MODELLING OF SEABED INTERACTION IN FREQUENCY DOMAIN ANALYSIS OF MOORING CABLES

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Offshore structures rely heavily on the performance of mooring cables to stay in position.

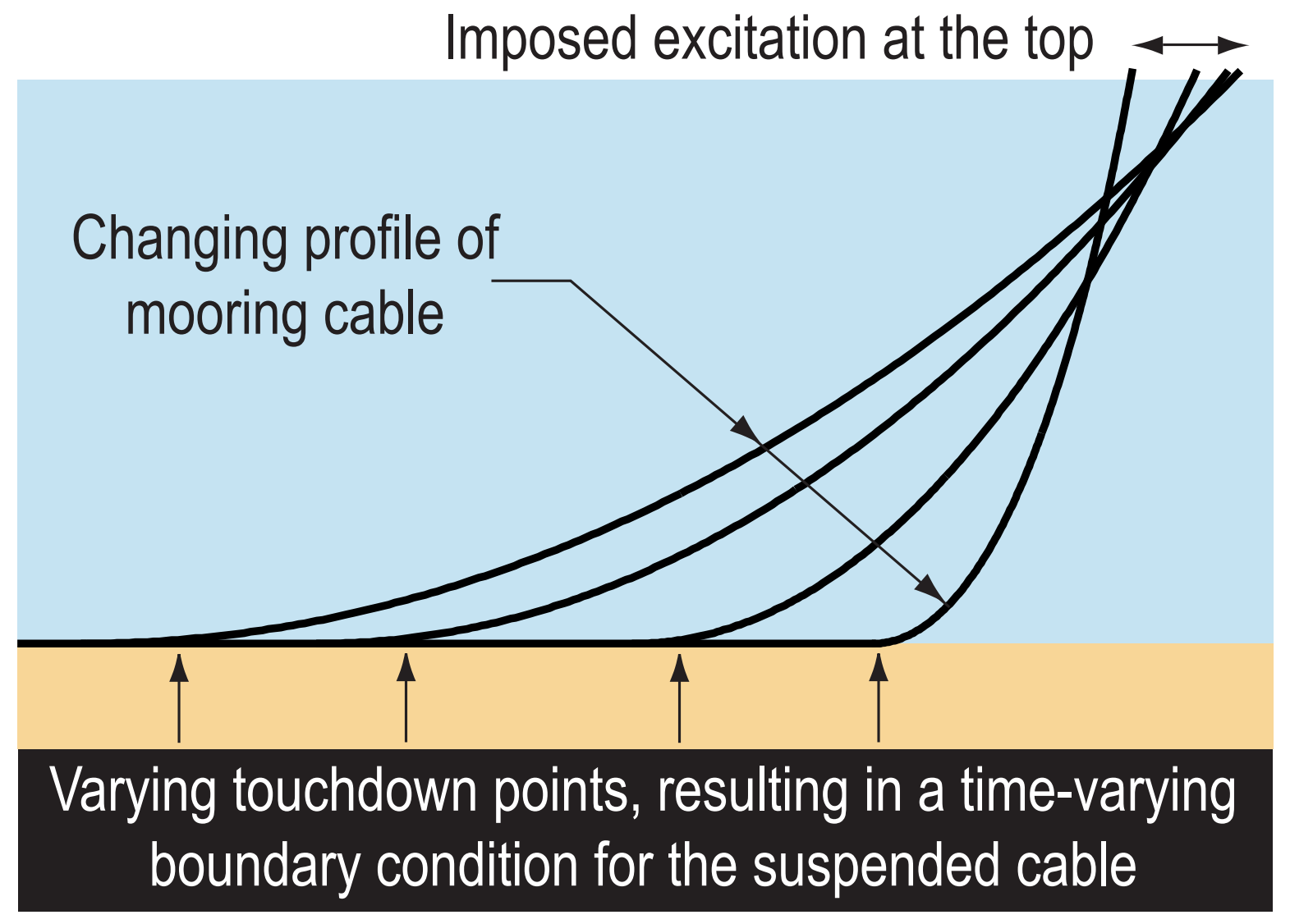
Accurate modelling of mooring cables however requires lengthy non-linear computations. For example, seabed interaction results in a time-varying boundary condition which is difficult to model in full.

