Practical methods to estimate the non-linear shear stiffness of fine grained soils

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ABSTRACT: The use of databases in geotechnical engineering allows engineers to make a priori estimates of soil behaviour. Based on a study of the published literature, a database of 20 clays and silts is presented that allows predictions to be made of the strain-dependent stiffness of fine-grained soils, based on simple soil parameters. The significance of rate effects is discussed and corrections are made. The use of a reference strain γ_{ref} to normalize shear strain values γ in relation to modulus reduction G/G_0 is discussed. Empirical formulations are presented based on a rigorous regression analysis, and design charts are constructed.

1. INTRODUCTION

In many applications of geotechnical engineering an estimate of the stiffness degradation of clays and silts is required. In earthquake engineering, prediction of the strain level that indicates modulus reduction is crucial in the prediction of damping. The seismic response is considered undrained in fine grained soils.

2. DATABASE

A database of clay and silt stiffness degradation was sourced from ten publications (listed in the Appendix). The data is presented in the terms of secant stiffness, the typical cyclic response being defined in Fig. 1. All the authors measured G_0 values directly apart from Teachavorasinskun et al (2002) who used the correlations in Hardin & Black (1968).

The samples derived from various countries and were tested in a variety of conditions from normally consolidated to heavily overconsolidated, in various laboratories and shear testing devices, over a period of 30 years. It should be recognised that most of this data relates to cyclic testing in which the immediately preceding strain history is one of reversal of the principal strain directions. The initial behaviour exhibited would therefore be expected to be one of maximum stiffness G_o (Atkinson et al., 1990).

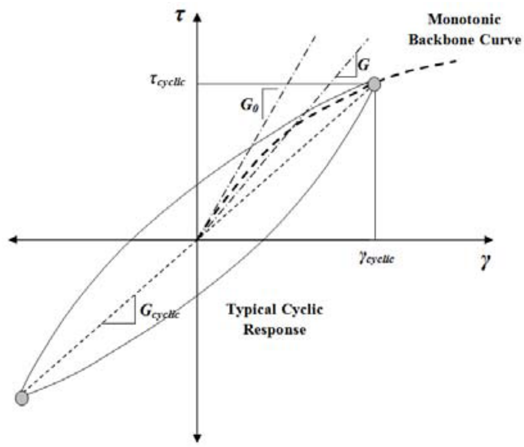

Figure 1. Secant stiffness

3. HYPERBOLIC MODELS

Konder (1963), Duncan & Chang (1970) and Hardin & Drnevich (1972) used hyperbolae to model shear stress-strain curves, being asymptotic to G_0 at zero strain and to τ_{max} at infinite strain. By defining a reference strain ($\gamma_{ref} = \tau_{max}/G_0$) it was possible to rewrite the equation of a hyperbola as a normalised secant shear modulus (G/G_0) reducing with normalised shear strain (γ/γ_{ref}):

$$\frac{G}{G_0} = \frac{1}{\left(1 + \frac{\gamma}{\gamma_{ref}}\right)} \tag{1}$$

On the other hand, Fahey and Carter (1993) adopted the formulation (2):

$$\frac{G}{G_0} = 1 + f\left(\frac{\tau}{\tau_{\max}}\right)^g \tag{2}$$

This is a quasi-hyperbolic relation written in terms of shear stress rather than shear strain, and employing an exponent g to adjust the shape of the curve (Fahey, 1992).

Darendeli (2001) and Zhang et al. (2005) similarly raised the normalised shear strain (γ/γ_{ref}) to a power α in order to better fit the data of small strains: equation (3). This definition retains the feature that secant shear stiffness reduces to half its initial maximum value when $\gamma = \gamma_{ref}$. The current study will adopt the same family of modified hyperbolae in order to find an optimum fit for each soil.

$$\frac{G}{G_0} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{ref}}\right)^\alpha} \tag{3}$$

Darendeli (2001) presented a database of clay, sand and silt data. Using a Bayesian analysis, the curvature parameter α was found to be 0.92 for the normalised strain data. The reference strain was found to be a function of OCR, plasticity index and mean effective confining pressure. Zhang et al. (2005) presented a database of sandy to clayey soils from South Carolina, North Carolina and Alabama. The curvature parameter α was shown to vary from about 0.6 to 1.55 for un-normalised shear strain data.

