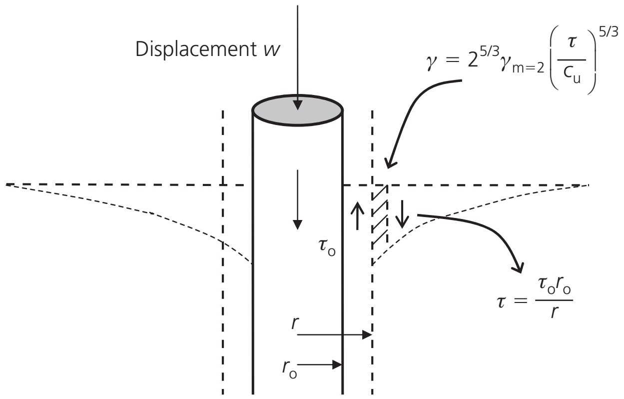

Figure 10. Displacement of a single pile (after Randolph, 1977; Fleming *et al.*, 2009)

32. $$\gamma = 2^{5/3}\gamma_{M=2}\left(\frac{\tau_o r_o}{rc_u}\right)^{5/3}$$

The downward displacement of the pile w is equal to the integral of the shear strain with respect to the radii of the concentric surfaces (Randolph, 1977)

33. $$w = \int_{r_o}^{\infty}\gamma \mathrm{d}r = \int_{r_o}^{\infty} 2^{5/3}\gamma_{M=2}\left(\frac{\tau_o r_o}{rc_u}\right)^{5/3}\mathrm{d}r$$

$$w = 2^{5/3}\gamma_{M=2}\left(\frac{\tau_o r_o}{c_u}\right)^{5/3}\int_{r_o}^{\infty}\frac{1}{r^{5/3}}\;\mathrm{d}r$$

34. $$\frac{w}{r_o} = 4{\cdot}76\gamma_{M=2}\left(\frac{\tau_o}{c_u}\right)^{1{\cdot}67}$$

Substituting $\gamma_{M=2}$ from Equation 30 and replacing τ_o/c_u with the inverse of the mobilisation factor

35. $$\frac{w}{r_o} = \frac{4{\cdot}76}{1000}(-2{\cdot}84\ln z + 15{\cdot}42)\left(\frac{1}{M}\right)^{1{\cdot}67}$$

Equation 35 is used to compute various values of settlement ratio for a variety of pile length values and mobilisation levels. Figure 11 summarises these calculations, and it should be noted that the w/r_o values here are taken to be the contribution of the soil to the total pile settlement.

4.2 Pile compressibility

Pile compressibility is the other component of total pile settlement. For a cast-in-situ concrete pile, and assuming linear-elasticity of the pile itself, and a constant rate of load transfer in shaft friction, with the axial thrust in the pile reducing linearly from Q_h at the head to zero at the toe, the following equation can be written

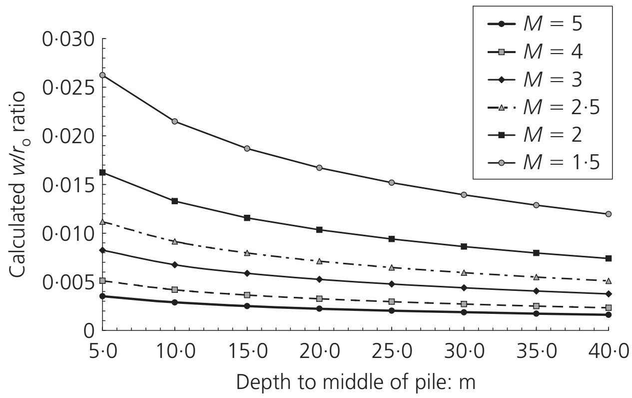

Figure 11. w/r_o ratios for London Clay calculated using Equation 34

36. $$\Delta w = \frac{Q_h 4\,L}{2\,\pi D^2 E_c}$$

where Δw is the compression of the pile under applied load; Q_h is the pile head load (kN); D is the pile diameter (m); E_c is the elastic modulus of reinforced concrete (kPa); and L is the length of the pile (m).