A new method of modelling seabed interaction using a linearised computational scheme is proposed, which will improve the overall accuracy of mooring cable analysis.

INTRODUCTION

Mooring cables play an important role in anchoring offshore structures to the seabed. Their slender behaviour in water are often difficult to understand and model. Their interaction with the seabed further complicates the problem.

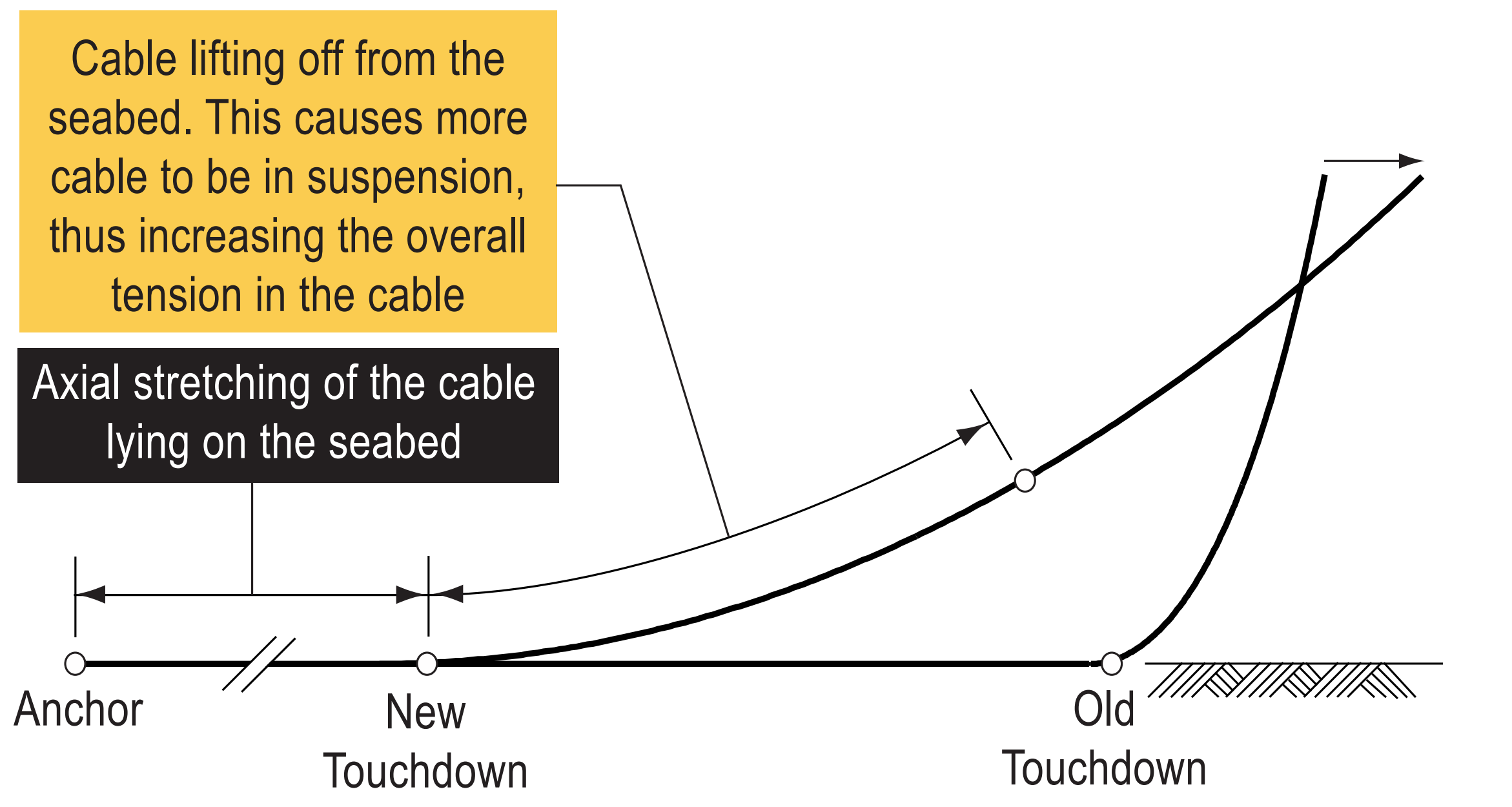
When excited under wave loading, these cables interact dynamically with the seabed, creating a boundary condition that varies in time and space, see right.



