DEM simulations of crushable grains and soils

D. Robertson & M.D. Bolton
Cambridge University Engineering Department, U.K.

in Powders and Grains 2001, Sendai, Japan

ABSTRACT: Distinct element models of crushable grains have been made using the PFC^{3D} computer code by bonding elementary spheres in "crystallographic" arrays. Random flaws were introduced to give realistic Weibull statistics of the crushing strength of grains tested singly between parallel platens. Cubical triaxial samples of the crushable grains were then formed, and routines were written to permit one-dimensional or isotropic crushing and relaxation followed by stress path tests. The yielding and hardening of the simulated soil was quite similar to the behaviour of crushable sand, and correspondingly to advanced constitutive models of soil plasticity. Contours of the proportion of broken bonds in the sample resemble Modified Cam Clay yield surfaces, but stress paths "inside" a yield surface are also seen to be capable of rearranging the grains and producing additional damage and compaction. DEM simulations may be useful in unifying our understanding of sands, clays and other clastic materials.

1 INTRODUCTION

Many granular materials are clastic in nature, in that grains crush, fracture and fragment rather than flow plastically under stress. It is evident from particle size analyses that quartz sand grains are clastic; extra "fines" are produced under moderate stresses, and copiously during penetrometer and pile insertion. Crushable carbonate sands do the same under smaller imposed stresses. New evidence shows "plastic yielding" and "plastic hardening" of sands which correlate with grain breakage. Yet plastic clays, for which such behaviour is stereotypical, are not widely thought of as crushable materials.

While Discrete Element Models (DEM) have provided many insights into the interaction of perfectly elastic and infinitely strong grains, they have generally ignored crushability. Correspondingly, the well-known physical behaviours of normal consolidation, over-consolidation, plastic yielding, and critical states have not been replicated. This paper aims to show that all these phenomena should become available to DEM with crushable grains, pointing the way to the modelling of clays as an aggregate of crushable, porous grains each made up as agglomerates of clay platelets.

The DEM method of Cundall and Strack (1979) uses an explicit time-marching scheme to solve the equations of motion directly. It is embodied in the PFC^{3D} program, Itasca Consulting Group (1995), which has been used in the following simulations.

2 MODELLING CRUSHABLE GRAINS

PFC3D allows elementary particles to be bonded together to form arbitrary shapes. Simple contact bonds are used here, fixing the distance between the centres of bonded particles subject to a maximum tensile force and a maximum shear force at which breakage occurs. Non-bonded contacts display linear elastic contact stiffness limited by frictional sliding. The agglomerates considered in this chapter are made from regular "crystalline" assemblies of balls.

Crushing tests were first performed on single agglomerates in which all bond strengths were identical: see Figure 1. Where the axis of symmetry of the grain coincides with the axis of compression, as here, multiple fracture events tend to occur due to the trapping of the "hemispheres" produced by first fracture. Some restricted variation in grain strength was produced by introducing a random axial orientation before crushing.

Further random variation was necessary to produce a more realistic continuous distribution of strengths conforming approximately to a Weibull distribution, as observed by Nakata et al (1999) in tests on sand grains. Various random allocation of bonding flaws, and random omissions of balls, were considered in relation to 57-ball agglomerates.

