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Velocity fields of granular flows down a rough incline: a DEM investigation

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Abstract This paper describes a statistical method to explore the velocity profiles of granular flows down rough inclines. Using 3D Discrete Element Method (DEM), granular material is released from a box onto a slope and allowed to flow indefinitely. Fluctuating velocity fields are observed, as particle motions become more dynamic and agitated. Linear regression is used to decompose the fluctuating velocity field into a best-fit velocity profile and a fluctuating component. Analysis shows that the slope inclination has a considerable influence on the rheology, in terms of both the fluctuating velocity and the shear rate of the flow.

Keywords Granular flow · Rheology · Discrete element modelling

1 Introduction

One of the most intriguing aspects of rapid granular flows is that the granular material may behave as a solid, liquid or gas [1]. This multi-phase nature poses tremendous difficulties on the correct modelling of constitutive behaviour and mechanics. The problem is also complicated by the need to address the microscopic interactions between the flowing grains, as collisions and friction at contacts have a critical influence on the flow characteristics.

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In spite of the huge research efforts of physicists and engineers over the last few decades, the rheology and mechanics of rapid granular flows down inclines, such as rockslides and debris flows, are still not well understood [2–5]. The obstacles mainly come from: (1) physical experiments failing to mimic the full-scale event; (2) numerical modelling being undermined by incorrect rheological models and input parameters and (3) field examinations being obtainable only *a posteriori*. As a result, there have been diverse observations and/or formulations of velocity profiles of such flows [6–8].

The most widely known yet misleading rheological model for rapid granular flows originates from Bagnold's shear-cell experiments [9]. Bagnold discovered that the shear stresses resulting from grain collisions are proportional to the square of the shear rate, and that the velocity profile u across the flow direction W can be described as:

$$u(W) = C_{\text{bag}}\left[h^{\frac{3}{2}} - (h - W)^{\frac{3}{2}}\right] \tag{1}$$

where C_{bag} is the Bagnold's constant and h is depth below the flow surface.

However, critics have pointed out the inaccurate scaling due to experimental setup conditions and that stress measurements were seriously affected by vortices [10]. Also the idealised steady, dense, non-gravity-driven flow conditions in Bagnold's experiments are generally incompatible with natural granular flows.

This research studies the velocity profiles of granular avalanches down a rough inclined plane using the Discrete Element Method (DEM). The aim is to understand the evolving rheology of flows down planes with various inclination angles θ. A statistical approach is used to analyse the fluctuating grain motions and to systematically study the rheology of avalanche motion.

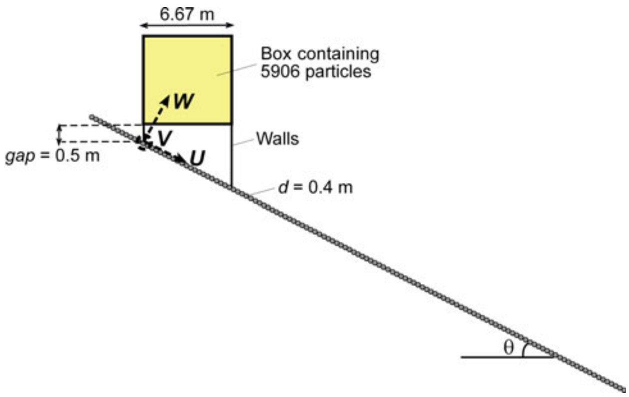

Fig. 1 Schematic layout of the numerical experiment. The axis system (*U, V, W*) is defined with respect to the *sloping plane*

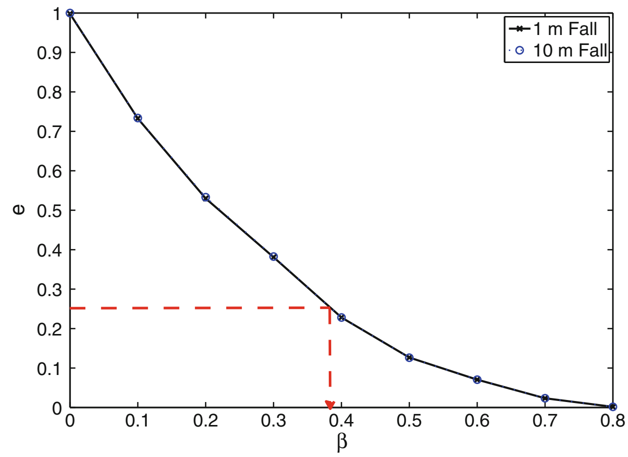

Fig. 2 Calibration test to determine the relation between coefficient of restitution e and critical damping ratio β. The dashed arrow indicates how the input critical damping ratio $\beta \sim 0.38$ is interpolated from the chosen value of restitution coefficient $e = 0.25$ in our simulations

2 Discrete element simulations

The simulations are performed using the discrete element program, PFC^{3D}. A more detailed description of the code formulation can be found in the PFC^{3D} Manual [11].