Varying touchdown points, resulting in a time-varying boundary condition for the suspended cable

To model this interaction in full, one would have to carry out a lengthy time domain analysis. A more efficient approach would be to model in the frequency domain, which is however a linear method of analysis.

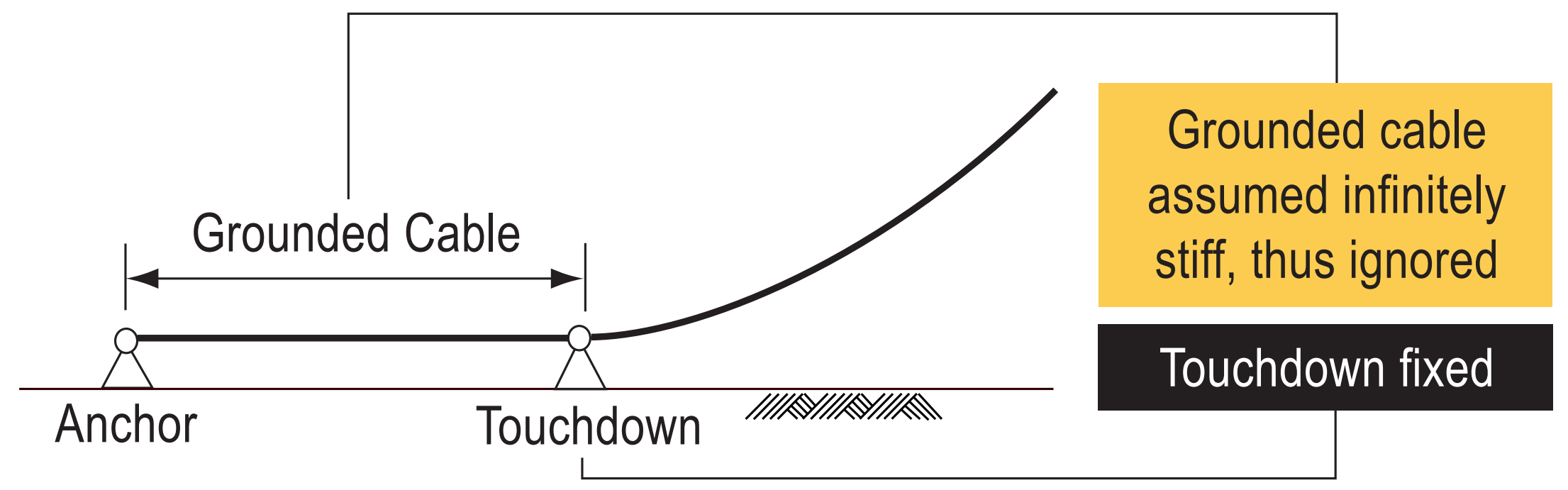
The problem can be better understood by dividing the interaction into two separate yet dependent mechanisms, occuring simultaneously whenever a cable is excited:



Cable lifting off from the seabed. This causes more cable to be in suspension, thus increasing the overall tension in the cable

