Mechanical Behaviour of an idealised "Wet-Clay"

by K. H. Roscoe, M. C., M. A., and A. N. Schofield, M. A., Ph. D.,
Department of Engineering, Cambridge University

Summary

This paper gives a new presentation of some recent theories of the mechanical behaviour of homogeneous isotropic saturated remoulded "wet" clays, which are concerned with the compressibility, the stress-strain properties and the prediction of pore pressures of such soils. The term "wet" clays is defined.

Compression under increasing effective isotropic stress, except in the case of virgin consolidation, is considered to be elastic and therefore recoverable, without any slip at grain contacts. Under increasing deviator stress the soil is deemed to be rigid, until the deviator stress reaches a value at which plastic yield occurs with slip at grain contacts to obtain closer packing of the grain structure.

The phenomenon of isotropic virgin-consolidation of wet clay is seen as an occurence under a certain intensity of mean normal stress at which the material becomes meta-stable and slip at grain contacts may occur spontaneously with possible distortion despite the absence of deviator stress.

General expressions relating stress and strain in the triaxial test are applied to predict the development of excess pore-water pressure as the destortion of an undrained sample is increased.

Résumé

On présente d'une façon nouvelle certaines théories récentes sur le comportement mécanique d'argiles «mouillées» remoulées homogènes isotropes saturées, sous le rapport de la compressibilité, des caractéristiques contrainte/déformation, ainsi que de la prédiction des pressions interstitielles pour des sols de ce genre.

A l'exception des cas de consolidation vierge, on considère que la compression sous l'effet de l'augmentation d'une contrainte isotrope effective est élastique, et peut donc se récupérer, sans aucun glissement des grain à leurs points de contact. Lorsque la différence entre les contraintes principales augmente, on estime que le sol demeure rigide, jusqu'au moment ou cette différence acquiert une valeur telle qu'un écoulement plastique, se produit avec glissement aux points de contact entre les grains, ce qui entraîne un tassement plus prononcé de la structure granulaire.

On considère la consolidation vierge isotrope de l'argile mouillée comme un événement qui se produit lorsque la contrainte moyenne normale atteint une certaine intensité, à partir de laquelle le matériau devient métastable, le glissement aux points de contact entre les grains peut se produire spontanément, et il y a possibilité de distorsion en dépit de l'absence de différence entre les contraintes principales.

On se base sur des expressions générales du rapport contrainte/déformation dans l'essai triaxial pour prédire la formation de pressions insterstitielles excessives quand la distorsion d'un échantillon non-drainé augmente.

Zusammenfassung

Im vorliegenden Aufsatz werden einige neuere Theorien über das mechanische Verhalten von homogenen, isotropen, gesättigten, gestörten „nassen" Tonen in bezug auf die Zusammendrückbarkeit, die Spannungs-Dehnungs-Eigenschaften und die Voraussage des Porenwasserdrucks dieser Böden neu dargestellt. Die Bezeichnung „nasser" Ton wird definiert.

Es wird angenommen, daß der Vorgang der Zusammendrückung unter zunehmender wirksamer isotroper Spannung, außer für den Fall der Erstverdichtungskurve, elastisch und deshalb umkehrbar ist, ohne daß hierbei ein Gleiten an den Berührpunkten der Körner auftritt. Bei zunehmender Hauptspannungsdifferenz wird der Boden als starr angesehen, bis bei einem bestimmten Wert plastisches Fließen eintritt und durch Gleitbewegungen an den Kornberührpunkten eine dichtere Lagerung der Kornstruktur erhalten wird.

Die Erscheinung der isotropen Erstverdichtung eines nassen Tones wird als ein Vorgang betrachtet, der bei einer bestimmten Stärke der mittleren Normalspannung, bei der das Material metastabil wird, auftritt. Hierbei können von selbst Gleitbewegungen an den Kornberührpunkten und möglicherweise Verformungen trotz fehlender Hauptspannungsdifferenz auftreten. Es werden allgemeine Formeln für Spannung und Dehnung im Triaxialversuch angewandt, um die Entstehung des Porenwasserüberdrucks bei zunehmender Verformung einer undränierten Probe vorauszubestimmen.

