Discussion of "Accuracy of Digital Image Correlation for Measuring Deformations in Transparent Media" by Samer Sadek, Magued G. Iskander, and Jinyuan Liu

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The authors present an interesting application of digital image correlation techniques to the measurement of deformations in transparent media, and comment upon its accuracy in image-space (i.e., in units of pixels). This valuable contribution demonstrates that modern image analysis techniques are superior to conventional methods in which film photographs or x-rays are used to track target markers. With digitial image analysis, target markers need not be embedded in the soil, and many more measurement points can be established in the model. We wish to share some of our own experiences validating a similar image analysis system (White et al. 2003; White and Take 2002) and describe methods by which the accuracy, or more correctly the precision, of the measurements presented by the authors' technique can be improved.

Accuracy, Precision, and Resolution

A clear distinction should be drawn between accuracy, precision, and resolution when assessing the performance of an image-based measurement system. Accuracy is defined as the systematic difference between a measured quantity and the true value. Precision is defined as the random difference between multiple measurements of the same quantity. Resolution is defined as the smallest interval that can be present in a reading.

The accuracy of a digital image-based measurement technique should be assessed systemwide. In making a single displacement measurement, (1) an image is acquired, (2) a displacement is calculated in units of pixels, and (3) that measurement is converted using some calibration into units with physical relevance. System accuracy, therefore, is a function of the errors associated with (1) the optics of image formation, (2) the image-processing algorithm, and (3) the transformation by which image-space coordinates (pixels) are converted into object-space coordinates (e.g., mm).

It is often assumed that a single image scale can be applied across the entire image to convert from image-space coordinates to object-space coordinates. However, this simple correction can only be applied in the case of an ideal camera that provides a perfect perspective projection, in an experiment in which the camera axis is exactly orthogonal to the plane of observation. Digital cameras are not ideal, and cannot be positioned so accurately. Radial and tangential lens distortion is always present, and the camera axis is never perfectly orthogonal to the experimental plane, which usually lies behind a viewing window that refracts the incident rays. These sources of distortion lead to a systematic variation in image scale across the image. Close-range photogrammetry can be used to model this systematic variation (Atkinson 2001), as acknowledged by Taylor et al. (1998), who first applied these techniques to geotechnical physical modeling. In our experience, even with careful attempts to align the camera axis perpendicular to the image plane, a typical digital image taken by a Kodak DC280 (which has similar optics to the authors' Nikon Coolpix 990) has an image-scale variation of 2%. In other words, observed movements of 100 pixels in different parts of an image with a mean scale 10 pixels/mm could correspond to actual movements of 9.9 and 10.1 mm, respectively. If a constant image scale were assumed, these movements would both be erroneously reported as 10 mm. The authors quote image scales of 9 and 27 pixels/mm for their two validation experiments; how were these chosen, and were they checked at multiple points within each image?

Of the three measurement stages described above, the authors' paper is concerned with errors that arise during stage (2), the image-processing algorithm. The authors present the random scatter within measurements of an artificially created image movement (Figs. 10 and 11). These measurements are indicative of the precision (not the accuracy) of the cross-correlation method used for the DIC analysis. The accuracy of the cross-correlation method is indicated by the difference between the mean measured displacement and the actual (artificial) image movement. Since cross-correlation methods are free of sources of systematic error for integer pixel movements, the mean measured vector will be equal to the artificial value, indicating a systematic error of zero.

This distinction between accuracy and precision is important. System accuracy is governed by the method used to account for variations in image-scale arising from optical distortion. System precision depends on the digital image correlation technique. Both precision and accuracy should be quantified as measurements of system performance. The improved precision (reduced random error) offered by DIC techniques should be coupled with the improved accuracy (reduced systematic error) offered by photogrammetric correction methods to account for optical errors and provide a robust framework for the translation of image movements to real displacements (White 2002; Take 2003). It is only by this coupling that the full potential of the digital image correlation techniques can be realized.

Improved Precision of Cross-Correlation Algorithm

The precision of cross-correlation algorithms is a function of image texture (i.e., the spatial frequency of the soil image), the form of the cross-correlation equations, and the subpixel interpolation scheme. As the first component is covered at length in White et al. (2003), the remainder of the discussion focuses on the latter two components.