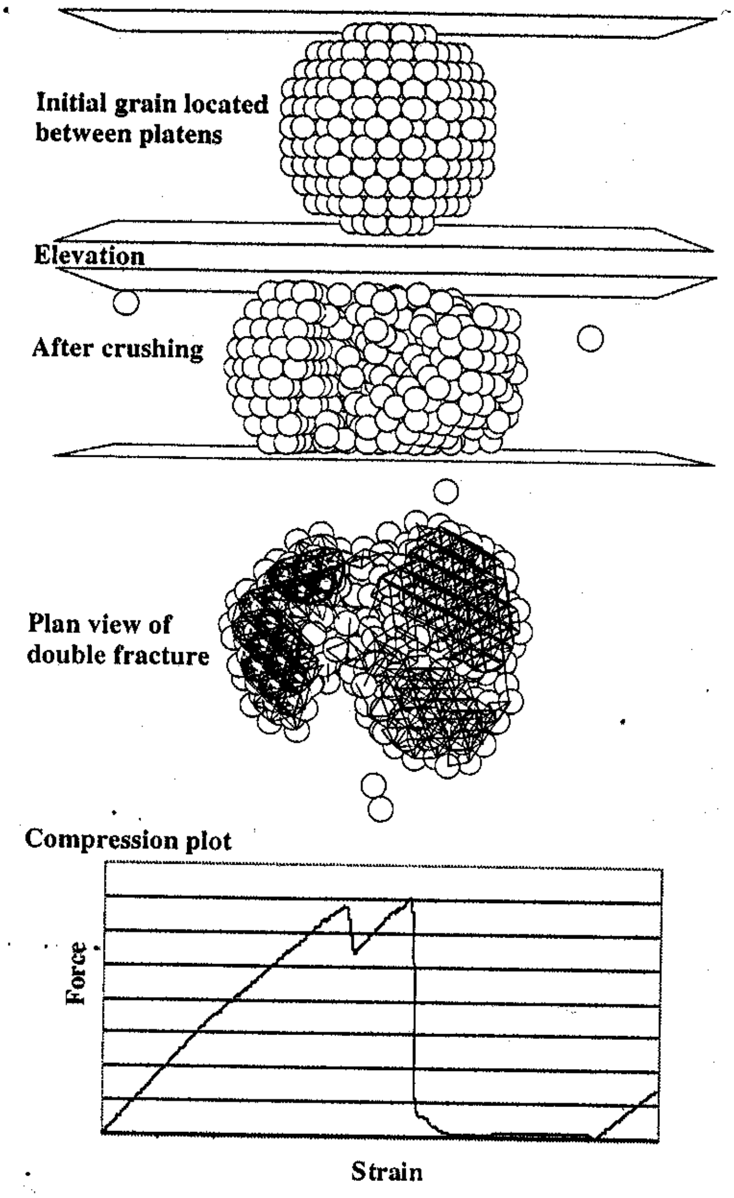


Figure 1. Crushing of a "crystalline" agglomerate

Figure 2 shows typical strength distributions in which the flawed bonds had 5% of the strength of unflawed bonds and where the probability of good bonding in a particular agglomerate was itself taken as a uniformly distributed random variable. Grains therefore differ in their probability of encountering unflawed bonds. Once the unflawed-bond probability had been selected, the bond strengths in that grain were allocated accordingly, at random. Weibull statistics demands a linear relation between log(-log(survival probablity)) and log(strength), such as that seen for an unflawed-bond probability range of 0.4 to 1.0 in Figure 2. The slope of the plot is the Weibull modulus, 2.8 here, and which is often found to lie in the region of 2.5 to 5 for sand grains. Accordingly, this statistical scheme for the strengths of 57-ball agglomerates, representing quartz sand grains, was used in subsequent trials of the behaviour of soil elements. Further details on this, and all other modelling methods used in this paper, can be found in Robertson (2000).