DEM simulations are performed on a sloping plane as sketched in Fig. 1. A granular sample of 5,906 spherical particles is prepared in a cubical box of side length 6.67 m. The micromechanical parameters used in the sample are listed in Table 1. Here, the normal and shear contact stiffnesses are treated as linear, and taken to be equal. In quasi-static loading of a dense aggregate it is recognised that this ratio influences the Poisson's ratio [12]. In the current work, the dense granular sample is released at the beginning, and the regime that develops is collisional. In this case, the mechanics of stable contacts at long duration is not thought to be relevant. The coefficient of restitution e determines the energy loss on collision. This was obtained by specifying the critical damping ratio β of a contact system idealised with contact springs and a contact dashpot. Calibration between e and β was confirmed using single-particle drop-tests with 1 and 10 m fall distances (see Fig. 2).

The sloping plane is made of grains layered in a hexagonal closed packing. These grains are identical to those in the sample box, except that their stiffnesses are 100 times those in the sample. This produces a bumpy slope surface such that some rolling resistance is offered to free particle, by virtue of the collisions it must have with the fixed particles. Although it is recognised that the explicit provision in DEM of rolling resistance can improve the modelling of dense aggregate [13], this was considered unnecessary in the present study of collisional flows.

Table 1 Micromechanical parameters of the granular sample

Particle diameter, d	0.4 m
Spring stiffness (normal and tangential), k	8×10^9 N/m
Inter-particle friction, μ	0.50
Coefficient of restitution, e	0.25
Initial porosity, p_0	0.33

Upon completion of the sample preparation, the side walls of the cubical box are extended to the inclined plane, followed by the removal of box base to allow particles to settle onto the slope. Once the particles have settled to an equilibrium state, granular flow is initiated down the slope by deleting all bounding walls. Periodic boundaries are immediately imposed at the lateral extremes ($V = \pm 3.335$ m) to model flow of infinitely wide extent.

A total of 6 granular flow simulations are performed down slopes of various θ ($\theta = 7.5°, 15°, 22.5°, 30°, 45°$ and $60°$) in this study, with an aim of understanding how the slope inclination influences the flow rheology.

3 Results

Figure 3 shows successive snapshots of a granular avalanche down a slope with $\theta = 30°$. An initially stationary rock mass is gradually turning into a dilute granular flow, with its front tail progressively becoming agitated and behaving like a rapid granular gas. Unlike other studies which may have a continuous supply of flowing materials at the inlet to promote steady uniform flow conditions, this study models the flow characteristics of a finite volume of rock particles being released onto an incline. Thus flows tend to be thin, dilute and unsteady in nature. In Fig. 3, the colouring of the initial sample in five equal vertical layers also helps trace the mass distribution throughout the flowing process.