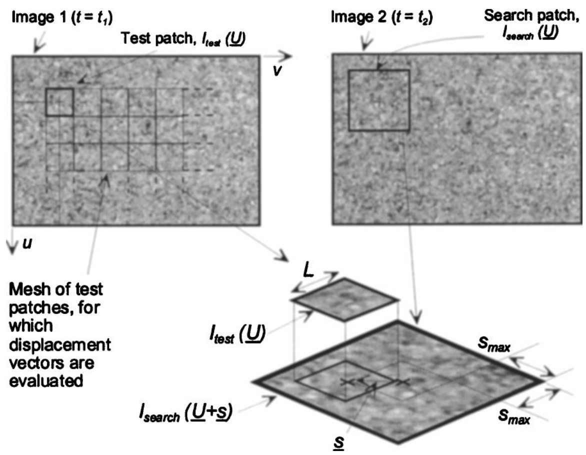

Fig. 1. PIV/DIC procedure used by GeoPIV (White and Take 2002; White et al. 2003)

The authors describe various cross-correlation methods for the implementation of digital image correlation (DIC), referred to in our publications using the experimental fluid mechanics term *particle image velocimetry* (PIV). These correlation algorithms serve the purpose of establishing the "degree of match" between a test patch taken from an initial image, $f(m,n)$, and a search patch in a subsequent image, $g(m,n)$. These test and search patches are shaded in Fig. (3) of the paper. The "degree of match" is evaluated at integer pixel shifts (x,y) from the location of the test patch. The shift at which the highest "degree of match" is recorded (x_p,y_p) indicates the displacement vector of the test patch between the two images.

In the PIV/DIC implementation described by the authors, the functions $f(m,n)$ and $g(m,n)$ are truncated to have zero value outside the domain of the test patch (this domain is called the interrogation window) and renamed $f_e(m,n)$ and $g_e(m,n)$. Therefore, if the material visible in $f_e(m,n)$ has moved to outside the interrogation window, it cannot be identified since it lies in the region of $g(m,n)$ that has been truncated to zero value in $g_e(m,n)$. This truncation of the function $g(m,n)$ limits this implementation to the measurement of movements smaller than the interrogation window size.

The solution to this drawback is to truncate only the test patch, $f_e(m,n)$. The search patch can remain the full image, $g(m,n)$, or can be limited to a zone close to the test patch to reduce the computational burden. This procedure is shown schematically in the discussers' Fig. 1, which should be contrasted with Fig. (3) of the paper. The two input functions for this PIV/DIC cross-correlation procedure are I_{test} (the patch of soil for which a displacement vector is sought) and I_{search} (the region of a subsequent image in which the patch may now lie).

White et al. (2001a, b, 2002, 2003) describe a PIV/DIC system, called GeoPIV, that uses a cross-correlation procedure similar to the NCC method referred to by the authors. The NCC algorithm is reproduced incorrectly in Eq. (4); in the denominator, g_e should be in $(x+m,y+n)$, not (m,n). The modified version of the NCC algorithm used in GeoPIV is shown here in Eq. (1) following the notation of our Fig. 1, and is illustrated in a one-dimensional form in Fig. 2. The mask matrix $\mathbf{M}(\mathbf{U})$ is of unit intensity over the domain of $\mathbf{U}$ from which I_{test} is taken. The vector $\mathbf{U}$ denotes the pixel coordinates (u,v), which are equivalent to (m,n). Aside from the different truncation procedure, the correlation method used by GeoPIV does not include the square