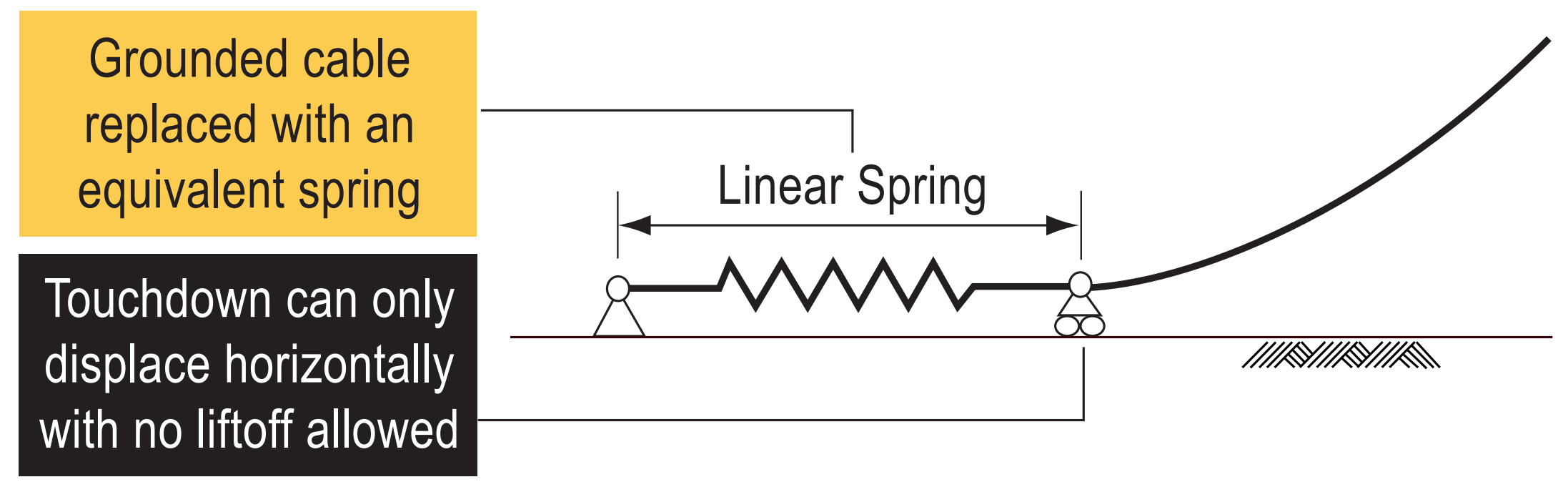
Axial stretching of the cable lying on the seabed

Traditionally, two methods of modelling seabed interaction exist in the frequency domain--(i) pinning the touchdown; or (ii) replacing the grounded cable with an equivalent spring:



Grounded cable assumed infinitely stiff, thus ignored

Touchdown fixed



Grounded cable replaced with an equivalent spring

Touchdown can only displace horizontally with no liftoff allowed

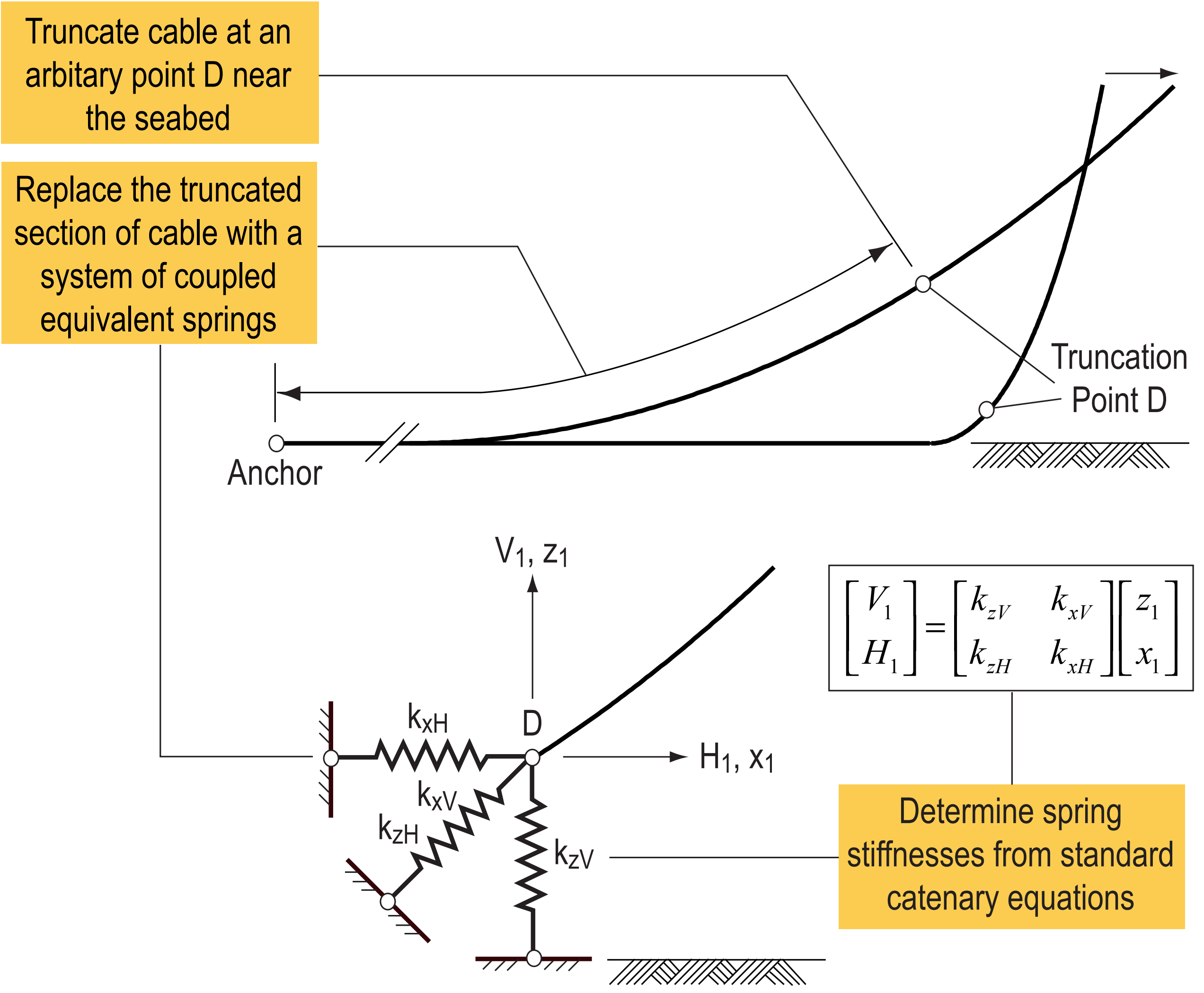
None of the existing methods takes into account the effect of liftoff and touchdown of the cable during excitation. It is observed that neglecting this effect, the frequency domain analysis would yield inaccurate cable tensions under large excitations.

