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Casagrande's concept of Critical Density, Hvorslev's equation for shear strength, and the Cambridge concept of Critical States of soil

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Bu yazıda, şekil değiştirmeye zorlanan zeminde bir kritik durumun ortaya çıkması ile ilgili bugünkü bilgimizin ışığı altında, Casagrande ve Hvorslev'in önemli bazı çalışmaları yeni baştan değerlendirilmektedir.

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In this paper some notable work of Casagrande and Hvorslev is re-examined, in the light of our present understanding of the development of a critical state *in soil subjected to deformation.*

* * *

1. In a notable contribution to the Journal of the Boston Society of Civil Engineers A. Casagrande wrote (1),

«If we observe carefully the volume changes of samples of sand during shearing tests we find that **dense sand expands** and **very loose sand reduces its volume.**

In a dense sand, as in Fig. 1 a, the grains are so closely interlocked that deformation is not possible unless accompanied by a loosening up of the structure, as indicated by Fig. 1 b. If dense sand is so confined that it cannot expand, then the shearing strength is determined by the resistance of the grains to crushing, and therefore it acts essentially like a rigid stone. This was observed fifty years ago by Osborne Reynolds, who demonstrated it by filling a water-tight bag first with dry sand, in which condition the bag could be deformed easily, and then replacing the air in the voids of the sand completely with water, after which the bag turned rigid because the sand could no longer change its volume.

When the horizontal displacement and the volume change during a shearing test on dense sand are plotted against the corresponding shearing stress (Fig. 1e and 1f), it is noticed that the shearing stress reaches a maximum S_{i} - corresponding to the point B on the curve, - and if the deformation is continued, the shearing stress drops again to a smaller value, S_{L}, at which value it remains constant for all further displacement. During this drop in shearing stress, the sand continues to expand, as shown in Fig. 1 f, curve E-G; finally reaching a critical density at which continuous deformation is possible at the constant shearing stress S . This critical density corresponds for very coarse, well-graded sand and for gravel approximately to the loose state of the material. For medium and finer grained sands it lies between the loosest and densest state. In addition, it depends to a large extent on the uniformity of the material. The more uniform a soil the lower the critical density.

The above also furnishes the explanation why most sands in their loose state have a tendency to reduce their volume when subjected to a shearing test under constant normal pressure. The shearing stress simply increases until it reaches the shearing strength S_{l} and if the displacement is continued beyond this point the resistance remains unchanged. Obviously, the volume of the sand in this state must correspond to the critical density which we had finally reached when performing a test on the same material in the dense state. Therefore the curves representing the volume changes during shearing tests on material in the dense and the loose state must meet at the critical density when the stationary condition is established.»

Having thus discussed density changes in shear tests at constant effective stress p, Casagrande continues to discuss the pore water pressure changes in shear tests at constant density as follows.

«While the normal stresses in the soil are partially or fully carried by the water, the pressures acting between the individual grains are reduced by a corresponding amount, since the total stress must remain equal to the overlying loads. Simultaneously, with this reduction, the frictional resistance between the grains is reduced in the same proportion. The amount of this reduction can be analyzed with the help of Fig. 2. Let us consider a volume element in the mass of sand in which the stress conditions correspond to point B. This would indicate that the sand has been deposited originally in a loose state and was subsequently compressed by a static pressure equal to p_1. Now the mass of sand shall be

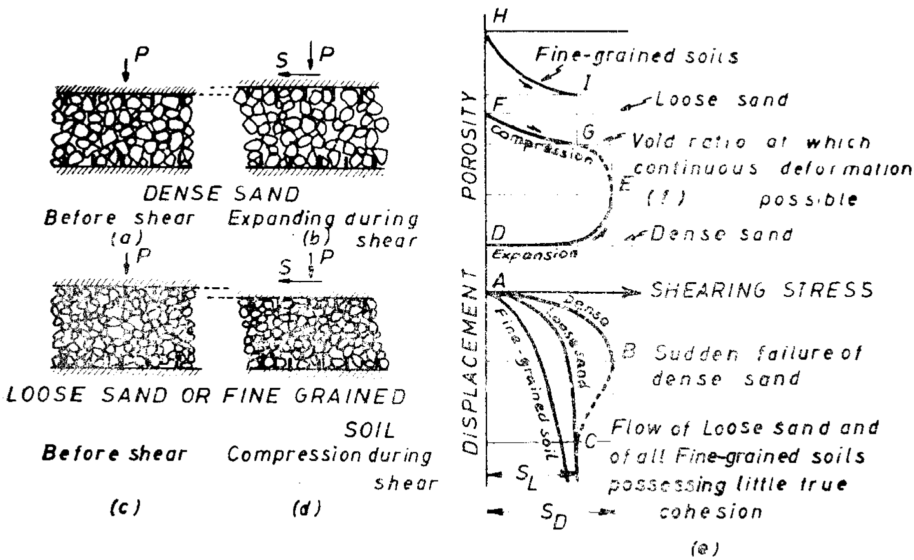


Fig. 1 Effect of shearing on volume of soils (after A. Casagrande)

exposed to horizontal forces which tend to deform it without, however, changing the vertical pressures in the mass. That portion of the sand which is affected by the deformation will tend to reduce its volume. If the deformation is sufficiently large, a new state of internal equilibrium will be established after the critical density n_0 is reached in point C. The compression curve through point C reflects the change in the structure of the sand which was produced by the deformation.