Introduction

(1.1) This paper presents a simple but complete theory of the mechanical behaviour of an ideal continuum called "wet clay". The term "wet clay" has been properly defined by Roscoe, Schofield and Wroth (1958). A briefer and more approximate definition is given in Section 3, see also fig. 2. The continuum material is considered to be an isotropic aggregate of irregular clay grains, randomly packed in mechanical conctat with each other and forming a redundant structure. The theory is concerned with the macroscopic isotropic behaviour of this material and, for simplicity, stress and strain of a triaxial-test sample will be considered. Let the sample material come into equilibrium under certain effective stresses: the sample and the triaxial loading apparatus together then form a system at rest.

It is assumed that the system is stable. The problem is to predict the small strain increment that will occur, in the material of the loaded sample, when the overall system of sample and loading apparatus is probed by an external agency. The external agency applies a small probing stress increment to the material of the sample in parallel with the existing relatively large stresses imposed by the loading apparatus.

(1.2) Suppose the material to be in equilibrium under effective stresses $(\sigma'_1, \sigma'_3, \sigma'_3)$, let a probing stress increment $(\delta\sigma'_1, \delta\sigma'_3, \delta\sigma'_3)$ be applied and a consequent strain increment $(\delta\varepsilon_1, \delta\varepsilon_3, \delta\varepsilon_3)$ be measured. To separate effects related to compression from effects related to distortion six quantities are defined:

$p = \left(\frac{\sigma'_1 + 2\sigma'_3}{3}\right)$ mean normal stress,

$q = |\sigma'_1 - \sigma'_3|$ deviator stress:

$\delta p = \left(\frac{\delta\sigma'_1 + 2\delta\sigma'_3}{3}\right)$ mean normal stress increment,

$\delta q = |\delta\sigma'_1 - \delta\sigma'_3|$ deviator stress increment:

$\delta v = (\delta\varepsilon_1 + 2\delta\varepsilon_3)$ compression increment,

$\delta\varepsilon = \frac{2}{3}|\delta\varepsilon_1 - \delta\varepsilon_3|$ distortion increment

Compressive stresses and strains are positive.

The ratio of volume of voids to volume of solids in the material is called the voids ratio e, which is related to the compression increment δv by the equation

$$\delta v = -\frac{\delta e}{1 + e} \quad \ldots\ (1)$$

(1.3) The material is considered to be elastic-plastic, in the sense that if a stress increment is applied and then removed there will be, in general, some recoverable elastic strain (denoted by the superfix e) and some permanent plastic strain (denoted by the superfix p) where

$$\left.\begin{aligned} \delta v &= \delta v^e + \delta v^p \\ \delta\varepsilon &= \delta\varepsilon^e + \delta\varepsilon^p \end{aligned}\right\} \quad \ldots\ (2)$$

(1.4) During the probing action the existing stresses $(\sigma'_1, \sigma'_3, \sigma'_3)$ imposed by the loading apparatus do work $\delta E'$ on unit volume of the material, where

$$\sigma'_1\,\delta\varepsilon_1 + 2\,\sigma'_3\delta\varepsilon_3 = \delta E',$$

which can be re-written by introducing (p, q) to give

$$\delta E' = p\delta v + q\delta\varepsilon \quad \ldots\ (3)$$

The work $\delta E'$ can be divided into a recoverable part δU and a dissipated part δW

$$\left.\begin{aligned} \delta U &= p\delta v^e + q\delta\varepsilon^e \\ \delta W &= p\delta v^p + q\delta\varepsilon^p \end{aligned}\right\} \quad \ldots\ (4)$$

(1.5) If the overall system of sample and loading apparatus is to be stable then, in any conceivable cycle of application and removal of a probing stress increment to this system, the external probing agency must never be able to extract work from the system, see Drucker (1959). Thus, for stability,

$$\delta p \cdot \delta v^p + \delta q \cdot \delta\varepsilon^p \geqq 0 \quad \ldots\ (5)$$

(1.6) The material is assumed to be, and to remain, isotropic and homogeneous. Four constants $(\lambda, \varkappa, M, \Gamma)$ will be introduced below to define the fundamental scalar isotropic properties of the material.