It must be recognised that the value of α will bear no relation to the strain rate used in the test. Fig. 2 shows equation (3) plotted with various values of the curvature parameter α. It is observed that increasing α causes an increase in normalised stiffness at small normalised strains but decreases the stiffness at high strains. This behaviour is a feature of the modified hyperbolic model but it does not represent the typical behaviour of soil tested at different strain rates. Fine-grained soils typically show stiffness and strength enhancing at all strain rates, for strains in excess of the linear elastic limit: Vucetic & Tabata (2003).

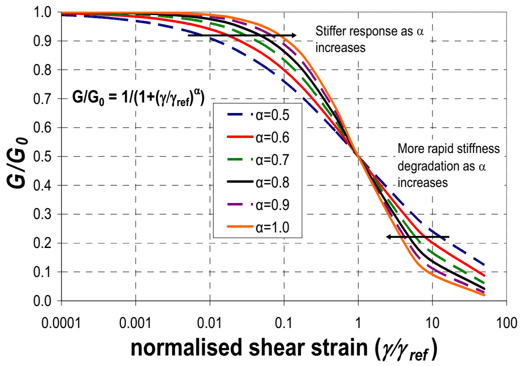

Figure 2. G/G_0 versus normalized strain for various values of the curvature parameter

4. DATABASE ANALYSIS

4.1 Curve fitting parameters

The best-fit values of parameters α and γ_{ref} for each of the studied soils were determined together with the corresponding coefficients of determination, R^2. The number of digitised data-points available, n, was also determined. A higher R^2 is needed in order to describe a correlation as significant when fewer data-points are used in its derivation.

The maximum, minimum, mean and standard deviation (σ) values for these key parameters are shown in Table 1. The statistical fit for equation (3) is very good, with R^2 for individual tests ranging from 0.914 to 1.0.

Table 1. Curve fitting parameters (original data)

Statistic	γ_{ref}	n	α
max	0.00540	73	1.69
min	0.00041	4	0.54
mean	0.00138	18	0.97
σ	0.00098	13	0.26

4.2 Rate effect corrections

It is has been known for many years that the stiffness and strength of clays is rate-sensitive. Richardson and Whitman (1963), for example, used triaxial tests with variable strain-rates. They demonstrated for normally consolidated plastic clay that an increased strain-rate led to enhanced stiffness at moderate strains without any change of pore pressure. In a recent review on the effect of strain rate on cyclic shear modulus at small strains (up to a shear strain of 0.01%) Vucetic & Tabata (2003) reported that the enhancement in stiffness per $\log_{10}$ cycle of strain rate increased with plasticity index I_p from about 2% for very low plasticity clays (I_p < 10%) to about 5% for high plasticity clays ($I_p \approx 40\%$). For large strains, however, Kulhawy & Mayne (1990) obtained a good correlation ($R^2 = 0.802$) for 26 clays by taking the undrained shear strength to increase by 10% per $\log_{10}$ cycle of strain rate. Lo Presti et al (1997) and d'Onofrio et al (1999) offer evidence for low to medium plasticity clays (10% < I_p < 30%) which supports the proposition that the strain rate effect on stiffness may be negligible for very small strains, but can rise to about 5% per $\log_{10}$ cycle at a strain of 0.01% and to about 10% per $\log_{10}$ cycle at a strain of 1%. Detailed reviews of the influence of rate (viscous) effects at intermediate strain levels can be found, for example, in the keynote lectures of Tatsuoka & Shibuya (1992) and Tatsuoka et al (1997).

A carefully conducted undrained triaxial test achieving peak strength at an axial strain of about 2% (and therefore a shear strain of about 3%) after 8 hours would have a shear strain rate $\dot{\gamma} \approx 10^{-6}\text{s}^{-1}$. On the other hand, a resonant column vibrating under maximum excitation with a cyclic shear strain amplitude of 0.1% at 50 Hz would have a peak shear strain rate $\dot{\gamma} \approx 0.3\,\text{s}^{-1}$, which is 5.5 $\log_{10}$ cycles faster than the triaxial test.