If the quantity of water which could escape during deformation is negligible, then the change in the structure without change in density causes a drop in the pressure acting between the grains from p_1 to p_2, the latter being determined by the intersection D between the abscissae through B and the compression curve through C. The difference $(p_1 - p_2)$ is now carried temporarily by the water and does not produce frictional resistance, since the shearing strength of water is zero. The shearing resistance in the zone of deformation, being proportional to the pressure actually transferred between the grains, is now reduced in the ratio p_2 / p_1. The pressure p_2 can be a small fraction of p_1, and may even be equal to zero, so that temporarily the soil can lose a large portion of or its entire shearing strength.

For very large pressures, which are not normally encountered in problems of earth and foundation engineering, even a loose sand may be compressed to its critical void ratio, as, for example, point E, and then deformation will not cause any reduction in shearing resistance.

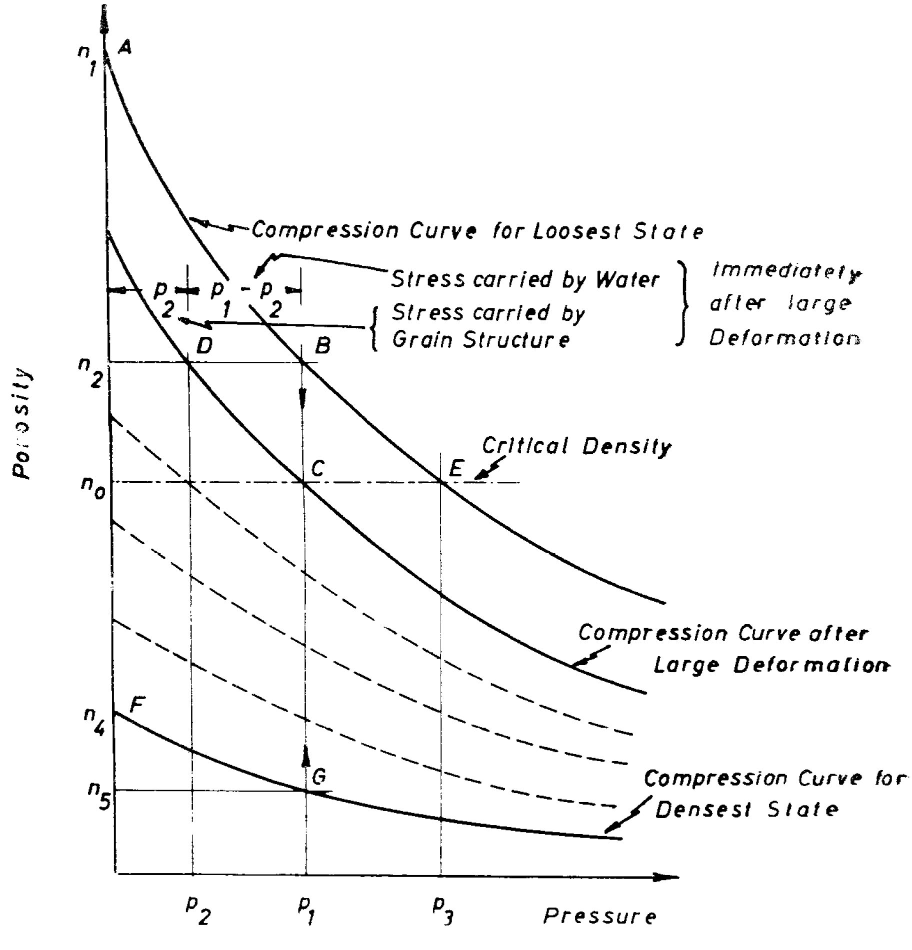


Fig. 2 Pressure-density relationship for sand (after A. Casagrande)

If the density at the beginning of the deformation was below the critical density, as is always the case with dense sand, then deformation of a saturated mass will temporarily create tension in the water and a corresponding increase in the pressure between grains. Hence deformation of a mass of dense, saturated sand may result in an increase of the shearing resistance beyond the normal value. In other words, the mass seems to be bracing itself, to become temporarily more stable.»

2. In the second passage quoted above and in his Fig. 2 it is clear that for Casagrande the «Compression curve after large deformation» was not a unique curve for the soil material. Fig. 3 shows an additional compression curve from an initial state of lower porosity then Casagrande's n_1 in Fig. 2. Material on such a curve will reach the porosity n_2 at a state indicated by point B′ with a pressure p_1' slightly less than Casagrande's p_1. If sheared at constant effective pressure p_1' the material would pass through states that settle from B′ to C′ in Fig. 3, coming into Casagrande's critical density n_0 at C′. If sheared at constant porosity n_2 it would pass through states that move from B′ to D′, according to Casagrande's construction, and at D′ come into a state of internal equilibrium at which the effective pressure p_2' was determined by the compression curve through C′ and the abscissa through B′.