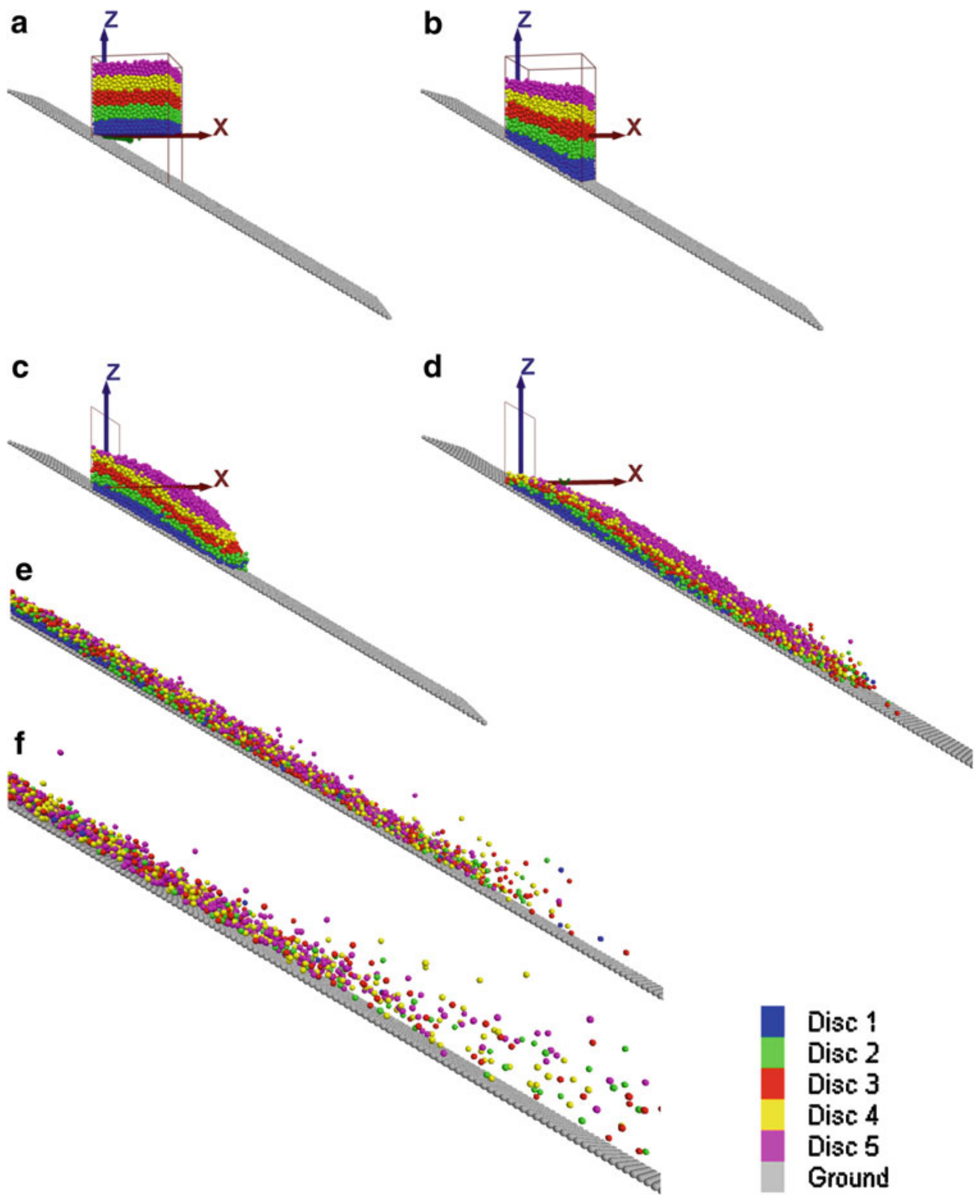

Fig. 3 Snapshots of granular flow down an incline of $\theta = 30°$ with $\mu = 0.5$ and $e = 0.25$ at characteristic times T: **a** After sample preparation (initial stage), **b** After resolidification onto the slope and the sample was brought to equilibrium, when T is defined, **c** $T = 6.6$, **d** $T = 15$, **e** $T = 25$, **f** $T = 45$. The colour of the grains corresponds to the origin of the vertical layers they belong to at initial stage, as illustrated in the key next to **f**. The magnification of each snapshot remains the same, and therefore not all grains are necessarily captured in the diagrams

Considerable flattening and mixing between vertical layers are observed once the flow is mobilised.

3.1 Data analysis

In order to investigate the local rheology of the flows, we need to extract velocity data at a representative sampling location within the moving granular mass. Here, local velocity distributions are obtained from a region of 20 particle diameters wide about the centre of mass of the flow. A similar sampling technique was adopted in [14,15].

Information can then be extracted regarding the flow rheology during the course of the simulations. Figure 4 shows one such example showing snapshots of the velocity distribution at successive time instants for the case of $\theta = 30°$, with the axes depicting the velocity and height data of the particles falling in the sampling zone.

Note that the velocity u, height W and time t are normalised by the groups $(gd)^{0.5}$, d and $(d/g)^{0.5}$ respectively, where d is the particle diameter and g is the acceleration due to gravity. For simplicity, we denote the normalised time as capitalised letter T.