The focus of this paper is stiffness at moderate strains. Accordingly, all stiffness data will be normalised to a standard test rate of $\dot{\gamma} \approx 10^{-6}\text{s}^{-1}$, by assuming a strain-rate effect of 5% per $\log_{10}$ cycle consistent with the findings of Lo Presti et al (1997) and d'Onofrio et al (1999). In doing so it is accepted that the stiffness of very low plasticity clays at low cyclic strain amplitudes in resonant column tests is likely to be underestimated, and that the stiffness of high plasticity clays at large strain amplitudes in resonant column tests may remain overestimated. Nevertheless, the disparity in stiffness between dynamic and static test results should have been reduced.

Table 2 shows the transformed metrics for comparison with Table 1, rate-effects having been allowed for. The assumed test frequencies (unless given in the original publication) are given in the Appendix. Note the general reduction of the curvature parameter α.

Figure 3 shows the original data compared with the rate-corrected data. The resonant column test curves are depressed to show a less-stiff response in the rate-corrected plot.

Table 2. Curve fitting parameters (rate corrected data)

Statistic	γ_{ref}	n	α
max	0.00336	73	1.01
min	0.00025	4	0.52
mean	0.00094	18	0.76
σ	0.00065	13	0.11

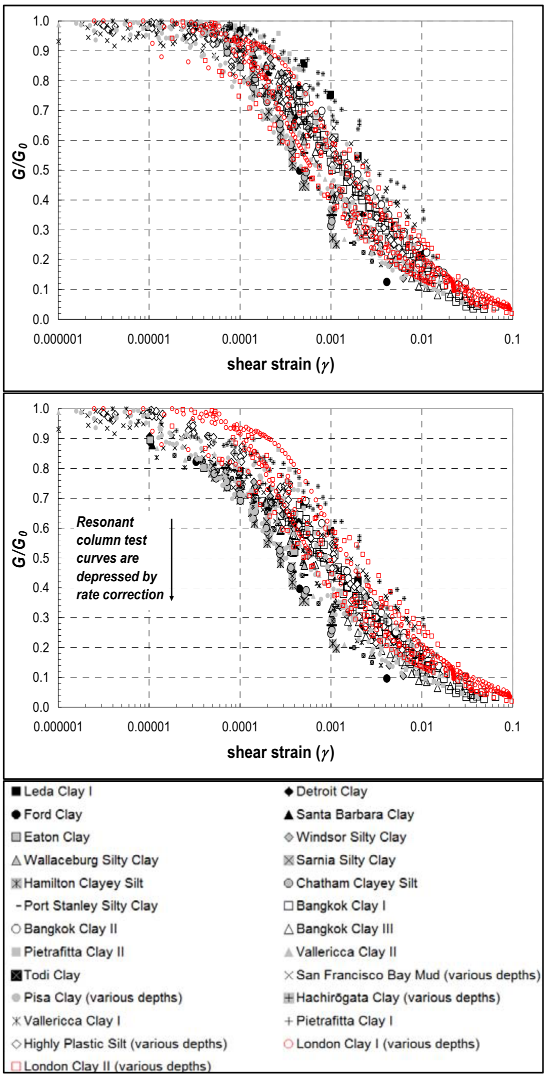

Figure 3. G/G_0 versus shear strain (γ) raw data (top) and with rate correction (bottom)

Fig. 4 shows the hyperbolic fit to the normalised data once it has been rate corrected in the aforementioned manner. The following equation results:

$$\frac{G}{G_0} = \frac{1}{1 + \left(\dfrac{\gamma}{\gamma_{ref}}\right)^{0.74}}$$

R^2=0.96, n=1105, S.E. = 0.0130, p<0.001 (4)

The R^2 for this correlation is very good with 96 percent of the variation being explained by the model. The standard error (*S.E.*) is low and the probability of a correlation not existing (*p*) is less than 1 in 1000.

Further empirical correlations must now be obtained for γ_{ref} in terms of readily available soil properties.