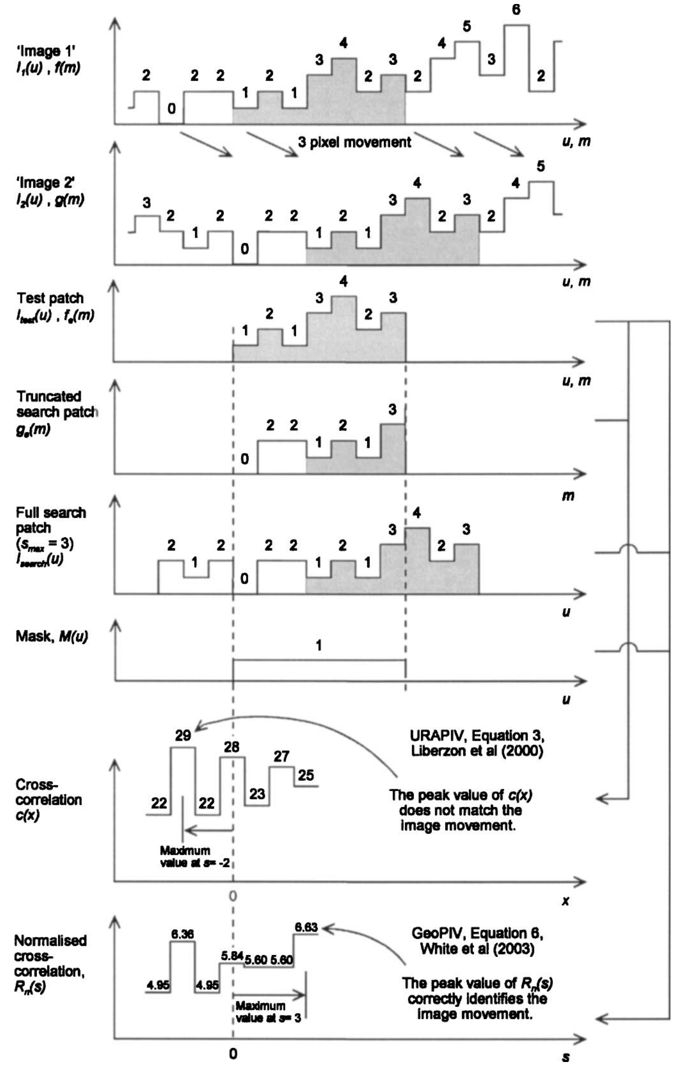

Fig. 2. Comparison of correlation methods

of the test patch in the denominator [as the NCC method does, Eq. (4) of the original paper], since this term is independent of (s) and so does not influence the shape of the correlation plane, and is therefore an unnecessary additional computation.

$$R_n(\underline{s}) = \frac{\sum_U I_{\text{test}}(\mathbf{U}) I_{\text{search}}(\mathbf{U}+\underline{s})}{\sqrt{\sum_U [I_{\text{search}}(\mathbf{U}+\underline{s})]^2 \mathbf{M}(\mathbf{U})}} \tag{1}$$

We were surprised to find that although the authors were using a similar NCC procedure to ours, the reported accuracy (Figs. 10 and 11) was lower than our own, even over small displacement increments for which the truncation of the search patch has minimal influence. White et al. (2003) report a precision better than 0.05 pixels for test patches larger than 16×16 pixels in a variety of validation tests using GeoPIV. The authors report comparable precision only when using 256×256 pixel patches, and record errors of 0.3 pixels or more for 16×16 pixel patches.

An explanation for this discrepancy lies in the URAPIV software used by the authors, which is distributed over the Internet by Liberzon et al. (2000) as an open-source MatLab application (http://urapiv.tripod.com). This software uses a nonnormalized cross-correlation algorithm, as shown in Eq. (3) of the paper, rather than the normalized version shown in Eq. (4). As the authors note, referring to Gonzalez and Woods (1992), this approach has the disadvantage of being sensitive to changes in the amplitude (intensity) of the images. The result is that "wild vectors" occur where a high value of the correlation function $C(x,y)$ is generated by the high intensity, rather than due to a high "degree of match" between the test and search patches.

It is this lack of normalization, coupled with the truncation of the search patch, that is responsible for the "wild vectors" apparent in Fig. (9) and the resulting large errors in Figs. (10) and (11). The same two images have been used in all of the validation experiments reported by the authors. Therefore, had an NCC routine been used, with the truncation procedure as described in Fig. 1, the shape of each correlation plane would be identical, regardless of the artificial image shift, "S." Therefore, there would be no variation in the measurement error with the magnitude of the artificial image shift, as is shown in Figs. (10) and (11).

However, when truncating the search patch as described by the authors, and using a nonnormalized correlation algorithm, the reliability of the correlation process is reduced. Fig. 2 illustrates this effect in a simple fashion using a pair of one-dimensional "images," with pixel intensities in the range 0 to 6. The second image has been shifted by a distance of 3 pixels. The nonnormalized correlation function [Eq. (3), URAPIV] gives a "wild vector," since the truncated search zone does not contain sufficient of the displaced test patch to produce the highest value of the correlation function. The normalized cross-correlation function with a search patch containing the entire displaced test patch [Eq. (1), GeoPIV] correctly identifies the true displacement vector.