It follows that

37. $$\frac{\Delta w}{D} = \frac{2}{\pi}\frac{Q_h}{E_c}\frac{L}{D^3}$$

Considering that the head load is transferred through shaft friction at the average undrained shear strength of the soil stratum surrounding the pile, factored down by the mobilisation factor M, the following can be written

38. $$Q_h = \frac{\bar{c}_u}{M}\pi DL$$

From Equation 34, the following equation may also be written

39. $$\frac{w}{D} = \frac{4{\cdot}76}{2}\frac{\gamma_{M=2}}{M^{5/3}}$$

The head displacement (w_h) of the pile is taken approximately to be the sum of Equations 39 and 37, with Equation 38 substituted into Equation 37

$$\frac{w_h}{D} = \frac{4{\cdot}76}{2}\frac{\gamma_{M=2}}{M^{5/3}} + \frac{\bar{c}_u}{M}\frac{2}{E_c}\left(\frac{L}{D}\right)^2 \quad (40)$$

A characteristic value of $\gamma_{M=2} = 0{\cdot}008$ has been adopted for London Clay. For example, a 0·6 m diameter pile, 15 m long, with a typical mobilisation factor $M = 3$, and with E_c taken as 20×10^6 kPa and an assumed c_u variation with depth of $50 + 7{\cdot}5z$ (Patel, 1992), the following is obtained

$$\frac{w_h}{D} = \frac{4{\cdot}76}{2} \times \frac{0{\cdot}008}{3^{5/3}} + \frac{50 + 7{\cdot}5(15/2)}{3} \times \frac{2}{20 \times 10^6}\left(\frac{15}{0{\cdot}6}\right)^2$$

$$\frac{w_h}{0{\cdot}6} = 0{\cdot}00304 + 0{\cdot}0022 \quad (41)$$

This computes a settlement at the pile head $w_h = 3{\cdot}14$ mm, which is 0·53% of the pile diameter.

The major variable that the engineer should consider is the M factor that is applied. Design curves are given in Figure 12. Here it should be noted that $1/M$ must be no greater than α; otherwise Equation 1 would indicate undrained shaft failure at the softened pile interface. This ULS criterion sets a lower limit of $M = 2$ at $\alpha = 0{\cdot}5$, as used for London Clay. Hence, if $\alpha = 0{\cdot}6$, M would have a lower limit of 1·67. The definition of a lumped factor of safety F, as was adopted in Equation 3, provides for a *further* enhancement in M, so that

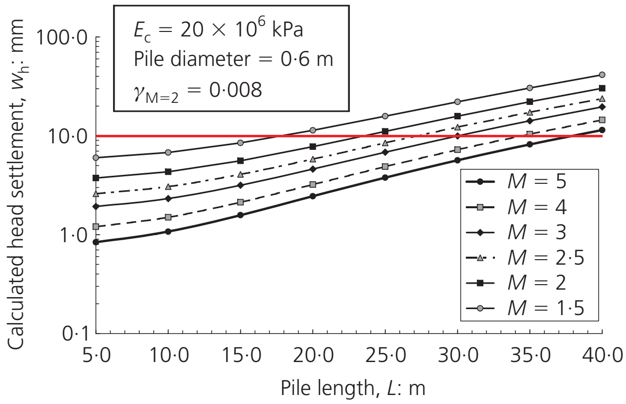

Figure 12. Head settlement ratios for London Clay calculated using Equation 39

$$F = \alpha M \quad (42a)$$

$$M \geqslant 1/\alpha \quad (42b)$$

It is now evident that the very small pile head settlement of 3·14 mm calculated above for $M = 3$ actually corresponds to a global safety factor $F = 1{\cdot}5$, which is considerably smaller than the factor $F = 2{\cdot}5$ typically demanded by current codes of practice.

4.3 Comments on uncertainty of pile settlement calculations

Figure 13 shows the range of head settlements of bored piles of various lengths L in London Clay and various diameters D, taken

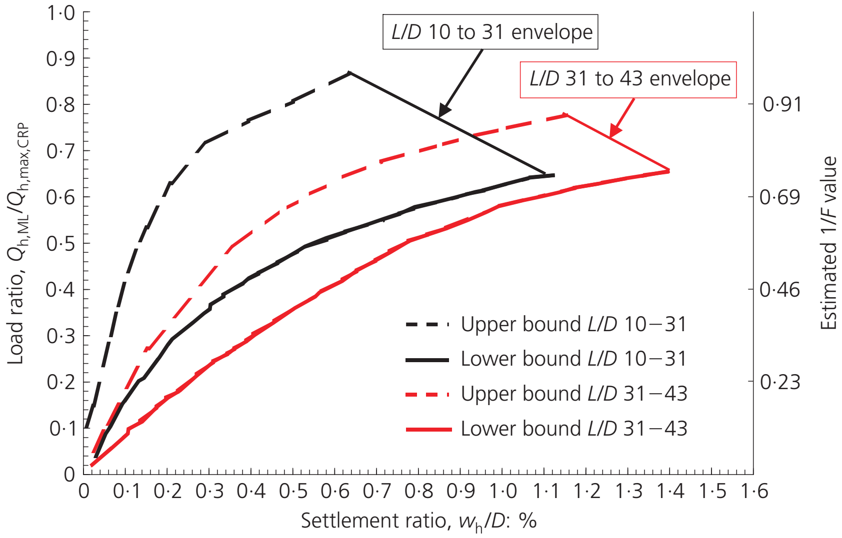

Figure 13. Load ratio plotted against settlement ratio (after Patel, 1992)