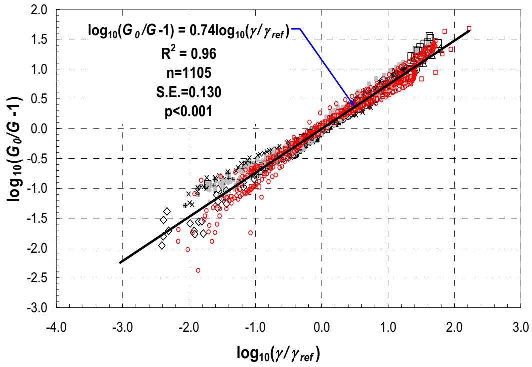

Figure 4. $\log_{10}(G_0/G\text{-}1)$ versus $\log_{10}(\gamma/\gamma_{ref})$ rate corrected data (for key see Fig. 3)

4.3 Prediction of reference strain

Linear regressions were performed using the following variables: plasticity index, liquid limit and plastic limit and voids ratio. Fig. 5 shows the scatter-plots that display the data and the following regression equations:

$$\gamma_{ref}=2.17(I_p)/1000 \quad R^2=0.75,\ n=61,\ S.E.=0.00031,\ p<0.001 \qquad (5)$$

$$\gamma_{ref}=1.25(w_L)/1000 \quad R^2=0.75,\ n=61,\ S.E.=0.00029,\ p<0.001 \qquad (6)$$

$$\gamma_{ref}=2.73(w_P)/1000 \quad R^2=0.57,\ n=61,\ S.E.=0.00039,\ p<0.001 \qquad (7)$$

$$\gamma_{ref}=0.56(e_0)/1000 \quad R^2=0.75,\ n=61,\ S.E.=0.00030,\ p<0.001 \qquad (8)$$

Reasonable R^2 values are obtained for these correlations though an error band of ±50% (shown as dashed lines on Fig. 5) is commonly observed. In each case five of the London clay tests were deemed outliers to the trend. This may be due to the presence of fissuring in the samples: Gasparre (2005).

5. DESIGN CHARTS

5.1 Plasticity Index

Vucetic & Dobry (1991) presented commonly used design charts for seismic engineering. They emphasize the importance of plasticity index. A shortcoming of these charts is that they do not give a mathematical formulation for the degradation curves that they indicate. Fig. 6 compares the curves drawn using equations (4) and (5) to predict reference strain using plasticity index. It is clear that Vucetic & Dobry's curves (shown dashed) display a stiffer response at each strain level. This is understandable, as much of the data in Vucetic & Dobry's database is from fast-cyclic testing (e.g. resonant column) and no rate-effect corrections were made. A rate-effect adjustment can be made with a faster test as a standard, and this would yield curves similar to Vucetic & Dobry's. For foundation design in static situations the curves presented using the formulation in this paper are more applicable.

5.2 Liquid Limit

The liquid limit (fall-cone) test is semi-automated and requires much less judgment on the part of the operator than is the case with the plastic limit test. A correlation with plasticity index ($I_P = w_L - w_P$) calls for both tests to be performed. The adoption of liquid limit alone as the parameter for new design charts should lead to greater reliability in practice. The Atterberg Limits w_L and w_P both relate to the capacity of clays to maintain an open stable structure with a high voids ratio. It is therefore no surprise that the correlations shown in Fig. 5 were found.

Voids ratio requires undisturbed samples in order to minimize water migration, but it offers no statistical improvement. Hence, w_L is favoured. Active clays have stronger intergranular attractions leading to the formation of well-bonded agglomerates. They accordingly tend to have high Atterberg Limits, and it is reasonable that they have been discovered to require higher strains to reduce their initial linear-elastic stiffness.

Fig. 7 shows new design curves for the degradation of clay and silt stiffness plotted against shear strain for a variety of liquid limits based on equations (4) and (6).

5.3 Accuracy of the model

Using equations (4) and (6), G/G_0 ratios were predicted for all the strain values in the database. Fig. 8 shows the plot of the predicted versus the measured data. Apart from the London clay outliers (reference strain values shown in Fig. 5) the modified hyperbola and the liquid limit predict G/G_0 within a bandwidth of ±30%, with lower accuracy at high strains. The framework presented in this Paper is able to predict G/G_0 at any strain, for a clay or silt, to a reasonable degree of accuracy, with knowledge only of the liquid limit.