The precision of cross-correlation methods is also a direct function of the method of subpixel interpolation. As noted by the authors, cross-correlation functions can only be evaluated at integer pixel intervals. In order to resolve displacement vectors to subpixel resolution, an interpolation method is used between the integer values close to the correlation peak. The precision of the cross-correlation can be further improved by fitting a two-dimensional bicubic interpolation surface through the integer peak (as is done in GeoPIV), rather than the orthogonal Gaussian curves used by URAPIV.

Summary

In summary, the PIV/DIC technique described by the authors represents a powerful method for the measurement of deformation in geotechnical physical modeling. In this discussion, we have suggested improvements to the PIV/DIC technique reported by the authors. In addition, we emphasise the importance of robust image calibration. The improved precision (reduced random error) of PIV/DIC should be coupled with the improved accuracy (reduced systematic error) offered by photogrammetric correction methods.

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Closure to "Accuracy of Digital Image Correlation for Measuring Deformation in Transparent Media" by Samer Sadek, Magued G. Iskander, and Jinyuan Liu

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The writers would like to thank the discussers for the interest in our work and for sharing their related research with Geo-PIV.

Measurement of deformation using digital image correlation is becoming an invaluable tool in civil engineering. Digital image correlation (DIC) is a classic pattern recognition technique in which two images are compared to obtain the relative displacement between them. DIC is widely used in many engineering fields to obtain spatial deformation patterns, albeit with several names. It is also known as particle image velocimetry (PIV), image cross-correlation, block or region matching method, surface displacement analysis, and subregion scanning computer vision. In geotechnical engineering, DIC has also been used by Horii et al. (1998), Guler et al. (1999), and Rechenmacher and Saab (2002).

PIV utilizes DIC in measuring flow pattern and velocity distribution by embedding small particles referred to as seeds in the flow path. Measurements are performed using the cross-correlation function on two consecutive images of these particles. PIV often uses flourescent seeds along with a laser source to identify flow characteristics in specified orientations. We believe that the term PIV is a suitable description for DIC application in fluid dynamics, and that surface deformation analysis in geotech-

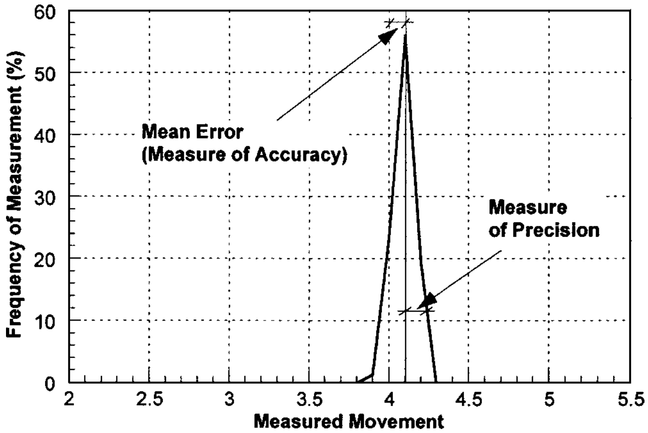

Fig. 1. Accuracy vs. precision of a measured 4-pixel movement using DIC (No. of measurements=128; interrogation window size =64×64 pixels)

nical engineering should be referred to by the umbrella term of the technology, i.e., DIC.

Accuracy and Precision

We agree with the discussers that precision is the random difference between multiple measurements of the same quantity. However, accuracy is commonly defined as the relationship between the value of a measurement and its true value. Our paper presents both the expected precision and accuracy due to the application of DIC algorithm in measurement of spatial deformations in transparent soils. For instance, the data for 4 pixels of movement and a window size of 64×64 pixels, shown in Fig. 10 of our paper, is replotted in Fig. 1 here to illustrate accuracy and precision. The paper also points out other sources of error, including imaging, out of plane, and rotational movement errors. These sources, together with the algorithm error, result in inaccuracy, not lack of precision. In any case, this is a semantic difference, not a germane one.