from Patel (1992). At the left-hand side, the vertical axis is that chosen by Patel and described by him as the load ratio, defined as the head load in an ML test divided by the plunge load determined from a corresponding CRP test. At first sight, this appears to be equivalent to an inverse safety factor. However, it might reasonably be supposed that ML tests would last 3 days whereas CRP tests might last 3 h, in which case rate effects would influence the operational shear strength, owing to the test duration ratio being of the order of 24. The typical strength increase for different strain rates, as discovered by Kulhawy and Mayne (1990) for 26 clays for which they had data, is 10% per factor 10 increase in strain rate interpreted on a logarithmic scale. A loading–rate ratio of 24 would then suggest a soil strength ratio of 1·14. The vertical axis at the right-hand side of Figure 13 accordingly plots the authors' best estimate of the inverse factor of safety F in the ML test. The horizontal axis plots the ratio between measured head settlement w_h and the pile diameter D for the various tests listed by Patel (1992), represented in bands of various L/D ratio. The L/D ratio has a major effect on the settlement, as expected from Figure 12.

Figure 14 reuses the right-hand axis $1/F$ in Figure 13 as its vertical axis, in order to replot Patel's data in comparison with the predictions of the simplified settlement model described earlier, and expressed in Equation 39. The safety factor, F, in Figure 14 is given by equation 42 for the purposes of the settlement calculation. This therefore discounts the base resistance. The difference between the load ratio, determined by Patel (1992) and $1/F$ will be insignificant for slender piles. For comparison purposes, a 0·6 m diameter pile was used, as this was judged to be the mean pile diameter in Patel (1992), and a strength profile of $c_u = 50 + 7{\cdot}5z$ was taken as typical for the sites he studied. Since $F = M\alpha$, as given by Equation 41, it follows that for $\alpha = 0{\cdot}5$ and $M = 3$, the value $F = 1{\cdot}5$ is obtained, so that the corrected load ratio in Figure 14 is 0·67. This sets what might be regarded as an upper limit on the advisable mobilisation of shaft friction. In the general region of interest for design ($F \geqslant 1{\cdot}5$) the simple formulation presented herein is a good estimate of pile settlement in London Clay. The increasing rate of settlement seen at lower factors of safety ($F < 1{\cdot}5$) in Patel's data, as indicated in Figure 14, is presumed to arise from additional softening due to slip in the upper region of the pile, allowing the local shaft friction to fall towards residual strength. A more rigorous, non-linear settlement model could employ a full load-transfer analysis in the fashion recommended by Fleming *et al.* (2009: section 4·2). In such an analysis, the contribution of the base resistance increasing rather slowly with pile toe settlement could also be included if desired. Apparently, however, the simplified version of Equation 40 may be sufficient for practical purposes in London Clay.

5. Discussion

The measurement of the peak undrained shear strength of stiff clay, c_u, is not precise. The early part of this paper has shown how changes in the interpretation of c_u profiles with depth alter a pile design, and how the definition of design values requires