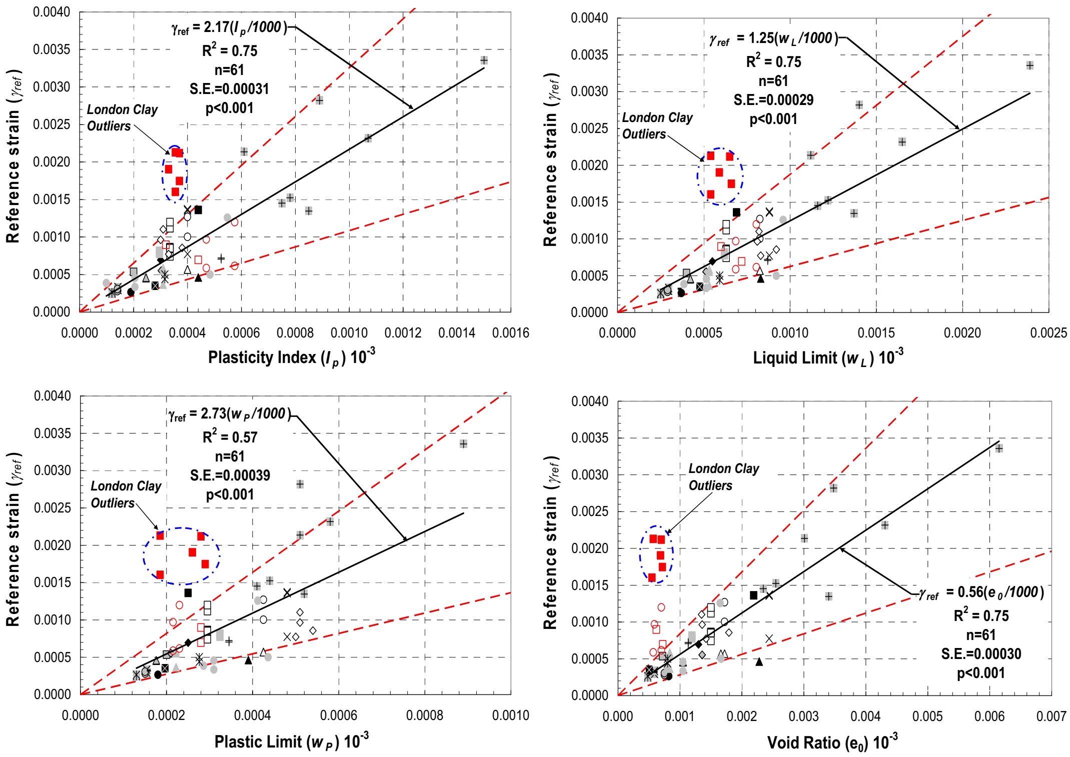

Figure 5. Clockwise from left - reference strain (γ_{ref}) versus plasticity index, liquid limit, plastic limit and voids ratio (for key see Figure 3)