2. The basic assumptions and the new work equation

(2.1) The problem of predicting the mechanical behaviour of clays is greatly simplified if it is assumed that there is never any recoverable distortion, so that

$$\left.\begin{aligned} \delta\varepsilon^e &= 0 \\ \delta\varepsilon &= \delta\varepsilon^p \end{aligned}\right. \quad \ldots\ (6)$$

and that the elastic compression increment δv^e satisfies a relationship

$$\delta v^e = \frac{\varkappa}{1 + e} \cdot \frac{\delta p}{p} \quad \ldots\ (7)$$

From equation (1) this relationship becomes

$$\delta e^e = -\varkappa\frac{\delta p}{p} \quad \ldots\ (8)$$

The family of lines in fig. 1 satisfy equation (8). Each line can be thought of as referring to one particular structural packing of the grains; it represents elastic swelling and compression of this grain structure without any irreversible slip at grain contacts.

Change of the packing can only occur if there is slip at grain contacts. When such slip does take place and the grains move into closer packing, the new state of the sample corresponds to a new elastic compression line which lies closer to the origin in fig. 1. The difference in ordinates between the two elastic compression lines is a measure of the plastic component of the change of voids ratio. The plastic compression increment can be calculated from the first equation (2)

$$\delta v^p = \delta v - \delta v^e.$$

(2.2) It will be assumed that, when there is slip of grains and a plastic compression increment, there is a functional relationship between the magnitudes of the distortion increment $\delta\varepsilon = \delta\varepsilon^p$ and the plastic compression increment δv^p. The particular relationship that will be taken to apply is

$$\frac{dv^p}{d\varepsilon} = \left(M - \frac{q}{p}\right) \qquad \ldots\ldots (9)$$

It is possible that this relationship could be derived from a consideration of the change of geometry of the random grain structure when grains slip under stress into a new packing. However in the following paragraph this relationship will be developed from an assumption concerning the work dissipated during relative slip of grains.

(2.3) Assume that the total rate at which work is dissipated in unit volume of space during slip at grain contacts is

$$\frac{dW}{d\varepsilon} = Mp \qquad \ldots\ldots (10)$$

The magnitude of the plastic compression increment δv_p depends upon the magnitudes of $\delta\varepsilon$, q and p, and the relationship between these four quantities can be found directly from equations (6), (10) and the second equation (4), namely

$$Mp\delta\varepsilon = p\delta v^p + q\delta\varepsilon,$$

which may be written

$$\frac{dv^p}{d\varepsilon} = \left(M - \frac{q}{p}\right) \qquad \ldots\ldots (9\ \text{bis})$$

(2.4) A simple expression can be derived for the maximum work that can be recovered from a sample of stressed soil. Introducing $\delta\varepsilon^e = 0$ and $\delta v^e = \frac{\varkappa}{1+e}\frac{\delta p}{p}$ into the first equation (4) gives

$$\frac{dU}{dp} = \frac{\varkappa}{(1+e)} \qquad \ldots\ldots (11)$$

Consider a sample with bulk volume V = (1 + e) under mean normal stress p, then the work done when this sample expands fully to a state of zero mean normal stress is

$$-\int_p^0 \varkappa dp = \varkappa p.$$

Equation (7) is therefore equivalent to an assumption that the maximum possible work that can be recovered from a unit volume of solid particles when under a mean stress p is simply $\varkappa p$.

(2.5) If the simplifying assumptions expressed by equations (3), (10) and (11) are taken together a new work equation is formed

$$p\delta v + q\delta\varepsilon = \frac{\varkappa\delta p}{1+e} + Mp\,\delta\varepsilon \qquad \ldots\ldots (12)$$

3. Critical states and yielding of wet clay

(3.1) From equation (9) it is clear that stresses for which q = Mp will bring the material into a state in which increments of distortion can occur without compression; i. e. the material will then flow as a frictional fluid.

Such critical states were described by Roscoe, Schofield, and Wroth (1958) who showed that for wet clay the critical states were governed by equations of the form

$$q = Mp \qquad \ldots\ldots (13)$$

$$e = \Gamma - \lambda \log p \qquad \ldots\ldots (14)$$

where Γ is the critical voids ratio under unit mean normal stress. They also suggested that samples on the "wet" side of the critical state line would "harden" during a test. Tests on such samples would be stable in the sense defined by equation (5).