OBJECTIVE

This research aims to develop a new method of modelling seabed interaction in the frequency domain, which will account for both the stretching of grounded cable and the liftoff and touchdown action of the cable.

METHODOLOGY

The approach adopted is to model the section of cable near the seabed with a system of linear coupled springs, by carrying out the following 3 steps:



Truncate cable at an arbitary point D near the seabed

Replace the truncated section of cable with a system of coupled equivalent springs

$$\begin{bmatrix} V_1 \\ H_1 \end{bmatrix} = \begin{bmatrix} k_{zV} & k_{xV} \\ k_{zH} & k_{xH} \end{bmatrix} \begin{bmatrix} z_1 \\ x_1 \end{bmatrix}$$

Determine spring stiffnesses from standard catenary equations

It is assumed that dynamic effects near the seabed are negligible and thus can be ignored. Static catenary equations are then used to determine the required spring stiffnesses through the following linearisation process:

$$Z = g(V,H) \quad X = f(V,H)$$

First, we take the above standard catenary equations and expand them in Taylor's Series up to the power of cubic

$$z_1 = \sum_{i=j}^{n=k} g_i H_1^{n-i} V_1^{i-j} \quad x_1 = \sum_{i=j}^{n=k} f_i H_1^{n-i} V_1^{i-j} \quad \begin{bmatrix} k \\ j \end{bmatrix} = \begin{bmatrix} 2 & 5 & 9 \\ 1 & 3 & 6 \end{bmatrix}$$

The above Taylor's expansions are then linearised using the Method of Least Squares to yield the following linear expression

$$\begin{bmatrix} z_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ H_1 \end{bmatrix}$$

The cofficients e_{ij} can thus be determined by satisfying the following criteria and the required stiffness matrix obtained from the inverse