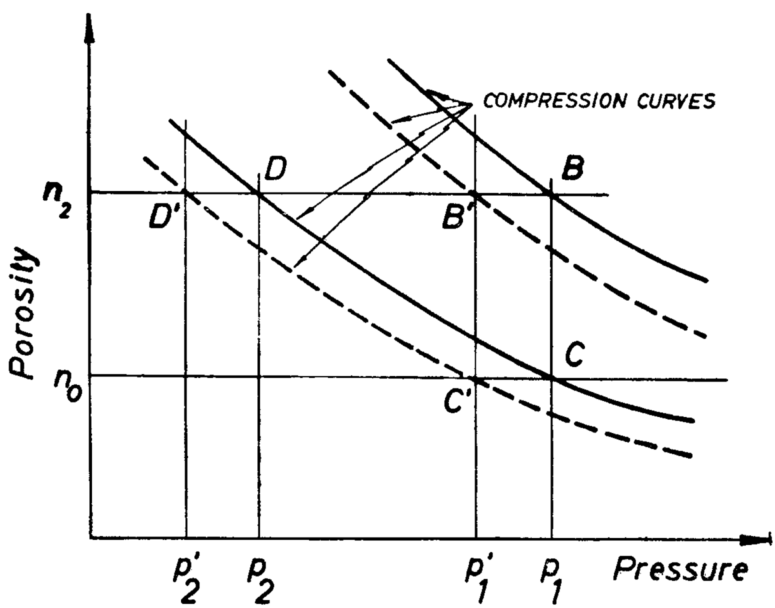


Fig. 3

This construction implies that in the shearing deformation of material at porosity n_2 the ultimate effective pressure p_2, p_2', etc. will depend on the initial state B, B', etc. from which the deformation began. However, if it is obvious in the first passage quoted that one ultimate critical density is reached after deformation at constant effective pressure of initially loose or dense soil, we may well consider it obvious that one ultimate effective pressure is reached after deformation at constant porosity of initially lightly or heavily compressed soil. That is to say, there is no obvious reason why p_2 and p_2' should differ.

3. The Cambridge concept of critical density as a function of pressure meets this dilemma. In Fig. 4 are shown three compression curves through B, C, and G. The states of critical density are now shown by a dashed line of critical states, where critical porosity is clearly seen to be dependent on effective pressure. The ultimate critical density of material sheared at constant effective pressure p_1 is, as before, represented by point C. Material in a loose state at B will settle during shear and material in a dense state at G will expand, and in either case there is ultimately no memory in the material at C of the initial state before shearing.

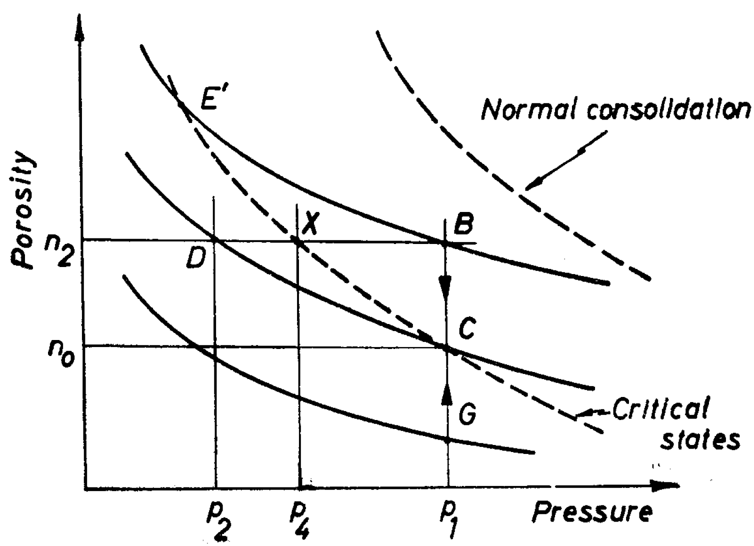


Fig. 4

However, there is not now a unique critical porosity n_0, since for any porosity there is some effective pressure such that material at that porosity can deform continuously. For the porosity n_2 the effective pressure at which the material will continuously deform is p_4, which pressure is a little higher than Casagrande's predictions p_2 and p_2'. Material at porosity n_2 and effective pressure p_2, a state represented by point D in Fig. 4, will now develop a small suction $(p_4 - p_2)$ during shearing deformation at constant porosity. The ultimate state reached after deformation of material at constant porosity n_2 is given by the intersection X of the curve if critical states and the abscissa of B and D, and in this state represented by point X the material has ultimately no memory whether it was lightly or heavily compressed, at D or at B, before shearing.

