Statistical Lifetime Predictions for Aramid Fibers

K. G. N. C. Alwis[1] and C. J. Burgoyne[2]

Abstract: This paper investigates the statistical procedures that can be used to analyze stress rupture data for aramid yarns, with a view to making reasonable predictions for the allowable prestress levels in parallel-lay ropes or fiber-reinforced polymer tendons for use as prestressing tendons in concrete structures. Two existing data sets are combined and used to illustrate the principles that are being discussed. Arrhenius, exponential, and inverse power physical models are used with Weibull and lognormal statistical distributions. A wide range of variations of the scale and shape parameters are analyzed. Various statistical criteria are used to reject models that are not statistically secure, and the Kullback-Liebler and Akaike information criteria are used to further limit and rank the possible models. The results show that there is still a large range of possible predictions for the long-term stress-rupture lifetime of aramid yarns, but also that statistical models are available for investigating further test work that needs to be carried out.

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Introduction

Aramid ropes have been proposed for use as prestressing tendons for concrete structures, and for a number of other structural applications (Burgoyne 1999). The attraction is their resistance to corrosion by water, which would allow their use as external tendons or with much reduced concrete cover. There is, however, reluctance within the industry because of the known phenomenon of stress-rupture. Prestressing tendons in concrete are most susceptible to this type of failure because they are tensioned against the concrete immediately after the concrete has hardened, to provide the required precompression, and the high force then remains in place for the lifetime of the structure (Burgoyne 1990).

The force in the tendon is altered very little by normal loadings, and if unbonded, as aramid tendons would be (Burgoyne 2001), the increase in stress when the structure is loaded to failure is small. Final failure would be expected to occur by crushing of the concrete, rather than by snapping of the tendon. A small reduction in stress in the tendon will occur due to creep of the concrete or relaxation of the tendon (Burgoyne 1993). Conventional bonded steel tendons are the most heavily stressed structural element in use; they are routinely stressed to 70% of the ultimate tensile strength (UTS), in the knowledge that creep will reduce that figure to about 60% UTS in the first few months, after which it remains constant (Abeles and Bardhan-Roy 1981). It is structurally sensible, and economically worthwhile, to make steel tendons out of very high strength steel (typical characteristic strengths are in the range 1,750–2,000 MPa), with carefully controlled microstructure to give good ductility (3.5% extension at failure is regarded as a minimum) (BSI 1980).

Aramid tendons are several times more expensive than steel (Jungwirth and Windisch 1995), but their resistance to corrosion, and the consequent assurance of long-term durability, can outweigh the additional first cost if whole-life costing is adopted (Balafas and Burgoyne 2003). The key figure required to determine the additional cost is the allowable long-term stress that can be applied to the tendon, which is governed by stress-rupture.

Structural lifetimes are measured in decades. Even the most ephemeral industrial building will be expected to last for 20 years, and 50 years is a common design lifetime for office buildings. Bridges are typically designed for 120-year lifetimes, but these figures are notional and society feels aggrieved if buildings or bridges need to be refurbished because of durability problems. It is clearly not feasible to conduct tests for these durations before using new materials, so extrapolation of short-term test data must be carried out. Tests carried out in testing machines rarely last for more than a few hours because of the expense of tying up the machine, while tests using dead weights have high capital costs and take up valuable space; they rarely last more than a couple of years.

Industry wants predictions of loads to give various stress-rupture lifetimes, together with associated probabilities of failure. If these figures are not available, or the perceived uncertainty is too high, then conventional materials will be used instead, despite their shortcomings. In these days of "design, build, finance, and operate" contracts, industry must cost-in both the real risk, due to material variability, and the notional risk, due to uncertainty in the modeling. Uncertainty in the extrapolation method can thus have a very real economic cost and can mean that a less suitable material is used simply because there is more certainty about its properties.

There is still discussion about how design values should be obtained from extrapolated data for stress-rupture, but a reasonable approach would be to take the mean value of the load to cause failure at the design lifetime, less a number of standard deviations (typically 1.64), to give a characteristic design value

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(Budelmann and Rostasy 1993). This would then be reduced by a partial safety factor that would reflect confidence in the model. A partial safety factor of 1.15 is used for the short-term strength of steel bars (which is easily measured); 1.4 is often used for the strength of concrete, which is more variable (Commission 1989). These figures are the subject of considerable debate because they have great commercial effect; they may be lowered if the quality control can be demonstrated to be very high, but it is difficult to resist calls for higher figures if the extrapolation is uncertain or the consequences of failure would be catastrophic.