Equation (14) is plotted as the chain dotted line AA' in fig. (2) where it is superposed on the compression lines of fig. (1). From now on attention will be confined to wet clay i.e. material in states represented by points further from the origin than the line AA' in fig. 2.

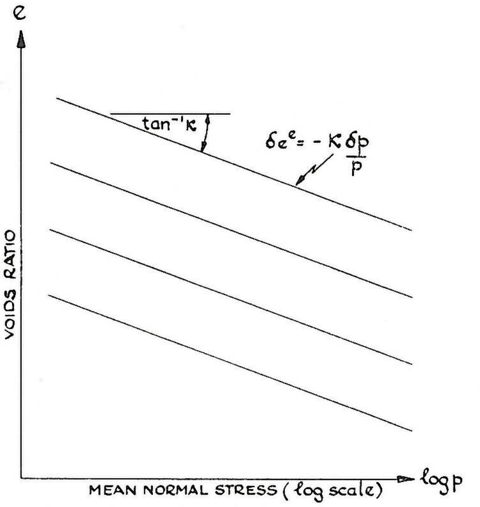

Fig. 1 Elastic compression and swelling lines

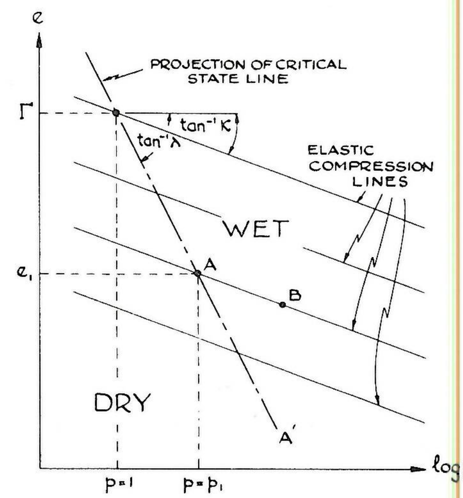

Fig. 2 Projection of critical state line on elastic compression lines

(3.2) Consider a sample of clay having a voids ratio e_1 when subjected to an isotropic stress p_1 and let the relationship between e_1 and p_1 satisfy equation (14): the state of the material may then be represented by point A in fig. 2 at the intersection of the elastic compression line AB and the line AA'. If this material is subjected to increasing deviator stress q at constant p it will remain rigid until q reaches the critical value specified by equation (13) when the material will flow without further changes of state.

Alternatively, suppose the same initial material is subjected to additional isotropic stress which brings it into a state represented by point B on the elastic compression line in fig. 2. If the material is then subjected to increasing deviator stress q, it remains rigid until q attains a limiting value at which yield occurs; on yielding there is an increment of distortion and the corresponding increment of plastic compression given by equation (9).

When q is below the limiting value a probing stress increment will cause only elastic strain increments, plastic strain increments will be zero, and in particular the equation

$$\delta p \cdot \delta v^p + \delta q \cdot \delta \varepsilon^p = 0 \qquad \ldots . \quad (15)$$

will be satisfied.

When q is at the limiting value a probing stress increment will cause plastic strain increments of magnitudes which satisfy equation (5). Generally the inequality sign in equation (5) will apply and the probing agency will do work on the system, but for one particular ratio of the stress increments, namely

$$\frac{dq}{dp} = -\frac{dv^p}{d\varepsilon} \qquad \ldots . \quad (16)$$

the equality signs in equations (5) and (15) are valid. Such stress increments cause changes from one limiting value of q to an adjacent limiting value of q without yield. Thus equations (15) and (9) can be combined to give an equation for the limiting values of q

$$\frac{dq}{dp} = \frac{q}{p} - M,$$

which can be integrated to give the family of curves

$$\frac{q}{Mp} + \log p = \text{constant} \qquad \ldots . \quad (17)$$

For any given value of this constant, equation (17) refers to material in one particular state of packing, and hence to one elastic compression line in fig. 2 such as, for example, the line AB through A.