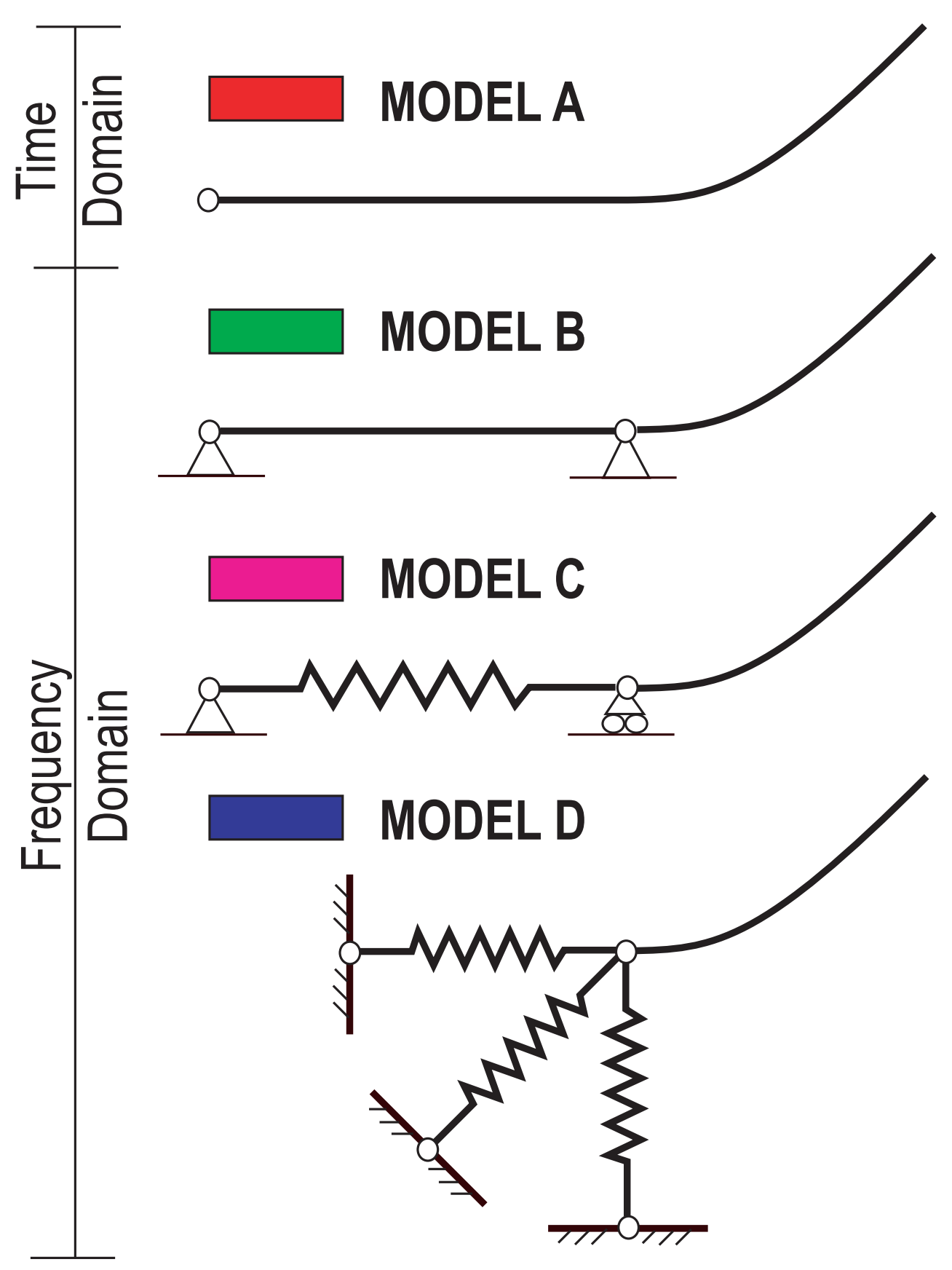
$$\frac{\partial}{\partial e_{11}} \int (z_1 - e_{11}V_1 - e_{12}H_1)^2 dt = \frac{\partial}{\partial e_{12}} \int (z_1 - e_{11}V_1 - e_{12}H_1)^2 dt = 0$$

$$\frac{\partial}{\partial e_{21}} \int (x_1 - e_{21}V_1 - e_{22}H_1)^2 dt = \frac{\partial}{\partial e_{22}} \int (x_1 - e_{21}V_1 - e_{22}H_1)^2 dt = 0$$