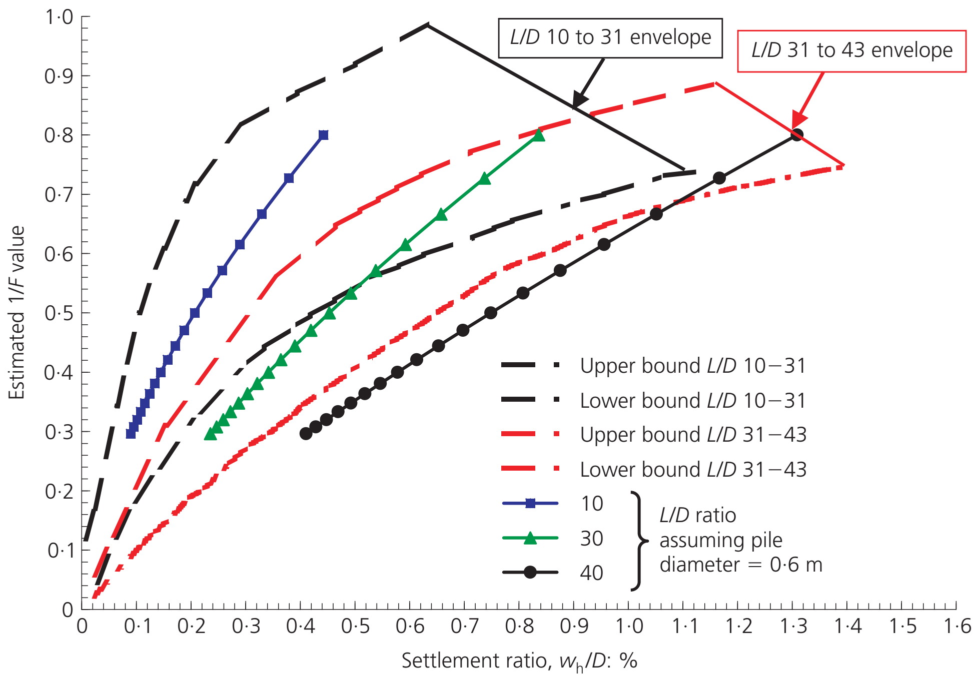

Figure 14. Rate-corrected load ratio ($1/F$) plotted against settlement ratio (calculated by Equation 39 for 0·6 m diameter piles; $c_u = 50 + 7{\cdot}5z$)

careful interpretation of the data. No doubt with this in mind, code-drafters apply factors to the calculated resistances, soil properties and loads in addition to specifying an α value to reduce the design value of c_u to a value which has proven typically to be mobilised on the shaft in undrained pile tests. Vardanega *et al.* (2012) have shown that the typical global factor of safety F implied by many codes is around 2·5. In London Clay it is also considered that $\alpha = 0{\cdot}5$ is required to avoid failure of a bored pile at the soil interface. This means that the degree of mobilisation of the intact soil strength in the region surrounding the pile is only one fifth. It has further been shown that with a mobilisation ratio $M = 5$, the shear strains in the clay, and the concomitant pile settlements due to soil strains, are very small.

Most of the settlement at the pile head in London Clay is caused by axial compression of the pile itself. For the longest and most compressible piles in Patel's database ($L/D = 40$), the adoption of a typical global factor of safety $F = 2{\cdot}5$ leads to a settlement ratio of about 0·58% as confirmed by the simple deformation model and Equation 40. The settlement at the pile head for a typical 1 m diameter bored pile in London Clay will therefore be only of the order of 6 mm. This is likely to be wasteful of concrete because a more carefully calibrated structural serviceability criterion based on *relative* settlement and building damage would be likely to set the maximum settlement of a single pile in excess of 20 mm, depending on the type of building and its finishes. If the global safety factor could be reduced to 1·5, for example, so that the design loads were increased by a factor of $2{\cdot}5/1{\cdot}5 = 1{\cdot}67$, Figure 14 indicates that the settlement ratio of the same pile would increase to 1·05%, which still corresponds to a settlement of only 10·5 mm.

Before making such a bold move, a designer would need to check whether all genuine safety concerns had been met. Such concerns should chiefly fall under the following four categories.

(*a*) *Design values of the working loads*. Both variable loads and permanent loads may be distributed between piles in different patterns, owing to redistribution through the structure in response to differential deformations. It could be considered an aid to good decision making if the axial pile head loads arising from structural permanent weight and variable load (G and V respectively in Figure 1) are first selected as conservative nominal values, and then multiplied by a load factor of 1·2 to make an allowance for load redistribution to the foundations.

(*b*) *Design values of undrained soil strength*. This paper has advocated the careful selection of a 50th percentile of undrained strength values obtained on site in order to establish a design profile of intact strength for the subsequent estimation of shaft resistance, and a 5th percentile in order to select an appropriate undrained strength value for base resistance calculations. On the basis of a history of pile CRP tests, a reduction factor $\alpha = 0{\cdot}5$ must be used on the intact value of undrained soil strength in order to match the observed shaft resistance. Having taken these reasonable steps, and having validated a separate settlement model for bored piles, it is no longer necessary to include additional strength reduction factors in an attempt to reduce settlements.