The current investigation is part of a larger study (Alwis 2003) into the stress-rupture behavior of aramid fibers, which includes the use of accelerated test methods such as time-temperature superposition (Markovitz 1975) and the stepped isothermal method (Thornton et al. 1998). The objective of this part of the study was to investigate the various statistical methods that could be used to extrapolate from an existing data set. It was desired to determine which technique gave the best fit to the existing data, and which gave the most useful predictions for the long-term stress-rupture behavior. Those two aims may not be mutually compatible.

It should be noted that other degradation mechanisms exist, most notably hydrolysis and other forms of chemical attack. This paper specifically does not address these issues, because creep and chemical action involve different processes with different activation energies. It is essential to understand the mechanisms separately before considering the effects of them acting together.

Data Set

The data used in this study is taken from work carried out on parallel-lay aramid ropes (Parafil), manufactured by Linear Composites Ltd. using Kevlar 49 fibers (Kingston 1988). These ropes retain virtually the full stiffness of the fibers (due to their parallel structure), and barrel and spike terminations are available that can develop the full strength of the rope (Kingston and Mattrass 1973). The ropes and terminations are in the same form as they would be used in prestressing applications, which is the principal reason for testing in this state rather than testing yarns or fibers.

Chambers (1986) carried out a series of tests on ropes with a nominal breaking load (NBL) of 60 metric tons under hydraulic loading, with the forces kept constant by a hydraulic load-maintaining system. Some of the specimens were tested in a tension testing machine; others were tested inside a steel tube used as a reaction frame (Chambers and Burgoyne 1990). Guimaraes (1988) published data pertaining to 1.5 and 3 ton parallel-lay ropes. The ropes were subjected to constant loads by weights applied through a lever arrangement (Burgoyne and Guimaraes 1996). The times to failure were observed; both of these data sets are used in this analysis (Table 1).

Predictions of the load that would give a mean time to failure of 100 years were made by both Chambers and Guimaraes; a value of 50% of the short-term ultimate tensile strength is accepted as a reasonably conservative mean value for use in design, and they both quoted 5 and 95% confidence limits on their predictions. After safety factors have been added, a value of 40% of the strength has been used in codes, but it is noted that these values are based on very little data (Bakht and Faoro 1996).

The ropes have short-term strengths that vary with size as predicted by bundle theory from the known strengths of fibers (Daniels 1945; Phoenix 1978; Amaniampong 1992; Amaniampong and Burgoyne 1995). When subjected to a load, the weakest fibers fail first and shed load to the remaining elements; eventually these are unable to sustain the increase in load, which leads to complete failure. The strength of the rope is thus governed by the presence of weak fibers. Comparisons between distributions of rope strength and yarn strength indicate that the rope fails at a stress at which only about 2% of the yarns would have failed (Burgoyne and Mills 1996).

Table 1. Aramid Rope Data [from Chambers (1986) and Guimaraes (1988)]

Specimen (metric tons)	Applied stress (% ABL)	Time to failure, t_i (days)
60	85.0	0.0187
60	85.5	0.9410
60	86.0	0.1118
60	79.5	0.0833
3	81.6	0.4181
3	81.6	0.4930
60	80.0	0.7889
3	81.6	1.5851
3	81.6	3.6193
3	81.6	11.490
3	81.6	18.712
60	77.5	0.7153
3	75.8	3.1925
60	75.0	11.163
60	76.0	11.956
3	75.8	27.529
3	77.1	36.358
3	75.8	39.506
3	75.8	45.360
3	77.1	53.200
3	75.8	72.915
1.5	73.3	3.4526
1.5	73.3	4.9342
1.5	73.1	9.6789
1.5	73.1	17.302
1.5	73.3	25.307
1.5	73.3	45.648
1.5	67.8	35.274
60	68.0	155.154
1.5	67.6	164.759
3	66.8	255.317

To allow for size effects in the data, it has been found reasonable to normalize the stresses applied to the ropes by the actual breaking load (ABL) in short-term tests. This has been found to allow the stress-rupture data of various rope sizes to be compared without further adjustment (Amaniampong and Burgoyne 1995).