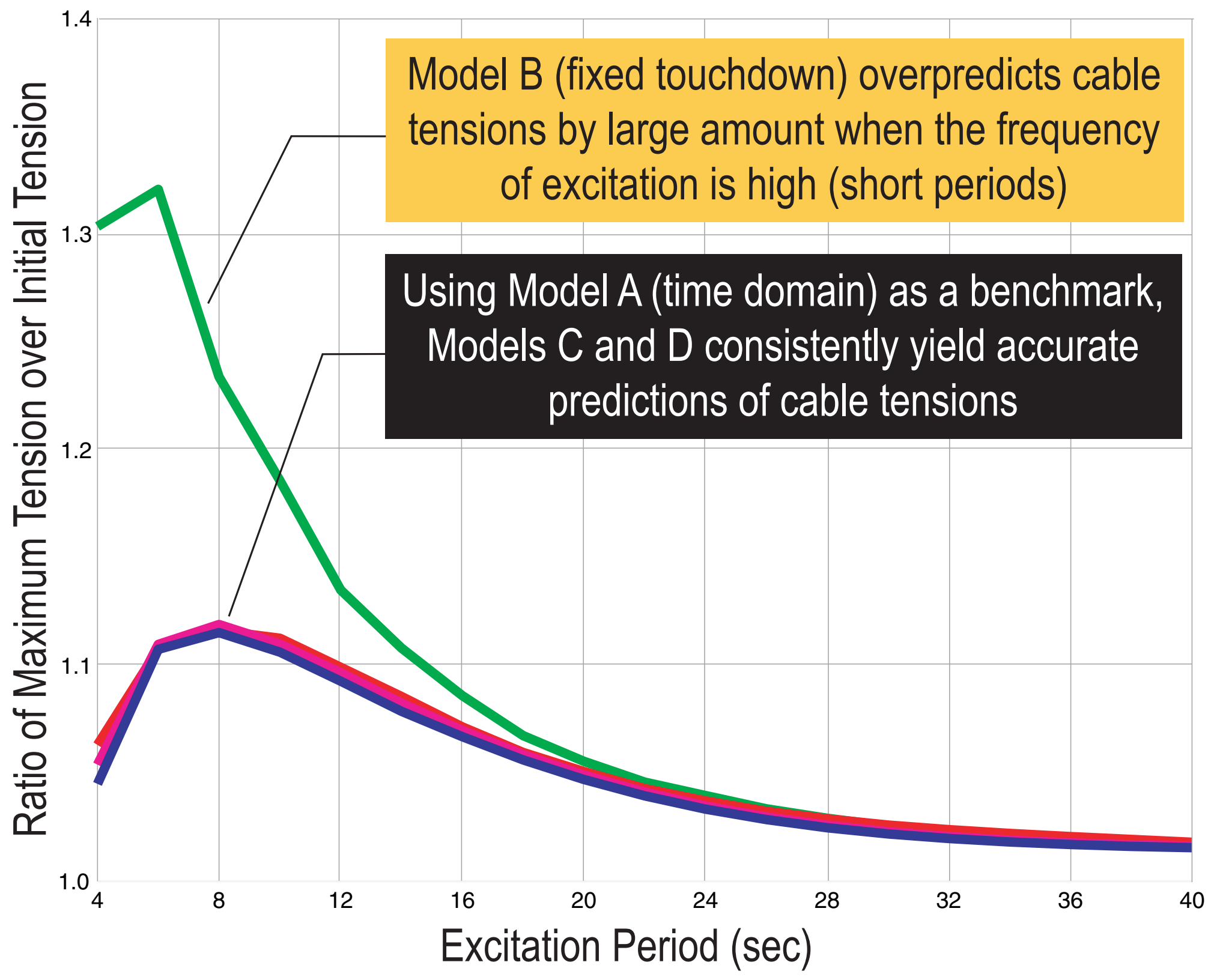
$$\begin{bmatrix} k_{zV} & k_{xV} \\ k_{zH} & k_{xH} \end{bmatrix} = (e_{11}e_{22} - e_{12}e_{21})^{-1} \begin{bmatrix} e_{22} & -e_{12} \\ -e_{21} & e_{11} \end{bmatrix}$$

RESULTS

The proposed new method, together with the two existing methods mentioned earlier, are tested and compared against a full time domain simulation. On the right is a legend of the four different models analysed. The comparison is carried out for both small and large excitations.