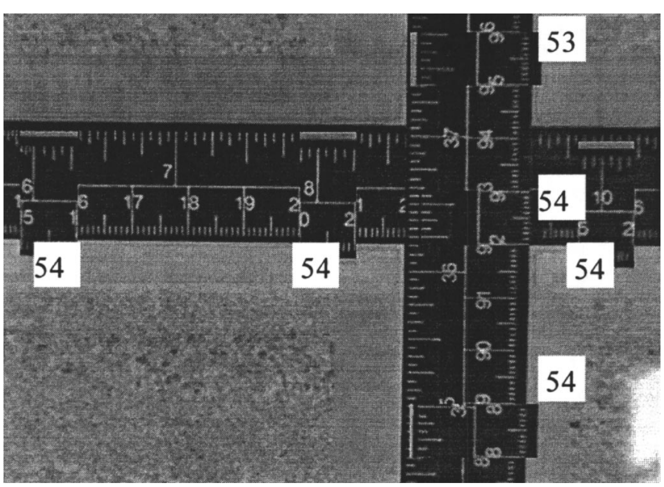

Fig. 2. Number of pixels corresponding to a fixed distance measured on the surface of the mold

Image Distortion and Movement Calibration

Indeed it is not physically possible to have no distortion in a captured image. We checked the distortion of our images, as shown here in Fig. 2, where a paper scale was taped to the test setup. Images were analyzed and the number of pixels corresponding to a fixed distance in various parts of the image was identified. The maximum error was one pixel. Note that the image shown in Fig. 2 was taken with our updated test setup (640 ×480 image size), which is described in Liu and Iskander (2004).

The distortion of image is mitigated by several factors. First, DIC compares similarly distorted images, thus limiting distortion errors to the final step of the analysis during the application of an imaging scale to DIC results to obtain final movements. Second, a potential ±1% error in displacement, as reported by the discussers, pales in comparison to the achievement of measuring continuous spatial displacements inside transparent soil models.

Regarding image scaling, the scales 27-pixel per mm and 9-pixel per mm represent the maximum and minimum zoom that could be achieved with the Nikon 990 camera and the experimental setup. The camera was set perpendicular to the model, 25 cm away from its center.

Perhaps a more important difference between our work and the discussers' work at Cambridge is their use of a set of markers with known space coordinates along with close-range photogrammetry to correct for distortions errors (White et al. 2003). The positioning of the camera in a typical centrifuge test setup may result in large fish-eye distortion in the captured images. The discussers report the distortion error in our system to be on the order of ±1%. Distortion errors are less important than other errors that may arise during measurement of spatial deformations in 3D transparent soil models, such as loss of speckle ID due to out-of-plane movements and particle rotation.

Normalized Cross-Correlation

We thank the discussers for pointing out the error in reproducing the normalized cross-correlation functions. Giatchetti (2000) presented two common methods for normalization, the normalized cross-correlation function (NCC) [Eq. (1)] and the Zero Normalized Cross-Correlation function (ZNCC) [Eq. (2)], of two gray scale functions f and g, having an M×N array of points given by

$$\mathrm{NCC} = \frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f_e(m,n)g_e(x+m,y+n)}{\sqrt{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f_e(m,n)^2 \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g_e(x+m,y+n)^2}} \qquad (1)$$

$$\mathrm{ZNCC} = \frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} [f_e(m,n)-\overline{f}][g_e(x+m,y+n)-\overline{g}]}{\sqrt{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} [f_e(m,n)-\overline{f}]^2 \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} [g_e(x+m,y+n)-\overline{g}]^2}} \qquad (2)$$

for $x=0,1,2,\ldots,\mathrm{M}-1$ and $y=0,1,2,\ldots,\mathrm{N}-1$, where $\overline{f}$ and $\overline{g}$ =mean values of f and g, respectively, and f_e and g_e=extended forms of f and g. The extended form can be defined as the original function and can be extended to have zero value everywhere else.