(3.3) The point A in fig. 2 has co-ordinates (p_1, e_1). The elastic compression line AB, satisfying equation (8), is

$$e + \varkappa \log p = e_1 + \varkappa \log p_1 \qquad \ldots\ (18)$$

The line AA' passes through (p_1, e_1) and from equation (14),

$$e_1 = \Gamma - \lambda \log p_1,$$

hence equation (18) becomes

$$e + \varkappa \log p = \Gamma - (\lambda - \varkappa) \log p_1 \qquad \ldots\ (19)$$

At A the limiting value of q is

$$q_1 = Mp_1,$$

so that the value of the constant in equation (17), for the state of packing corresponding to line AB, can be found, and for this state equation (17) becomes

$$\frac{q}{Mp} + \log p = \frac{q_1}{Mp_1} + \log p_1$$
$$= 1 + \log p_1$$

By elimination of log p_1 from this equation and equation (19) a general equation can be derived for the magnitude of deviator stress q at which material of any state (p, e) will yield. This general equation is

$$q = \frac{Mp}{\lambda - \varkappa}[\Gamma + \lambda - \varkappa - e - \lambda \log p] \qquad \ldots\ (20)$$

(3.4) If equation (20) is regarded as the equation of a curved surface in (e, p, q) space, such as is sketched in fig. 3, then this diagram may be used to give a simple explanation of the mechanical behaviour of wet clay.

Material with a packing corresponding to the elastic compression curve ABC in fig. 3 can be subjected to values of q represented by points in the wall ABCDE without yielding. The curve CDE indicates limiting values of q at which yield first occurs, the sample then begins to distort and the grains then begin to slip and become more closely packed. During yielding the material passes through states (e, p, q) represented by points on the curved surface CDE"D"C". When yielding has progressed to a point on the curve C'D'E', if the sample is unloaded the states of the material will be represented by points in the wall A'B'C'D'E', corresponding to the elastic compression curve A'B'C'. The surface CDEE"D"C" is called the state boundary surface.

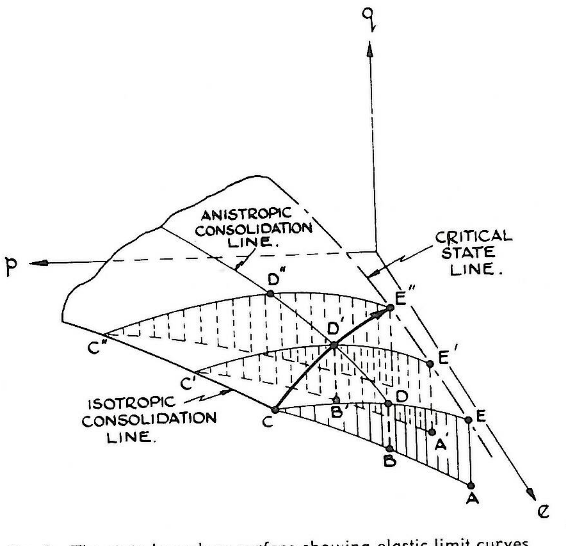

Fig. 3 The state boundary surface showing elastic limit curves (e. g. CDE) and the state path of an undrained test CD'E"

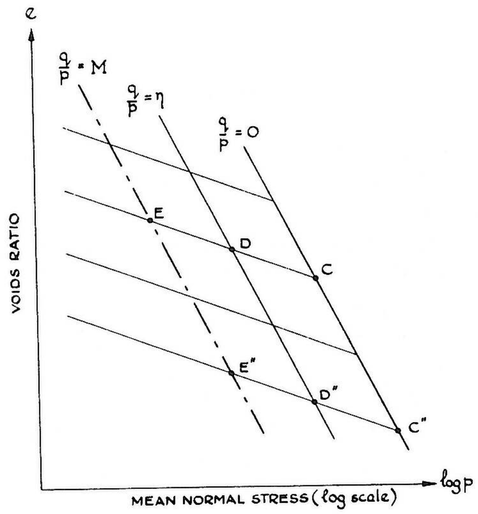

Fig. 4 Projection of consolidation lines and elastic limit lines on e-log p plane