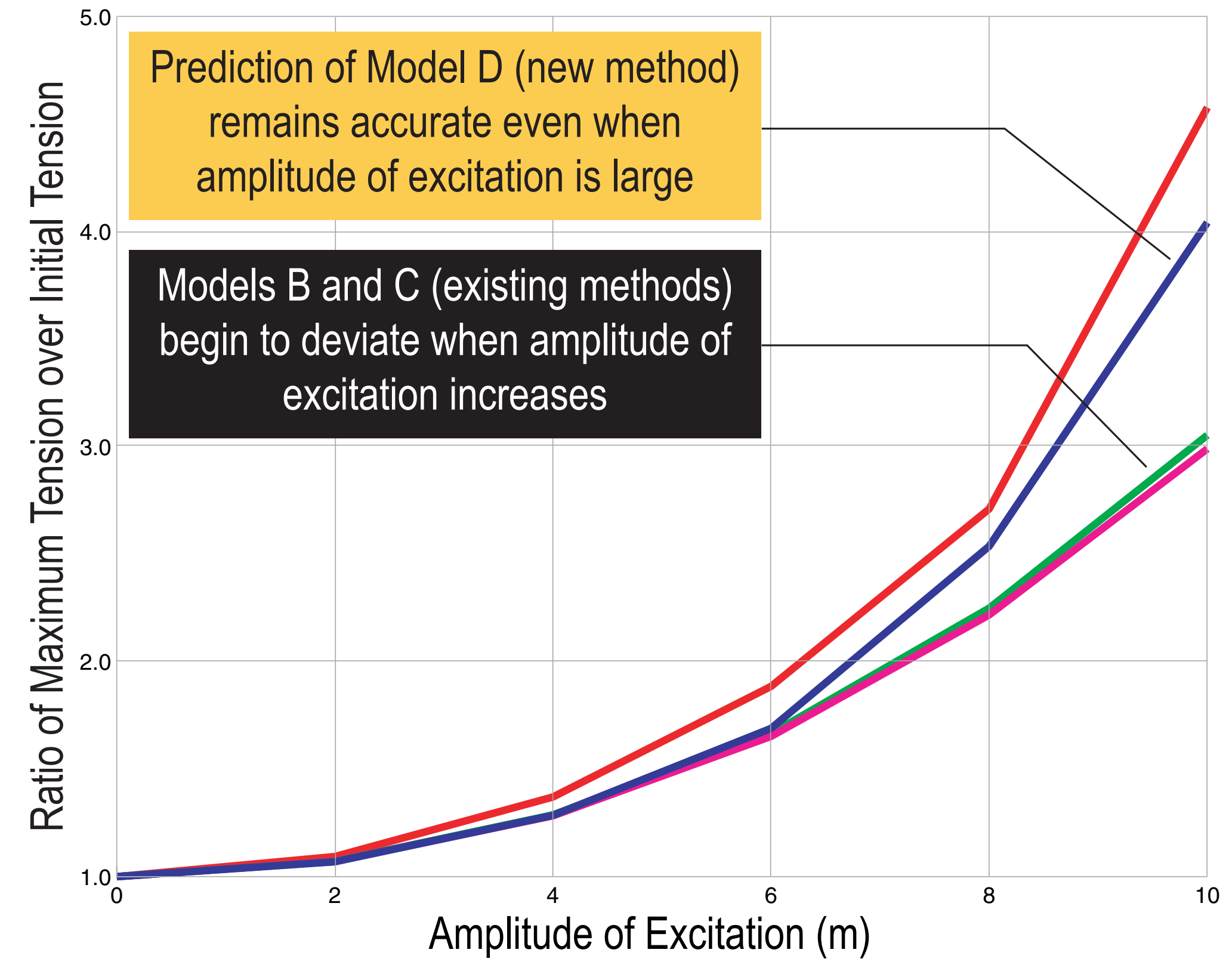


Comparing over a range of excitation periods with a fixed excitation amplitude of 1m.



Model B (fixed touchdown) overpredicts cable tensions by large amount when the frequency of excitation is high (short periods)

Using Model A (time domain) as a benchmark, Models C and D consistently yield accurate predictions of cable tensions



Prediction of Model D (new method) remains accurate even when amplitude of excitation is large

Models B and C (existing methods) begin to deviate when amplitude of excitation increases

Comparing over a range of excitation amplitudes with a fixed period of 10 seconds.

CONCLUSIONS

The proposed new method of modelling seabed interaction has shown to improve the overall accuracy of frequency domain analysis. Under large excitations, neglecting the effect of liftoff yields inaccurate cable tensions is also verified.

Besides mooring cables, this new approach can also be applied to the analysis of catenary risers or any problems involving cable-structure interaction.

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