4. One corollary of this Cambridge critical state concept is that the loose sand can be brought into a critical state not by compression to very large pressures but by substantial reduction of pressure from B to the state represented by point E′ in Fig. 4.

Increase of pressure carries the soil structure into a progressively more «overloaded» state, further beyond the curve of critical states. A second corollary of the Cambridge critical state concept is that Terzaghi's «normal consolidation» phenomenon that occurs in soft silty clay is viewed as a collapse of soil structure which is heavily overloaded. The constant separation distance between the curve of normal consolidation and the curve of ultimate critical states first observed by Casagrande and Albert, is in the Cambridge view seen to indicate that when a silty clay soil structure carries a certain proportion of «overload» above the critical state effective pressure, that structure collapses and the soil exhibits Terzaghi's consolidation as it settles to a compression curve of lower porosities.

5. After describing his notable experiments on some physical properties of remoulded cohesive soils M. J. Hvorslev wrote (2),

«... During the shearing tests the samples undergo further volume changes; as first shown by A. Casagrande, the void ratio of cohesive soils in a state of natural consolidation is decreased. Conversely to this, the author found that the void ratio of cohesive soils in a state of strong overconsolidation is increased during the shearing test (Figs. 5 and 6). This indicates that the shearing load will call forth a positive or negative excess pressure in the porewater, by Terzaghi called the hydrodynamic stress, according to the state of consolidation of soil.

In this case, however, the shearing load was applied at such a slow rate, that the hydrostatic excess pressures in the porewater were practically equalized at the moment of failure, and the measured stresses therefore equal to the effective stresses.

According to A. Casagrande, a cohesionless soil has a certain, critical void ratio, and a material with this void ratio, when subjected to pure shear, will not undergo changes in void ratio or porewater pressure. The author found that in the case of cohesive soils such a definite, critical void ratio does not exist, and that any void ratio can become critical if it is produced by a critical consolidation pressure.»

He then gives his expression for the shearing resistance of clay at failure as follows,

«To facilitate the analytical expression and graphical representation of the influence of the void ratio on the shearing resistance, the equivalent consolidation pressure, p_e was introduced, defined as the pressure in the virgin pressure-void ratio diagram, which corresponds to the actual void ratio, ε, of the sample. By means of Terzaghi's simplified equation for virgin consolidation, we obtain

$$p_e = p_1 e^{B(\varepsilon_1 - \varepsilon)}$$

By plotting the shearing resistance and the corresponding values of the equivalent consolidation pressure at failure, we obtain similar curves (Figs. 5 and 6), which suggest the following condition of failure:

$$\frac{s}{p_e} = \mu_0 \frac{p}{p_e} + \varkappa$$

As will be seen in Fig. 7, the points (p/p_e, s/p_e) lie almost exactly in a straight line, which gives directly the values of the coefficient of effective internal friction $\mu_0 = \tan\phi_0$ and the coefficient of effective cohesion $\varkappa$. By means of the relation between p_e and ε, and by introducing the coefficient $\nu = \varkappa\, p_1\, e^{B\varepsilon_1}$, we obtain

$$s = \mu_0\, p + \nu\, e^{-B\varepsilon}$$

or, expressed in words, the shearing resistance is a function of the effective normal stress on and the void ratio in the plane of and at the moment of failure, and this function is independent of the stress history of the sample.»