4. Consolidation

(4.1) Consolidation will be defined as a process in which clay yields under principal stresses which increase in constant ratio. One constant value of the ratio

$$\frac{q}{p} = \eta \qquad \ldots\ (21)$$

defines states in one consolidation test. For such a test equations (21) and (20) may be combined to give

$$e = \Gamma + (\lambda - \varkappa)\left(1 - \frac{\eta}{M}\right) - \lambda \log p \qquad \ldots. \quad (22)$$

The curve DD" sketched in fig. 3 satisfies equations (21) and (22). A consolidation process in which there is some deviator stress ($q \neq 0$) is called "anisotropic" consolidation: the particular case when there is no deviator stress ($q = 0$) is called "isotropic" consolidation. The word consolidation will not be used in association with any swelling or compression processes which do not entail yielding.

In fig. 4 a family of many lines such as DD" could be drawn on the wet side of the line EE", with each line corresponding to one anisotropic consolidation process. From equations (1), (7) and (22)

$$\delta v = \frac{\lambda}{1+e}\,\frac{\delta p}{p} = \frac{\lambda}{\varkappa}\,\delta v_e$$

and so

$$\delta v_p = \delta v - \delta v_e = \left(1 - \frac{\varkappa}{\lambda}\right)\delta v \qquad \ldots. \quad (23)$$

Equation (23) can be introduced into equation (9) to give

$$\delta v = \left(\frac{M - \eta}{1 - \frac{\varkappa}{\lambda}}\right)\delta\varepsilon \qquad \ldots. \quad (24)$$

hence in an anisotropic consolidation process the ratio of the principal (total) strain increments remains constant.

(4.2) The curve CC" in figs. 3 and 4 represents the equation

$$e = \Gamma + \lambda - \varkappa - \lambda \log p \qquad \ldots. \quad (25)$$

for isotropic consolidation. This curve forms a boundary in the (p, e) plane beyond which the family of elastic compression curves can not be continued, because at the points of their intersection with CC" the material suffers plastic compression despite the absence of deviator stress.

During plastic compression there is slip of grains. Introducing $q = 0$ into equation (9) and combining this with equation (23) it appears that in isotropic consolidation a distortion increment $\delta\varepsilon$ can occur, where

$$\delta\varepsilon = \left(\frac{1 - \frac{\varkappa}{\lambda}}{M}\right)\delta v \qquad \ldots. \quad (26)$$

even though there is no deviator stress.

In isotropic consolidation there are no principal axes of stress, and equation (26) merely predicts that a distortion of a certain magnitude can take place without specifying the direction of any principal axes of strain increment. The smallest possible deviator stress would suffice to impose definite directions for the principal stresses, but in the absence of any deviator stress it is conceivable that the local distortions in different parts of a sample are such that no overall distortion of the whole sample appears to take place.

Material in the state of isotropic consolidation must be regarded as meta-stable; it is sensitive to deviator stress, and the least difference of principal stresses will stabilize the material and impose definite directions of principal strain.

(4.3) If, alternatively, consolidation were defined as a process in which principal strains (as distinct from stresses) increased in constant ratio, then it could be predicted that, in all tests where this ratio is such that

$$\delta\varepsilon \leq \left(\frac{1 - \frac{\varkappa}{\lambda}}{M}\right)\delta v$$

the material would exert equal principal stresses in all directions. For example in the oedometer test, in which $\delta\varepsilon_3 = 0$ and $\delta\varepsilon = \frac{2}{3}\delta v$, the vertical and lateral effective stresses would not be expected to differ unless

$$\frac{1 - \frac{\varkappa}{\lambda}}{M} < 2/3$$

for the particular material under test.