The lifetime distribution is most simply shown by a plot of applied stress (% ABL) versus time to failure. The objective of this analysis is to investigate the possible models that can be fitted to failure data to produce curves for mean time to failure and other curves with 5 and 95% confidence limits. From a commercial viewpoint it would be desirable for the best model to give a narrow range between the confidence limits at the low stress levels that are likely to be used in practice. Models that give a wide range would imply that large partial safety factors would be required. However, it may be that the materials really are very variable and the model that best fits the data requires large factors of safety.

Other researchers (Glaser et al. 1984; Wagner et

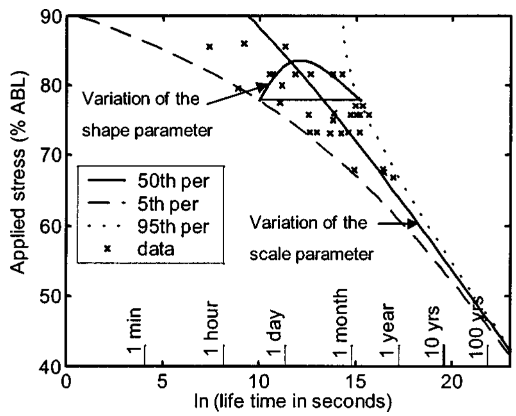

Fig. 1. Variations of scale and shape parameters with applied stress

al. 1986; Zeifman 2001) have used aramid fibers or fiber-impregnated composites to investigate the lifetime distributions, and their work is rarely compared with rope data. Extensive extrapolation procedures were needed to predict the long-term behavior at low loads.

In this analysis, aramid rope data are fitted statistically to different models. A model consists of a minimum of two parameters: the scale and the shape parameters. Fig. 1 shows the test data set used, together with notional views of the scale and shape parameters. Various forms of variation of these parameters with applied stress are considered subsequently.

The validities of the various models are checked using various model selection criteria: the Kullback-Leibler discrepancy criterion, an approximation to this criterion (the Akaike information criterion, or AIC), and the likelihood ratio confidence test.

Data can be broadly classified as observed (failed) and censored (nonfailed). Singly censored data are relevant for aramid rope testing, as censoring will be done if the specimen does not fail within a certain testing period. There are no censored values in the data set used in the present study, but the analysis is carried out in the knowledge that some censored data are likely to be available in the future. Initially the data are analyzed at a fixed stress level; that analysis is then extended to specimens loaded at different stress levels.

Statistical Distributions

Suppose that at a certain stress level, a number of samples have been tested and the corresponding failure times have been observed. It is then possible, by assuming a statistical distribution for the data and using the method of maximum likelihood, to estimate the distribution's parameters to optimally fit the data.

Many statistical distributions can be described by a minimum of two parameters: the scale and the shape parameters. The scale parameter has units of time or ln(time) and defines the time when a certain proportion of the specimens have failed. For some distributions the mean value is used; for others it represents a certain percentile. The shape parameter indicates the spread of data about the scale parameter.

Any statistical distribution can be expressed by well-defined functions such as the cumulative distribution function, the reliability function, and the probability density function. The cumulative distribution function $F(t)$ represents the probability of failing by a given time (t). An alternative is the reliability or survival function: $R(t)=1-F(t)$, which represents the probability of survival after a certain time t.

The probability density function $f(t)=[dF(t)]/dt$ is the first derivative of the cumulative distribution function. The probability of failure up to a time t is the integral of the function from zero to t.

The most appropriate statistical distributions for the analysis of life data of aramid ropes are the Weibull and lognormal distributions (Chambers 1986; Guimaraes 1988).

The Weibull distribution is suitable for handling weakest-link data (Nelson 1990). However, lognormal distributions have also been used in many applications to analyze life data (Glaser et al. 1984; Nelson 1990; Davies et al. 1999). In this analysis, aramid testing data are fitted to both distributions, and their suitability is checked.

Weibull

The Weibull distribution is used to describe product properties such as strength (electrical or mechanical), elongation, resistance, etc. Data which have a weakest link property can also be analyzed (Nelson 1990). The cumulative density function is defined by

$$F(t) = 1 - e^{[-(t/\alpha)]^{\beta}}, \quad t \geqslant 0 \tag{1}$$

where α=scale parameter (sometimes called the characteristic life); and β=shape parameter.

The exponential distribution, which is sometimes used, can be regarded as a special case of the Weibull distribution with $\beta=1$; it is not included separately in the present study.