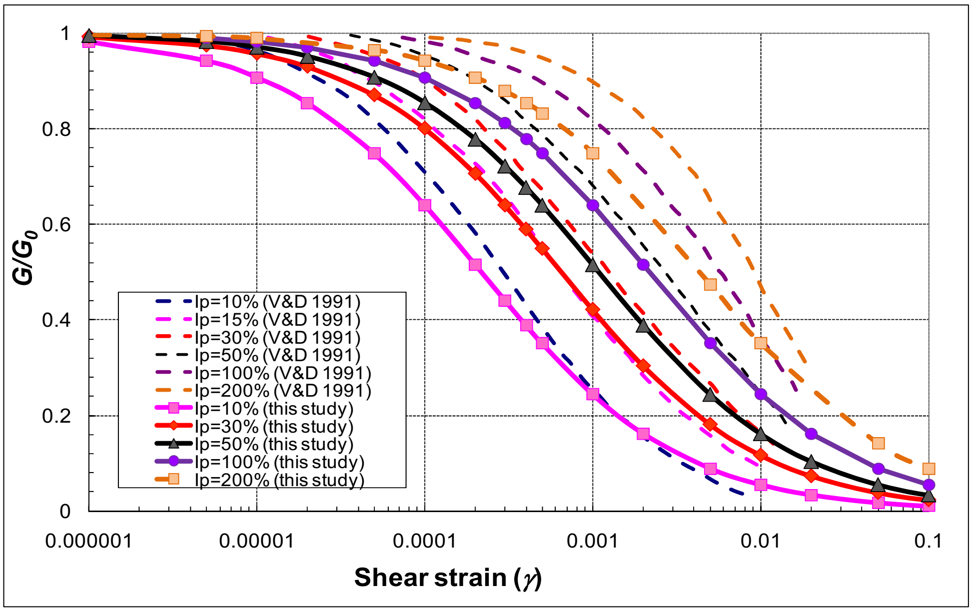

Figure 6. Comparison of predictions using equations (4) & (5) with those from Vucetic & Dobry (1991)

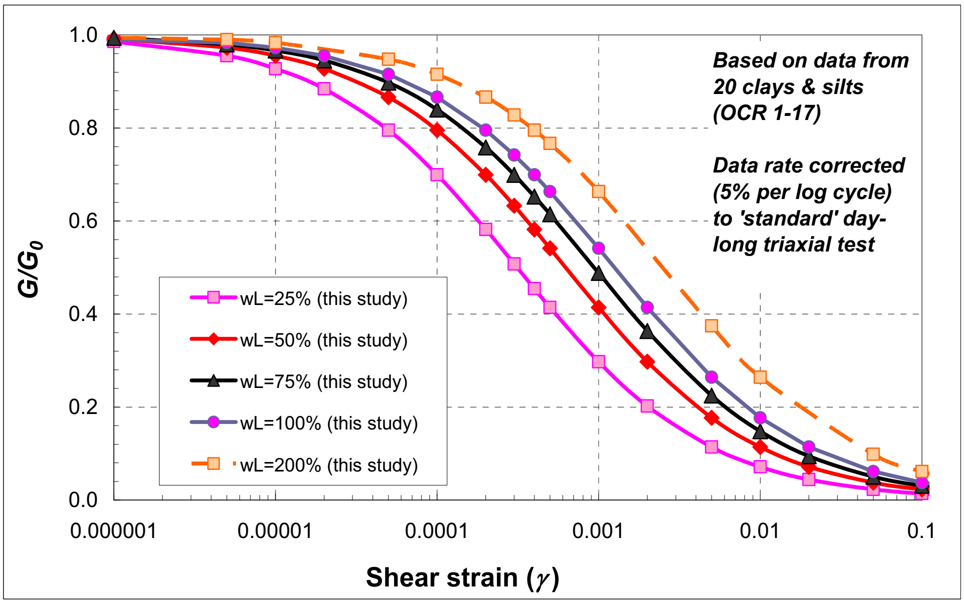

Figure 7. New design charts based on liquid limit (w_L)

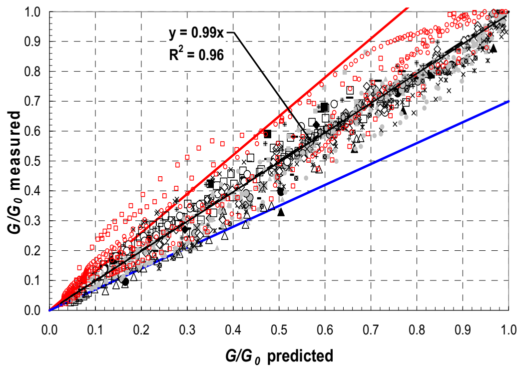

Figure 8. Accuracy of prediction model based on liquid limit (w_L) (for key see Fig. 3)

6. CONCLUSIONS

A detailed database of stiffness degradation has been analyzed to determine simple equations to estimate the behaviour of clays and silts for static, cyclic or dynamic applications. Liquid limit, plastic limit, plasticity index and voids ratio are shown to correlate well with reference strain (γ_{ref}).