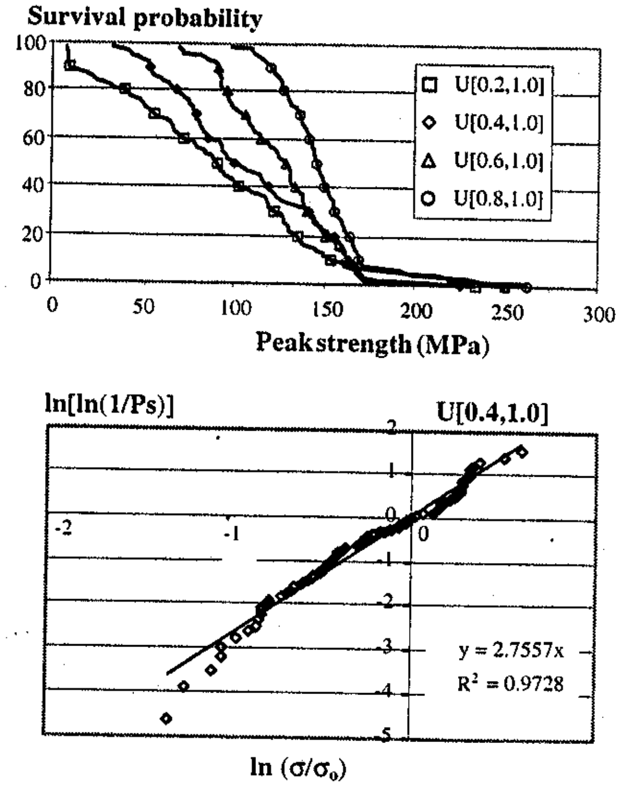


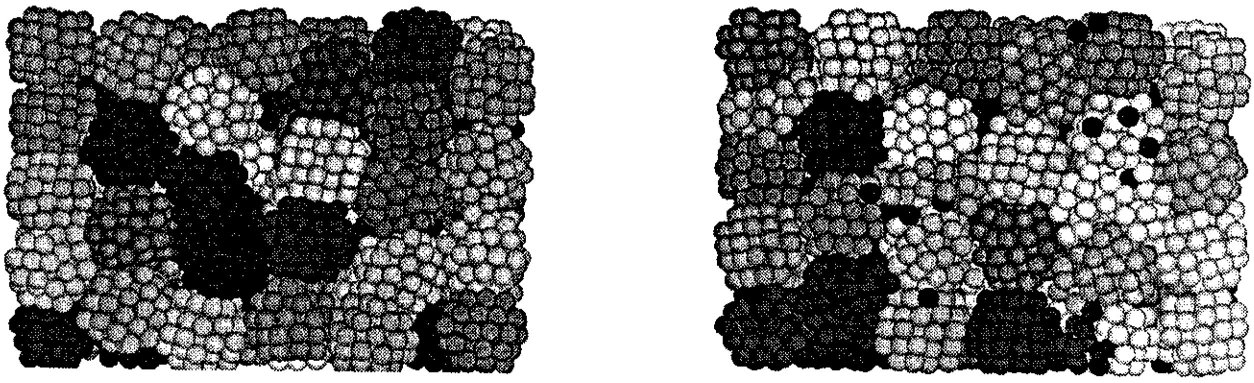
Figure 2. Achievement of Weibullian statistics of grain crushing using random bond strengths.

3 AN AGGREGATE OF AGGLOMERATES

For each of the tests reported here, an initial set of "exo-spheres" was first created at a size slightly smaller than the required agglomerates. They were placed at random, but excluding overlaps. Then they were expanded to the required size, and cycled to equilibrium so as to reduce unwanted gaps. During this process shear stiffness and friction were reduced to zero, while normal stiffness was increased 100-fold. A linked list storing the co-ordinates of their centres was then created and the exo-spheres deleted. Randomly rotated aggregates were then created in their place, centred at the co-ordinates in the list, and the assembly cycled to equilibrium again before commencing the tests. To reduce the likelihood of bonds between balls breaking during this stage, the strengths of the bonds were initially set very high and reduced after a number of cycles.

4 ONE-DIMENSIONAL COMPRESSION

An array of 159 agglomerates was set up with a macro-voids ratio of 0.5. The final shear and normal stiffnesses of the balls of the agglomerates were set to their final values ($4.0x10^{6}Nm^{-1}$) and their coefficient of friction set to 0.5 (corresponding to a contact friction angle of 26.5°). Finally, bond strengths were fixed either as constants or in some required statistical distribution as described earlier.