One can use Fig. 4 to reflect on the general mechanics of the granular flow as it moves down the 30° slope. The granular mass starts as a dense stationary material and becomes more dilute and agitated as it avalanches down the slope. Significant mixing occurs as collisions between particles promote a turbulent-like granular gas structure.

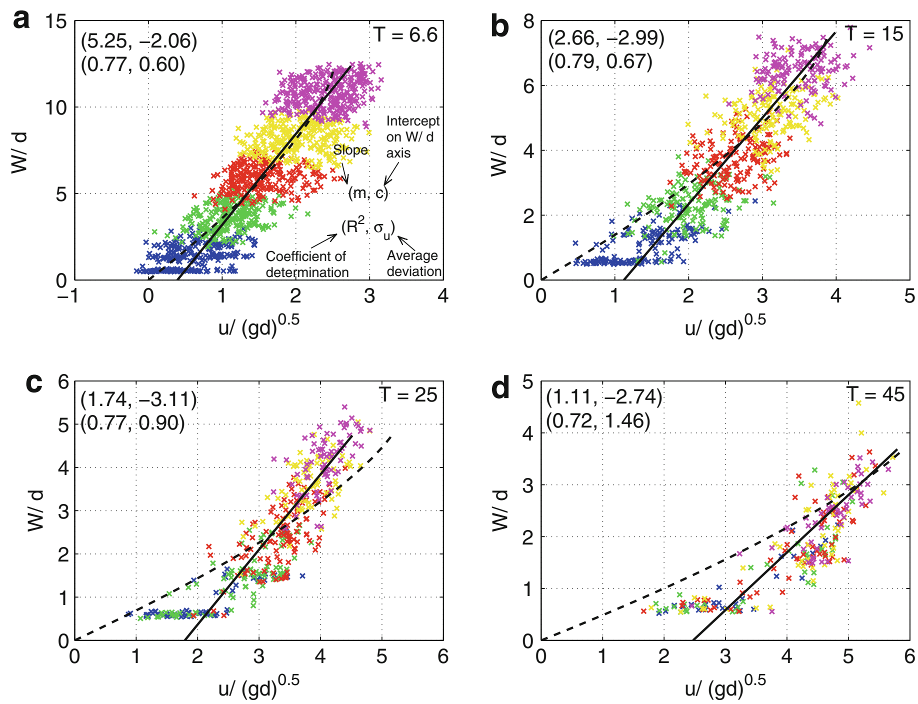

Fig. 4 Velocity profiles across the depth of slides through its centre of mass at characteristic times T. The statistical information of the profiles are given in the top left hand corner of each figure, according to the key given in (**a**)

3.2 Statistical description of the local velocity profiles

We investigate the local rheology of the granular flow by analysing the random fluctuating velocity field in a statistical approach.

Analogous to turbulent flow treatment in classical fluid mechanics, the turbulent-like downstream velocity field can be broken down into a best-fit velocity profile $\hat{u}(w)$ and a fluctuating velocity component u_i' (i.e. a measure of the velocity deviation from the best-fit velocity profile):

$$u_i = \hat{u}(W) + u_i' \quad (2)$$

The average deviation of the downstream *normalised* velocity field can be defined as

$$\sigma_u = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \frac{u_i'^2}{gd}} \quad (3)$$

where N is the total number of particles in the sampling zone.

In Fig. 4, we determine the best-fit linear profiles at different time instants of the granular flow, using linear regression. These are represented in the plots as solid straight lines. On the top left hand side of each snapshot are two brackets summarising the statistical information of the best-fit velocity profile. The first bracket gives the slope m and W-intercept c of the velocity profile, while the second bracket records the regression coefficient R^2 and the average deviation σ_u.

The velocity field is quite well defined using this statistical method; for example, the R^2 values lie above 0.7 at all times for $\theta = 30°$. Also plotted as dotted lines in Fig. 4 are the best-fit Bagnold profiles defined in Eq. 1, where C_{bag} is varied at each time instant to provide optimal fitting through the data set. Apparently, the Bagnold rheology gives a good scaling of the rheology when the granular flows are relatively dense at the beginning, but becomes more ill-fitting when the flow becomes more dilute and collisional at the later stages. This is understandable as Bagnold momentum transfer theory assumes a thick flow of constant high density, which is increasingly inapplicable as the flow progresses down the incline.