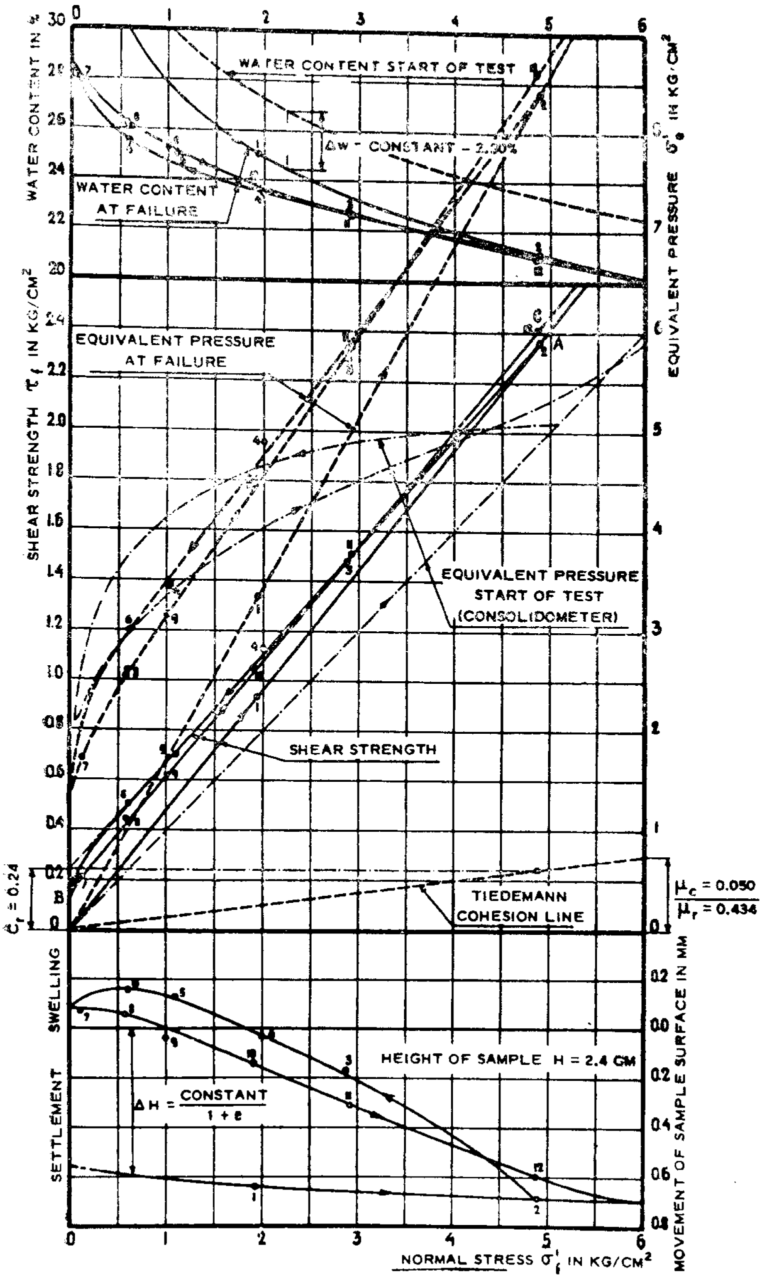


Fig. 5 Results of slow shear tests on Wiener Tegel V (after J. Hvorslev, ref. 4, Fig. 21)

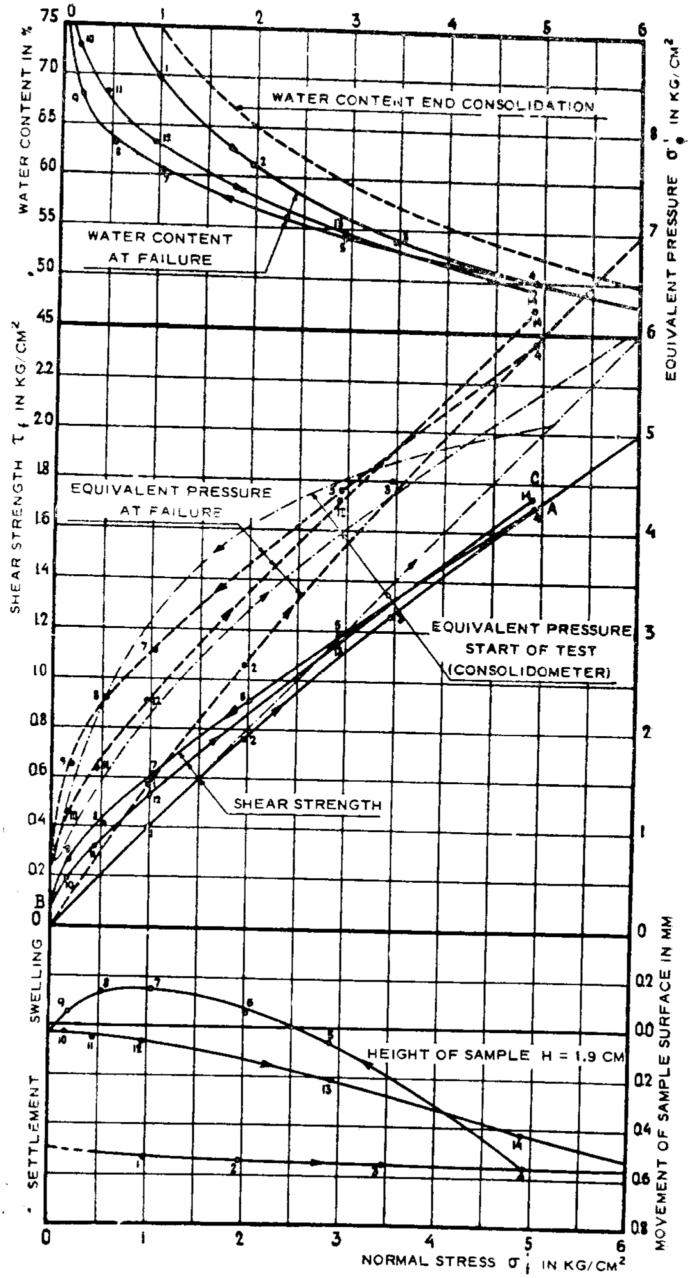


Fig.6 Results of slow shear tests on Klein Belt Ton
(after J. Hvorslev, ref. 4, Fig. 22)