5. Pore-water pressures developed in an undrained test

(5.1) The variation of pore-water pressure u during an undrained triaxial compression test at constant cell pressure was analysed by Roscoe, Schofield and Wroth (loc. cit.). In a sample of wet clay, initially consolidated under isotropic stress p_0, when a deviator stress q is applied the sample yields, the mean normal stress falls to p, while an excess pore water pressure is generated of magnitude

$$u = p_0 + \frac{1}{3} q - p \qquad \dots. \quad (27)$$

The voids ratio of the sample in the initial state may be obtained by putting $p = p_0$ in equation (25) which becomes

$$e = \Gamma + \lambda - \varkappa - \lambda \log p_0 \qquad \dots. \quad (28)$$

This value of e can be substituted into equation (20) to give

$$q = \frac{Mp}{1 - \frac{\varkappa}{\lambda}} \log \left(\frac{p_0}{p} \right) \qquad \dots. \quad (29)$$

which is an equation relating q and p in an undrained test. Elimination of p from equations (27) and (29) then gives the relationship between u and q. Experimental evidence appearing to confirm this predicted relationship has been cited by Roscoe, Schofield and Thurairajah (1963).

(5.2) To calculate distortion increments during an undrained test, put $\delta v = 0$ in equation (12), giving

$$\delta \varepsilon = \frac{\varkappa}{M(1+e)} \cdot \frac{-1}{1 - \frac{q}{Mp}} \cdot \frac{\delta p}{p}$$

and introduce the value of $\frac{q}{Mp}$ from equation (29) to obtain the equation

$$\delta \varepsilon = \frac{\varkappa \left(1 - \frac{\varkappa}{\lambda}\right)}{M(1+e)} \cdot \frac{-1}{\left(1 - \frac{\varkappa}{\lambda}\right) - \log p_0 + \log p} \, \frac{\delta p}{p}$$

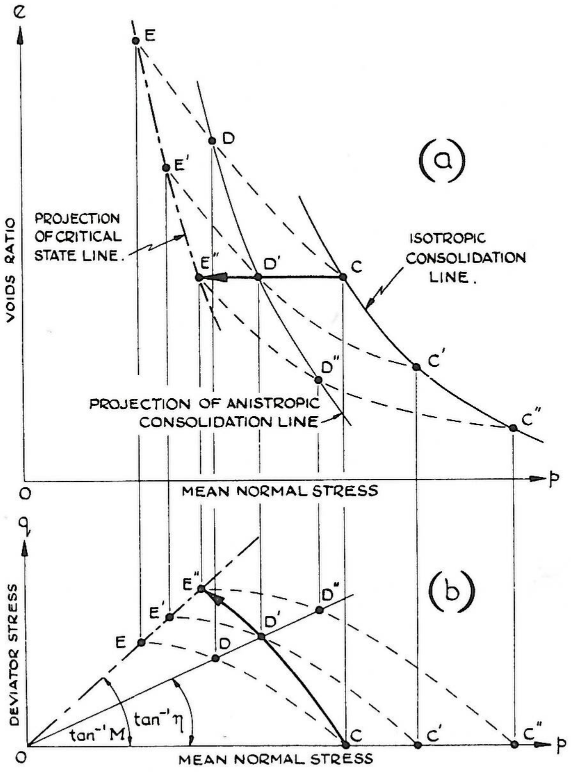

Fig. 5 Strain hardening during an undrained test

The sum ε of the distortion increments from the beginning of the test can be calculated by integration

$$\varepsilon = \frac{\varkappa\left(1 - \frac{\varkappa}{\lambda}\right)}{M(1+e)} \cdot \log \left[\frac{1 - \frac{\varkappa}{\lambda}}{1 - \frac{\varkappa}{\lambda} - \log \frac{p_0}{p}} \right] \quad \ldots. \quad (30)$$

The unique relationship between u and ε can now be found by eliminating p and q from equations (27), (29) and (30).

(5.3) The strain hardening of a sample during an undrained triaxial test is illustrated three dimensionally in fig. 3 and in plan and elevation in figs. (5 a) and 5 (b) respectively. The initial state of the sample corresponds to point C and its condition of packing is associated with the elastic wall ABCDE. In the undrained process the sample passes continuously through states represented by points on the curve CD'E", and the clay particles come successively into "closer" packings while the mean normal stress progressively diminishes. (The plastic volume reduction is equal and opposite to the elastic increase of volume.) When at a state represented by D', for example, the packing is associated with the elastic wall A'B'C'D'E'. This progressive hardening continues until the sample attains the critical state at E" when its packing is associated with the elastic wall A"B"C"D"E".

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