(*c*) *Design values of drained strength*. It has been shown that the use of the critical state angle of friction of the clay, both for shaft friction and for base resistance, together with appropriate normal effective stresses, leads to similar pile designs to those derived from the undrained strength with the conventional α factor of 0·5. It is tentatively recommended that the effective earth pressure coefficient to be used for the drained shaft resistance be based on the pressure of wet concrete during casting. Although there appears to be no extra reserve of strength to accompany soil drainage, there is equally no apparent need to make empirical strength reductions on those grounds. The β method can be recommended in strata where no prior data of empirical α values are available, or where shaft grouting is to be used to enhance shaft friction.

(*d*) *Long-term deterioration of soil strength*. There is ample evidence to show that soil creeps under constant shear stress, and that in an equivalent sense the shear strength of soil reduces as the shear strain rate falls, and that it does so at about 10% per factor 10 on strain rate. Geotechnical calculation models are calibrated against tests, whether triaxial tests or CRP pile tests, that take of the order of 8 h to reach failure. However, the constructions which are then created are intended to last without significant maintenance for at least 20 years. The ratio of these durations is 21 900, and this translates to a notional strength reduction factor of about 1·5. This could be considered a modelling correction. Logically, the neglect of this factor would generate an unexpected creep rate rather than an unexpected loss of equilibrium, so it could be seen to be an SLS matter rather than a ULS concern.

If these factors are combined to derive a new global safety factor, the following is obtained

$$F = 1{\cdot}2 \text{ (on loads)} \times 1{\cdot}0 \text{ (on soil strength data)} \times 1{\cdot}5 \text{ (model factor for creep effects)} = 1{\cdot}80$$

These factors are similar to those used in the Russian SNiP code, where the equivalent breakdown is

$$F = 1{\cdot}2 \text{ (on loads)} \times 1{\cdot}0 \text{ (on remoulded soil strength)} \times 1{\cdot}4 \text{ (extra geotechnical factor)} = 1{\cdot}68$$

Patel's database, and the fitting of the simplified settlement model in Figure 14, suggest that a global safety factor of 1·8 should lead to entirely acceptable settlements, while offering a saving of 28%

on the design pile capacity compared with Eurocode 7 design method DA1 following the UK national annex, for example. This objective approach also opens up the possibility later of reducing the proposed model factor of 1·5 for creep effects in the light of long-term monitoring data of structures as further information becomes available.

6. Summary

(*a*) Both the total stress (undrained) and the effective stress (drained) approach for the determination of pile capacity give reasonable answers in London Clay.

(*b*) A fully drained analysis of shaft capacity requires estimations only of the effective lateral earth pressure coefficient K_s, for which an objective procedure is recommended based on the pressure of wet concrete during casting, and the critical state friction angle. A safe lower bound approach to design is to take water pressures as hydrostatic, even though the particular site studied in London currently has high water pressures only about 60% of hydrostatic.

(*c*) The undrained analysis requires geotechnical judgement to exclude as outliers any invalid values of undrained shear strength. A procedure is then recommended for the selection of a 50th percentile strength profile for shaft capacity and a 5th percentile for base resistance. The amount of site investigation data available and the past experience of the geotechnical engineer should dictate the levels of conservatism that should be used in the assignment of these design profiles. This is where the geotechnical engineer can add significant value to the design process.

(*d*) The undrained analysis also relies on historic test data to determine the reduction factor α that applies to the intact undrained shear strength of clay to allow for softening in the thin soil zone that is influenced by pile installation.

(*e*) In a soil deposit where no prior knowledge of α is available, the β method would be preferred as less uncertainty exists in the determination of the soil strength parameters. ML tests would then be desirable to confirm the design.

(*f*) The stiffness of the concrete governs the settlement of long bored piles in London Clay as the mobilisation strain of London Clay is low. This means that in this particular soil deposit settlements are unlikely to be an issue at a global factor of safety even as low as 1·5.

(*g*) Factors of safety can be safely reduced for bored piles in London Clay provided that the soil data have been interpreted and the design parameters assigned with caution. A sensible regime would be to have the following partial load factors
- (i) permanent load – 1·2
- (ii) variable load – 1·2
- (iii) shaft capacity – 1·0 (with conservative soil properties)
- (iv) base capacity – 1·0 (with conservative soil properties)
- (v) model factor – 1·5 (long-term degradation of clay strength).

(*h*) Pile design in London Clay is simpler than in other materials because a large load-test database is available and a well-established value of $\alpha = 0{\cdot}5$ is known. Patel's database is also available to validate that settlement considerations can be done simply in this stiff deposit. Nevertheless, the objective replacement of arbitrary safety factors by explicit geotechnical mechanisms, statistical procedures based on the acquisition of routine ground data and the use in design of published databases, should be more widely attractive.

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