Unbreakable bonds **Random bond strength**

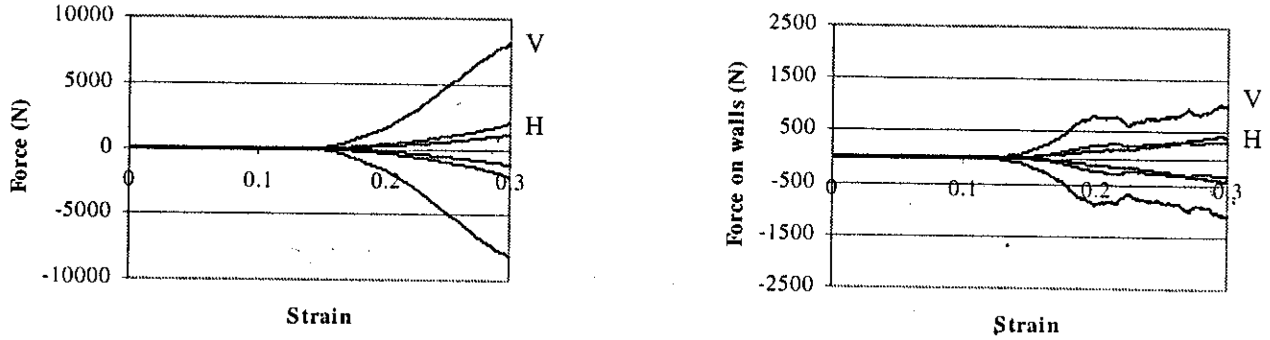


Figure 3. Cubical elements of uncrushable and crushable grains subject to 30% uniaxial compression

Figure 3 shows a comparison in the subsequent, very slow, one-dimensional compression of samples in which bonds were either unbreakable, or random so as to give grain crushing strengths in accordance with Figure 2. Forces only started to develop after about 15% vertical compression, when excessive gaps had closed.

The forces acting on the walls of the element increase steadily with the unbreakable grains, but crushability limits the increase in compressive force. Expressed in terms of earth pressure coefficient K_o in Figure 4, the initial K_o of about 0.2 corresponds to a Poisson's ratio of about 0.17 for elastic unbreakable grains. As grains crush, K_o increases and then achieves a plateau at about 0.5. This corresponds with $K_{o,nc} \approx 1 - \sin \phi$ when the internal friction angle $\phi = 30°$ which relates well to the selected contact friction angle of 26.5°.

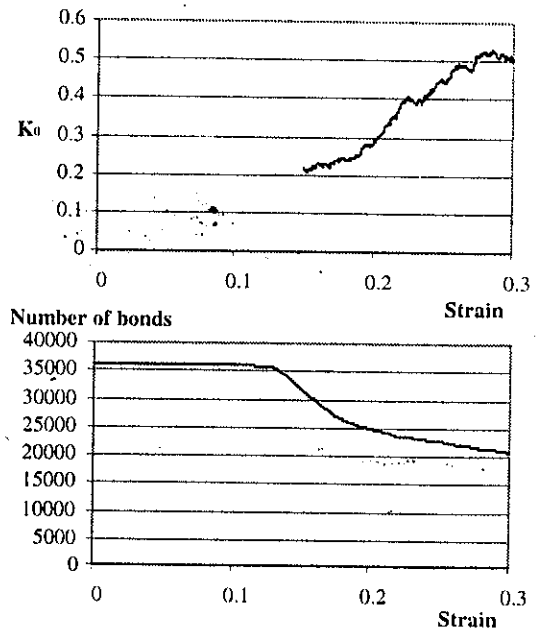


Figure 4. Lateral earth pressure coefficient K_o increasing to a plateau as bonds break

5 TRIAXIAL STRESS PATHS

Isotropic compression of a cubical arrangement of 389 agglomerates (containing 22173 balls) is shown in Figure 5. In Test 1 the bonds were identical, but in Test 3 they were randomised in strength in the way described above. It is clear that the Weibull strength distribution made the soil more crushable. The over-consolidation cycle from 40 MPa to 20 MPa was used to initialise the soil prior to stress-path testing; 31% of the bonds in Test 3 had then been broken.