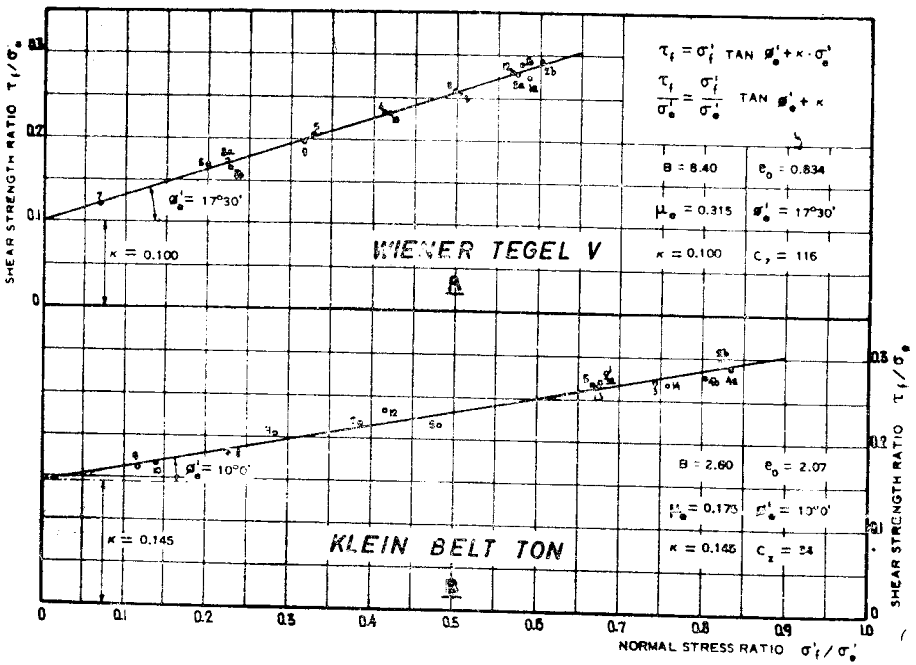


Fig. 7 Combined determination of friction and cohesion parameters (after J. Hvorslev, ref. 4, Fig. 24)

6. For Hvorslev's stress-controlled shear box tests «failure» was precisely defined to be occuring under that stress for which the slow rates of displacement first showed signs of acceleration. After his careful study of his own *rapid* shear test data he felt that he could not distinguish between thixotropic effects and pore-water pressure effects, but from his *slow drained* tests he was able to deduce the expression for shearing resistance of clay at failure quoted in the second passage above..

7. Examination of Hvorslev's data in Figs. 5 and 6 shows that there is region of water contents and normal pressures below the curve 12 in which can be plotted all points representing states in which «failure» occured. This region is also shown shaded in Fig. 8 a, and is indicated by the letter Z. The region is bounded alone by the curve 12, indicated

in Fig. 8 by the letter X. There is a region of higher water contents and normal pressures indicated by the letter Y in Fig. 8, between the curve 12 and the primary consolidation curve, and it is notable that no failure occured for specimens in that region.

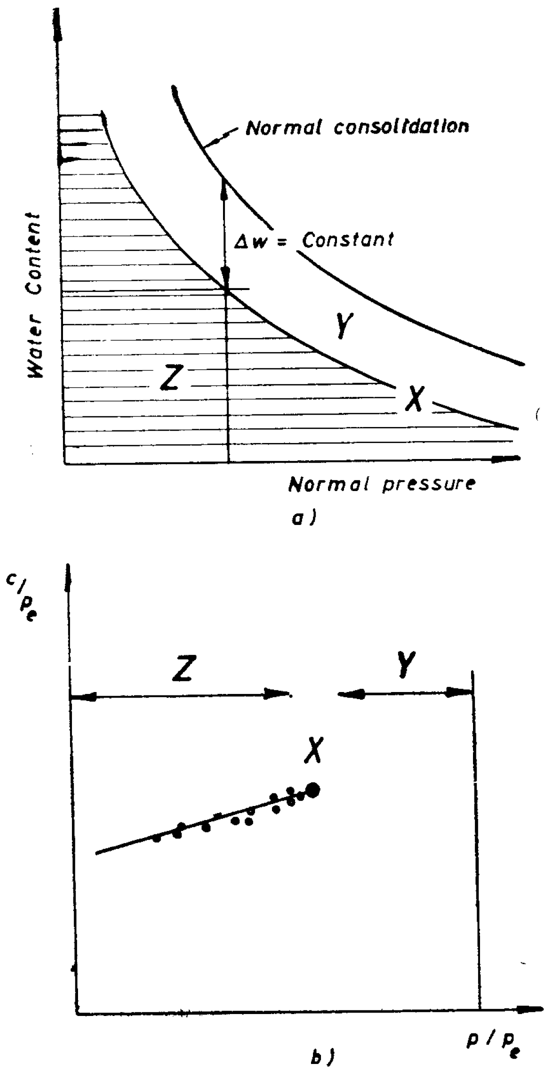


Fig. 8

8. For Wiener Tegel V in Fig. 5 the region Y has a constant separation Δw = constant between curve 12 and the primary consolidation curve, but for Kleinbelt Ton the separation of these curves (Fig. 6) is not constant. Consequently, in Fig. 7 when data of failure is replotted in ratio to the equivalent pressure, the data for Wiener Tegel V clearly lies in the region $p/p_e < 0.6$, while for Kleinbelt Ton there is data at higher values of p/p_e. In either case there is a region of highest values of p/p_e in which «failure» of samples was never recorded. For simplicity of subsequent discussion, and in view of the reported evidence of Casagrande and Albert, let us take the data of Wiener Tegel V to be typical, thus the line X in Fig. 8 a may be reduced to the point X in Fig. 8 b.