3.3 Influence of inclination angle

The influence of slope inclination on the flow rheology is explored through the best-fit velocity profiles in all our simulations. Figure 5 illustrates the downstream velocity profiles at characteristic times for different slope angles θ, with the depth $h = (W/d)_{\max}$ of the flow denoted by the top of the best-fit line and the average deviation σ_u represented by the shaded zones. This offers a clear and systematic breakdown of the fluctuating velocity fields in each case.

On shallow slopes such as $\theta = 7.5°, 15°$, the granular flow is steady or decelerating, with the velocity profile ultimately returning to a stationary state. When θ increases, the increased gravitational effect leads the flow to become

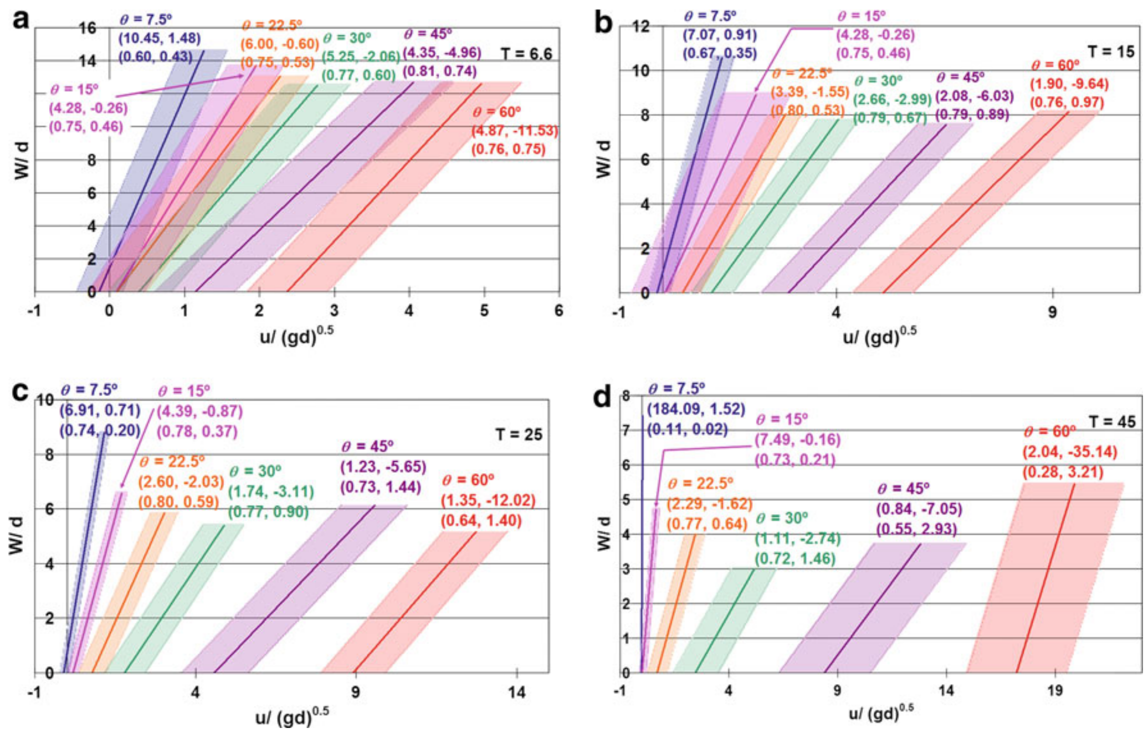

Fig. 5 Comparison of the instantaneous velocity profiles for flows down varying θ at characteristic times T

more dilute and increasingly collisional (shown by the average deviation σ_u). For steep slopes such as $\theta = 45°, 60°$ the agitated flow continues to accelerate, with both a higher 'shear rate' (given by the reciprocal of the slope m of the line) and a larger 'slip velocity' at the base (given by the intercept on the horizontal axis in Fig. 5). For the steepest slope, however, the shear rate ultimately reduces again as a plug of granular gas simply slides down the slope at high speed.

4 Conclusions

The evolving rheology of granular materials flowing down rough inclined planes has been examined through DEM simulations. The influence of slope inclination has been determined. Through the use of linear regression, both the trend-lines of the local velocity profiles and the fluctuating component of the velocity fields, are captured and analysed. It is shown that the slope inclination has a considerable influence on the local rheology, in terms of both the fluctuating velocity and the shear rate of the flow.

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