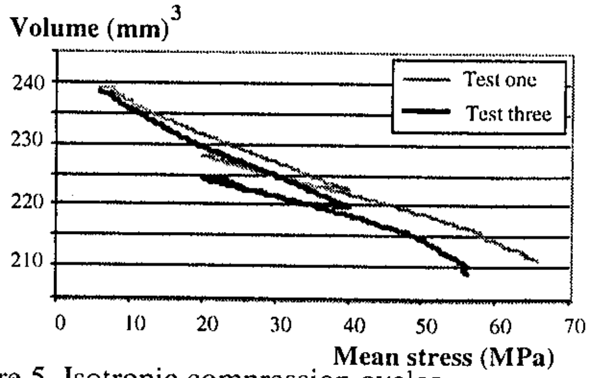


Figure 5. Isotropic compression cycles

A variety of stress paths **a** through **h** shown on the (q, p) diagram in Figure 6a were used to investigate the stress-strain response during a further 10% breakage of the previously unbroken bonds. Each path terminated at the tip of the arrow.

Path **h** unloaded mean stress, and led to almost zero further crushing. It terminated when the side walls pulled away from the specimen, leaving vertical "cliffs" in the interlocked "leggo-soil", carrying an unconfined stress of 15 MPa at q/p = 3. "True cohesion" resided *in* the agglomerates, not between them. The relatively small number of agglomerates, and their extreme roughness, prevented shear localisation.

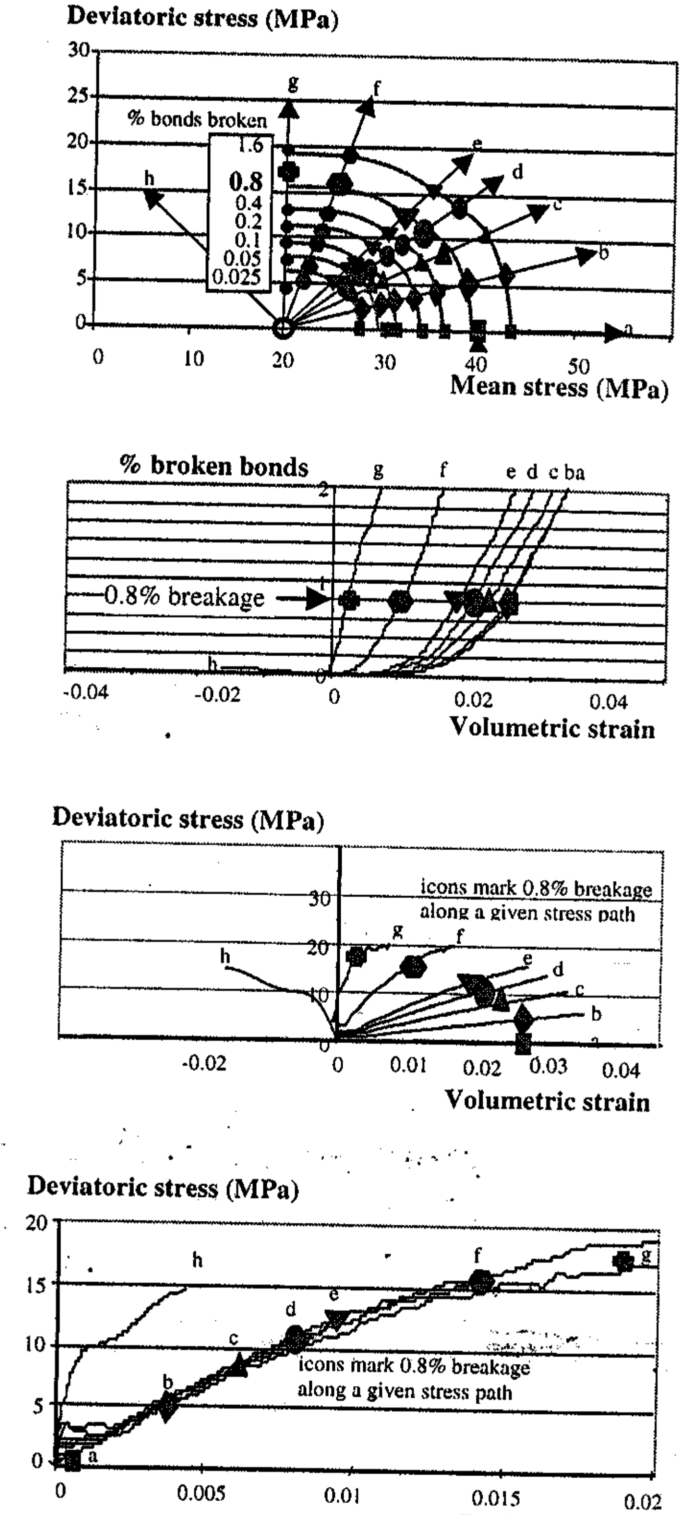
Path **g** reached a strength plateau, albeit with continuing volume reduction, at q/p = 1.25 corresponding to a mobilised angle of internal friction of 31° which again corresponds with what might have been expected at a critical state.