9. In Fig. 8 b the point X clearly marks the termination of a range Y in which data of failure was not recorded. There is an accumulation of data in the vicinity of $p/p_e = 0{,}6$ in Fig. 7 a, which seems to indicate that specimens originally at states where $p/p_e > 0.6$ must have first yielded and settled without failure, and then having traversed the region Y the specimens failed when they first entered the region Z at its boundary X.

It is clearly wrong to represent Hvorslev's results in the manner of Fig. 9 a in which the lines $s = \mu_0 p + \nu e^{-B\varepsilon}$ are extended into regions in which Hvorslev did not obtain data. The region Z of appli-

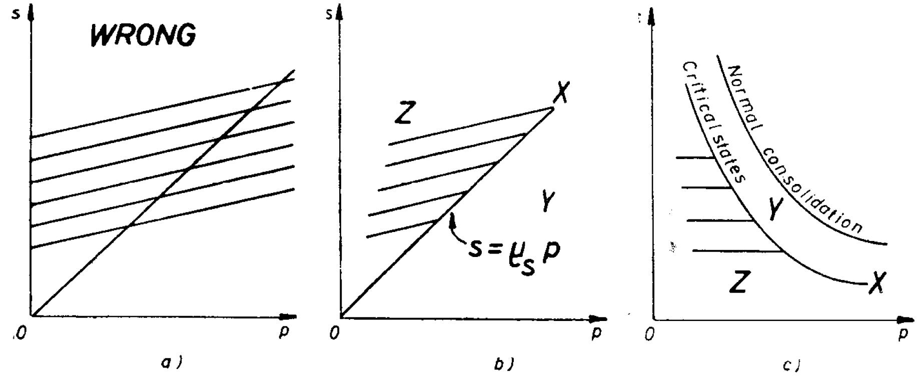
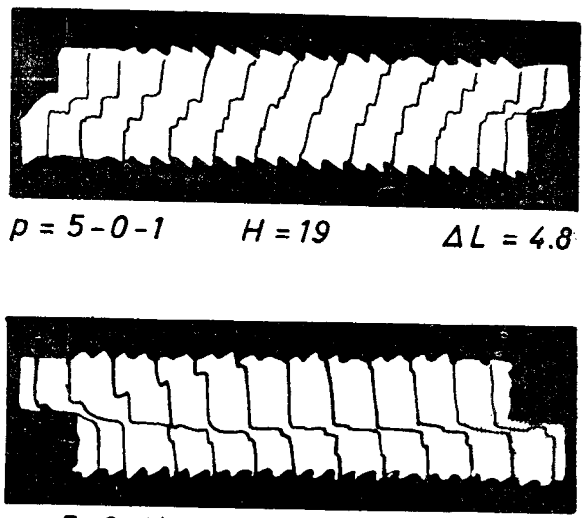


Fig. 9

cability of Hvorslev's equation should correctly be delineated in the manner of Fig. 9 b.

10. Hvorslev writes that for cohesive soils «any void ratio can become critical if it is produced by a critical consolidation procedure». The implication is that in Fig. 9-c specimens in region Y must settle, that their critical consolidation procedures must produce specimens that arrive and fail on the line X, and that specimens in region Z must expand. But it must not be inferred that the whole of a specimen in a state in region Z expands after failure. In Fig. 10 sketches of a specimen after failure show evidence of a large displacement localized in a narrow plane.



$p = 5-0-1$ $H = 19$ $\Delta L = 4.8$

$p = 5-0-1/2$ $H = 22$ $\Delta L = 4.0$

Fig. 10 Cross sections of two Wiener Tegel shear box samples (after J. Hvorslev)
H: Sample height, L: Horizontal displacement of sample,
Units: kg/cm^2 and mm.

Hvorslev determined water contents at failure by quickly dismantling the shear boxes immediately after failure, cutting a strip 2 cm.

wide and 3 mm. thick from «the failure zone», dividing in into 5 parts as shown in Fig. 11 and determining the average water content from these parts. From the sketches it seems that the surface of slip was very thin. Suppose in Fig. 12 that the thin slip surface is at the plane XX. Although the local water content might rise during expansion towards the critical voids ratio, this gain of water would temporarily be at the expense of a loss of water from the adjacent material - temporarily the water contents would be as sketched in Fig. 12. The rapid manner in which Hvorslev found the water content of the whole thick strip would indicate the general state of the material *just before* failure. If there had been at and about the failure plane a band or a region of homogenously deformed soil that was more than 3 mm. thick, and if the critical void ratio concept had applied to this material, then ultimate states of soil in the failure region could have been represented by points on the line X in Fig. 9 c. If Hvorslev had sampled material that had all softened towards the critical states then all data in 9 c would have been in the near vicinity of the line X and there would have been no data in the region Z: however there really was data in region Z.