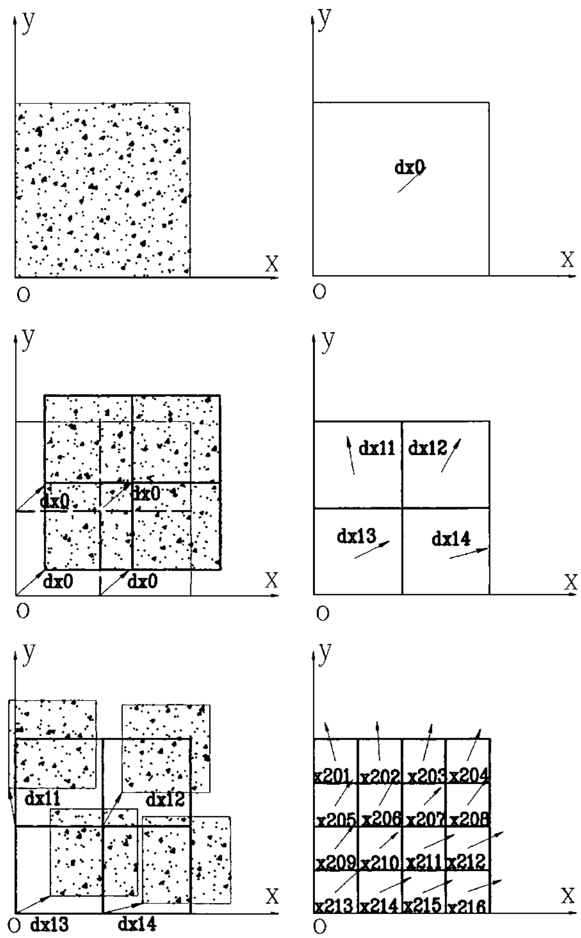

Fig. 3. Conceptual representation of adaptive cross-correlation

The writers' objective is to facilitate the use of transparent soils for modeling geotechnical problems. The setup for modeling with transparent soils involves lasers, molds, cameras, and framegrabbers, in addition to software. We chose an open-source matlab code (URAPIV) available free from Liberzon et al. (2000) in order to reduce the cost. URAPIV does not use normalized cross-correlation. There are many commercial packages that can be used for DIC that implement normalized cross-correlation along with other developments in DIC, as discussed in the next section. The difference in the accuracy reported by the discussers and the paper is probably caused by not normalizing the correlation equation in URAPIV. In any case, normalized cross-correlation is now a built-in function in Matlab, and can be easily implemented in URAPIV.

Improved Precision of Digital Image Correlation

The approach presented by the discussers obviously provides more accurate results than URAPIV. Liu and Iskander (2004) presented an advanced form of DIC referred to as adaptive cross-correlation (ACC).

Conventional DIC is calculated only once to obtain the relative displacement between any two corresponding interrogation windows. As a result, the correlated area in any two windows decreases as the displacement increases. ACC utilizes window shifting and variable window sizing to increase the size of the correlated area. In ACC, a large interrogation window size is first used to obtain the general direction of movement. Next, smaller windows are used to better define local displacements. ACC procedure also involves shifting one of the interrogation windows in the direction of the movement by a number of pixels that best approximates the deformation calculated in the previous iteration. This allows for most of the information in the original window to be included in the second window.

ACC is illustrated in Fig. 3. At the beginning, the image is divided into large interrogation windows with, say, a size 4×4 times the final window size. In this case, the analysis involves three steps. First, classical DIC is performed on these windows and the deformation is obtained for each window using either Eqs. (1) or (2). The second interrogation window is patterned in Fig. 3. The calculated movement from the first step, *dx0*, is shown as an arrow. The second step of the analysis involves dividing the original window into four smaller windows that are shifted from their original positions by the integer value of *dx0*. The calculated deformations for the four windows are shown as *dx1i*. This procedure is repeated for each of the smaller interrogation windows, until the final window size is reached. Verification criteria are implemented in each step to root out most of the spurious errors.

Conclusions

The paper's main objectives were to identify the practical limits of conventional DIC for measuring displacement, and to determine what window size should be used for the expected movement. The measured displacement is typically limited to $1/8-1/4$ of the window size in most reported DIC applications. Our paper confirms these limits for images of laser sheets captured inside transparent soil models.

We are glad that image correlation methods are gaining popularity in geotechnical engineering. We would like to thank the discussers for their thorough review of our paper. We encourage the readers to apply DIC and its derivatives for measuring deformation in soils. In particular we would like to encourage the readers to take advantage of transparent synthetic soil discussed in Iskander et al. (2002a,b), Sadek et al. (2002), and Liu et al. (2003) in measuring internal spatial deformations inside 3D transparent soil models.

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