Paths **a** through **f** provoked successive bond breakage from the outset, with the data shown in Figure 6a fitting a homologous set of elliptical breakage contours, of which the 0.8% contour happens to include the pre-compression point of 40 MPa. Here, breakage is exactly equivalent to the creation of new surface area on fractures.

Figure 6b shows how volumetric strain develops with breakage dependent on stress path, and Figure 6c shows deviatoric stress versus volumetric strain also dependent on stress path. Shear induces compaction which reduces from **a** to **g** and turns into dilatancy for path **h**. Figure 6d shows, in contrast, that there is a unique deviatoric stress-strain curve, and the absence of any clear "yield point". The behaviour encompassed in Figure 6 is striking in its similarity to real soil behaviour, usually modelled in continuum simulations using concepts of plasticity.

6 CONCLUSIONS

DEM simulations of crushable soil have been successful in reproducing plastic compression with an almost-recoverable unload and reload loop, but with some residual compaction. Earth pressure coefficient K_o has correctly been assigned different values for "sand" in the elastic and clastic ranges of its behaviour. In clastic (normal) compression the simulation produced $K_{o,nc} \approx 1 - \sin \phi_{crit}$. In triaxial test simulations a critical state friction angle of 31° was obtained for grains with a contact friction angle of 26.5°, which is rather realistic for typical quartz sands. "Plastic yielding" with compaction or dilation occurring progressively throughout test paths has been shown to be associated with breakage, whilst conforming to precepts of soil plasticity. This strongly supports the hypothesis that "plasticity" in soils derives from "clasticity" or crushability of grains, and challenges the conventional view of clay.



d) q versus ε_s

Figure 6. Development of breakage and strain in stress paths from OCR = 2; test 3 (random bonds).

REFERENCES

Cundall, P.A. & Strack, O.D.L. 1979. A discrete numerical model for granular assemblies', *Géotechnique*, 29, 47-65.

Itasca Consulting Group, Inc. 1995. *PFC3D (Particle Flow Code in 3D), Version 1.10*. Minneapolis: ICG.

Nakata, Y., Hyde, A. F. L., Hyodo, M., & Murata, H. 1999. A probabilistic approach to sand particle crushing in the triaxial cell. *Géotechnique*, 49, 567-583.

Robertson, D. 2000. *Computer simulations of crushable aggregates*. PhD Thesis, Cambridge University.