Simply using the liquid limit and the derived modified hyperbola, the G/G_0 ratio of a clay or silt at any strain level can be predicted within ±30%. New design curves are drawn.

Depending on the engineering application and the site data available, engineers can make predictions of clay and silt stiffness degradation with confidence in the hyperbolic stress-strain curve and the relationships between reference strain and basic soil parameters.

The values of G/G_0 from equations (4) and (6), and in the design charts of Fig. 7, all refer to the normalized strain rate $\dot{\gamma} \approx 10^{-6}s^{-1}$. If values of G/G_0 were required for a different strain rate $\dot{\gamma}$, and for moderate strain amplitudes, then the scaling $G^* = G + 0.05(\log_{10}(10^6\dot{\gamma}))$ could be used, following Vucetic & Tabata (2003).

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8. REFERENCES

Anderson, D. G. & Richart, F.E. (1976). Effect of straining on shear modulus of clays. *Journal of the Geotechnical Engineering Division (ASCE)*. Vol. 102, No. GT9, pp. 975-987.

Atkinson, J. H., Richardson, D. & Stallebrass, S. E. (1990). Effect of recent stress history on the stiffness of overconsolidated soil. *Géotechnique*. Vol. 40, No.4, pp. 531-540.

Darendeli, M. B. (2001). *Development of a new family of normalized modulus reduction and material damping curves*. Ph.D. thesis, University of Texas at Austin.

d'Onofrio, A., Silvestri, F. and Vinale F. (1999). Strain-rate dependent behaviour of a natural stiff clay. *Soils and Foundations*. Vol. 39, No. 2, pp. 69-82.

Doroudian, M. and Vucetic, M. (1999). Results of Geotechnical Laboratory Tests on Soil Samples from the UC Santa Barbara Campus. UCLA Research Report No. ENG-99-203.

Duncan, J.M. and Chang, C.Y. (1970). Non-linear analysis of stress and strain in soils. *Journal of Geotechnical Engineering (ASCE)*. Vol. 96, No. 5, pp. 1629-1653.

Fahey, M. (1992). Shear modulus of cohesionless soil: variation with stress and strain level. *Canadian Geotechnical Journal*. Vol. 29, No. 1, pp. 157-161.

Fahey, M. and Carter, J.P. (1993). A finite element study of the pressuremeter test in sand using a non-linear elastic plastic model. *Canadian Geotechnical Journal*, Vol. 30, No. 2, pp. 348-362.

Gasparre, A, (2005). *Advanced Laboratory Characterisation of London Clay*. PhD Thesis, Imperial College London.

Georgiannou, V. N., Rampello, S. and Silvestri, F. (1991). Static and dynamic measurements of undrained stiffness on natural overconsolidated clays. in *Proceedings 10th European Conference on Soil Mechanics and Foundation Engineering*, Firenze, Vol. 1, pp. 91-95.

Hardin, Bobby O. and Black, W. (1968). Vibration modulus of normally consolidated clay. *Journal of the Soil Mechanics and Foundations Division (ASCE)*. Vol. 94, No. SM2, pp. 353 to 369.

Hardin, Bobby O. & Drnevich, V. P. (1972). Shear modulus and damping in soils: design equations and curves. *Journal of the Soil Mechanics and Foundations Division (ASCE)*. Vol. 98, No. SM7, pp. 667 to 691.

Kim, T. C. and Novak, M. (1981). Dynamic properties of some cohesive soils of Ontario. *Canadian Geotechnical Journal*. Vol. 18, No. 3, pp. 371-389.

Konder, R. L. (1963). Hyperbolic stress-strain response: cohesive soils. *Journal of the Soil Mechanics and Foundations Division (ASCE)*. Vol. 89, No. SM1, pp. 115-143.

Kulhawy, F. H. and Mayne, P. W. (1990). Manual on estimating soil properties for foundation design. *Rep. No. EL-6800*, Electric Power Research Institute, Palo Alto, California.

Lo Presti, D. C. F., Jamiolkowski, M., Pallara, O., Cavallaro, A. and Pedroni, S. (1997). Shear modulus and damping of soils, *Géotechnique.* Vol. 47, No.3, pp. 603-617.