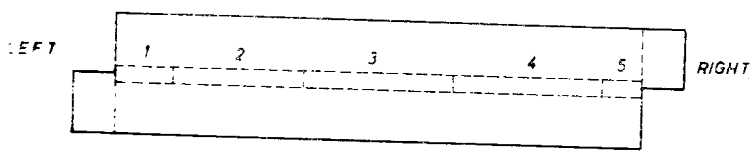


Fig. 11 Obtaining small samples for water content determination (after J. Hvorslev)

Hvorslev writes of a significant permanent change of structure at the failure plane - for Klein Belt Ton the sample could be seperated along the failure plane which «possesed a dull shine». Apparently displacements after failure proceeded sufficiently far to produce a thin sheet of oriented particles. Shearing resistance on this plane fell to residual values $s < \mu_s p$. Clearly the critical void ratio concept, or indeed the concept of water content itself, can only be meaningful when applied to a volume of very many particles in a locally homogeneous uniform state.

In our experience the Cambridge development of Casagrande's critical void ratio concept is helpful in understanding of all bulk

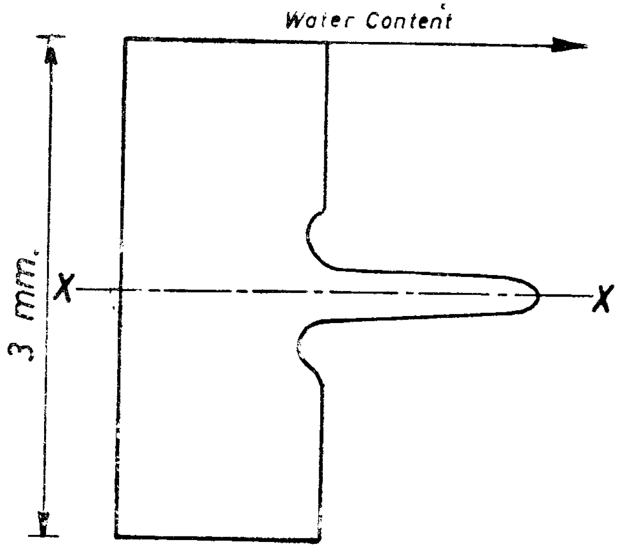


Fig. 12

changes of state of deforming granular media. The rates of dilatation or settlement of soil in any state depend on the separation of the point representing that state from the line representing critical states. Shear test data always shows an initial portion in which reasonably homogeneous deformation occurs as the loading programme is developed. For soil specimens looser or wetter or more heavily loaded than their critical states the specimens undergo large deformations and appear plastic; for such material the typical phenomenon is the settlement associated with Terzaghi's consolidation and plastic flow. Alternatively for soil specimens of less open structure, less water content, less heavilv loaded than their critical states, the specimens undergo relatively small uniform deformation and then appear brittle or unstable as slip planes are propagated through them: for such material the typical phenomenon is division of a large block into several blocks that slip relative to each other on Coulomb's slip surfaces.

In this view the curve X of critical states in Fig. 9 c is a watershed dividing the region Y of Terzaghi's consolidation and plastic flow from the region Z of Coulomb's slip surfaces. Both in Cambridge and Istanbul we have developed this concept in interpretation of triaxial compression tests with samples in the region Y. In this work we follow Hvorslev's notation and use q to denote the triaxial deviator stress. Hvorslev suggested that Mises expression

$$(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 = f\,(\sigma_1 + \sigma_2 + \sigma_3)$$

might be employed for soils. The effective stress parameters

$$q = \sqrt{\frac{(\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2 + (\sigma_1' - \sigma_2')^2}{2}}$$

and

$$p = \left(\frac{\sigma_1' + \sigma_2' + \sigma_3'}{3}\right)$$

were used by us. A yield function for consolidation and plastic flow of an ideal material called Cam-clay has the general form

$$q = f(p, \varepsilon),$$

which is a fulfillment of Hvorslev's suggestion.

12. Hvorslev concluded by writing, «There is, therefore, an urgent need for research with the object of establishing the fundamental laws governing the initial change of the hydrostatic pressure in the porewater and the ultimate change of the void ratio, which will occur when a soil, acted upon by an arbitrary system of stresses, is subjected to an arbitrary change of these stresses.». The discussion of this present paper shows how the Cambridge development of Casagrande's notable concept of critical states goes some way towards this object.

In the present reinterpretation of Hvorslev's experiments we are left with a feeling of curiosity about the manner in which Coulomb's slip planes are propagated through a deforming body of homogeneous material. Similar phenomenon of rupture and brittle failure are found in other materials, but Hvorslev's experimental success in defining so precisely a simple expression for states in which failure can occur in soils still seems a most notable achievement.

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