Rampello, S. and Silvestri, F. (1993). The stress-strain behaviour of natural and reconstituted samples of two overconsolidated clays. in *Geotechnical Engineering of Hard Soils-Soft Rocks*, Anagnostopoulos et al. (eds), Balkema, Rotterdam.

Richardson, A. M. and Whitman, R. V. (1963). Effect of strain-rate upon undrained shear resistance of a saturated remoulded fat clay. *Géotechnique*. Vol. 13, No.4, pp. 310-324.

Soga, K. (1994). *Mechanical behaviour & constitutive modelling of natural structured soils*. Ph. D. thesis, University of California at Berkeley.

Shibuya, S. and Mitachi, T. (1994). Small strain modulus of clay sedimentation in a state of normal consolidation. *Soils and Foundations*. Vol. 34, No.4, pp. 67-77.

Tatsuoka, F. and Shibuya, S. (1992) Deformation characteristics of soils and rocks from field and laboratory tests, Keynote Lecture, in *Proceedings 9th Asian Regional Conference on Soil Mechanics and Foundation Engineering*, Bangkok, 1991, Vol. 2, pp. 101-170.

Tatsuoka, F., Jardine, R. J., Lo Presti, D., Di Benedetto, H. and Kodaka, T. (1997). Characterising the pre-failure deformation properties of geomaterials, Theme lecture for the plenary session No. 1, in *Proceedings of XIV International Conference on Soil Mechanics and Foundation Engineering*, Hamburg, September 1997, Vol. 4, pp. 2129-2164.

Teachavorasinskun, S., Thongchim, P. and Lukkunaprasit, P. (2002). Shear modulus and damping of soft Bangkok clays (Technical Note). *Canadian Geotechnical Journal*. Vol. 39, No. 5, pp. 1201-1208.

Vucetic, M. and Dobry, R. (1991). Effect of soil plasticity on cyclic response. *Journal of Geotechnical Engineering (ASCE)*. Vol. 117, No. 1, pp. 89-117.

Vucetic, M. and Tabata, K., (2003). Influence of soil type on the effect of strain rate on small-strain cyclic shear modulus. *Soils and Foundations*. Vol. 43, No. 5, pp. 161-173.

Yimsiri, S. (2001). *Pre-failure deformation characteristics of soils: anisotropy and soil fabric*. Ph.D. thesis, University of Cambridge, U.K.

Zhang, J., Andrus, R.D. and Juang, C.H. (2005). Normalized shear modulus and material damping ratio relationships. *Journal of Geotechnical and Geoenvironmental Engineering (ASCE)*. Vol. 131, No. 4, pp. 453-464.

Appendix: Undrained Clay & Silt Database

Reference	Test Type	Test frequency Assumed	Soils Studied
Anderson & Richart (1976)	Resonant Column (RC)	50Hz	Detroit Clay, Ford Clay, Eaton Clay, Leda Clay & Santa Barbara Clay
Kim & Novak (1981)	Resonant Column (RC)	50Hz	7 Ontario Cohesive Soils
Teachavorasinskun et al. (2002)	Cyclic Triaxial (CT)	given in paper	Bangkok Clay (3 sites)
Georgiannou et al. (1991)	RC, T & TS	50Hz, 0.1Hz, 0.025Hz	Vallericca Clay, Pietrafitta Clay, Todi Clay
Soga (1994)	Cyclic Triaxial (CT)	given in thesis	San Francisco Bay Mud, Pancone Clay
Shibuya & Mitachi (1994)	Torsional Shear (TS)	given in paper	Hachirōgata Clay
Rampello & Silvestri (1993)	Resonant Column (RC)	50Hz	Vallericca Clay, Pietrafitta Clay
Doroudian & Vucetic (1999)	Direct Simple Shear (DSS)	0.025Hz	Santa Barbara Plastic Silt
Yimsiri (2001)	Triaxial (T)	No correction needed	London Clay I– Kennington Park
Gasparre (2005)	Triaxial (T)	given in thesis	London Clay II– Heathrow Terminal 5 project