Lognormal

The lognormal distribution is widely used for life data, including metal fatigue, solid-state components (semiconductors, diodes, etc.), and electrical insulations (Nelson 1990). The cumulative distributive function is defined by

$$F(t) = \phi\left\{\left[\frac{\ln(t) - \mu_e}{\sigma_e}\right]\right\}, \quad t \geqslant 0 \tag{2}$$

where $\phi(z)=(2\pi)^{-1/2}e^{[-z^2/2]}$.

This is a two-parameter distribution with a scale parameter μ_e and a shape parameter σ_e. In life distributions, σ_e is expressed as a function of the applied load. Both μ_e and σ_e are dimensionless, because they represent *ln*(time).

Different Life-Stress Relationships

It is to be expected that the scale and shape parameters will vary with applied stress (Fig. 1). Ideally, a large number of tests (say, 50) would be carried out at each load level, which would allow a proper determination of the parameters at each load; cost and time constraints preclude that possibility. It is more likely that there will be a small number of tests at each of a large number of stress levels as attempts are made to build up an overall picture of the material's behavior.

The variations of the scale and shape parameters with stress can be expressed by various life-stress relationships such as Arrhenius law, inverse power law, etc. There may be physical or chemical reasons why a particular law is appropriate, but in the present study only the statistical properties of the distribution are considered. It will be convenient in what follows to define a stress factor, X, which is a measure of the external applied stress x; the form of the relationship between X and x varies depending on the type of model used.

Arrhenius Life-Temperature Relationship

The Arrhenius relationship is widely used to model product life as a function of the temperature (Nelson 1990). It is best suited for chemical reactions where it is assumed that a certain amount of external energy must be supplied to initiate the reaction. In its normal formulation, it is assumed that the required energy is supplied by increasing the temperature, which leads to the formulation:

$$t_i = Me^{[\Delta H/kT]} \tag{3}$$

where t_i=time to failure; M is a constant that depends on the product geometry; ΔH=activation energy of the reaction; k=Boltzmann's constant; and T=absolute temperature in Kelvin.

However, the Arrhenius concept was modified by Zhurkov [as discussed in Wagner et al. (1986)] where it was assumed that both stress and temperature reduce the energy barrier. In the present analysis, stress takes the place of temperature in Eq. (3). This is not identical to Zhurkov's modified Arrehenius equation, but the idea of reducing the energy barrier with external applied stress is preserved. The rope would start to fail when the external load caused creep to reach a critical state. T would be replaced by the applied load, and a different constant equivalent to $\Delta H/k$ would apply. This would imply that the *ln*(time) taken for failure of an aramid specimen would apply inversely with the load. Thus, Arrhenius' law can be rewritten as

$$t_i = M' e^{[N/x]} \tag{4}$$

where M' is a constant that depends on the product geometry; x=external applied stress (% ABL); and N is a constant.

In the Arrhenius formulation the stress factor is conveniently given by $X=1/x$. Taking the natural log of Eq. (4) yields

$$\ln(t_i) = B_0 + B_1X \tag{5}$$

where B_0 and B_1 are coefficients.

The variations of the scale parameter can be described in the same way. As an example, consider variations of the Weibull scale parameter, α, with the external applied stress. Replacing t_i with the Weibull scale parameter, α, in Eq. (5) gives

$$\ln(\alpha) = B_0 + B_1X \tag{6}$$

Nelson (1990) and others (Meeker and Escobar 1998; Davies et al. 1999) have stated that all possible variations of the scale parameter should be considered separately. For the purposes of statistical analysis (and with no thought as to the physical justification there might be for such a form), the following variations have been used in the analysis:

- Linear: $\ln(\alpha)=B_0+B_1X$
- Exponential linear: $\ln(\alpha)=e^{(B_0+B_1X)}$
- Quadratic: $\ln(\alpha)=B_0+B_1X+B_2X^2$
- Exponential quadratic: $\ln(\alpha)=e^{(B_0+B_1X+B_2X^2)}$

Table 2. Definition of Stress Factor

Life relationship	Stress factor, X
Arrhenius	$X=1/x$
Inverse power	$X=\ln(x)$
Exponential	$X=x$

Inverse Power Relationship

An alternative to Arrhenius is the inverse power relationship, which is widely used to model product life as a function of an accelerating factor; here, it would be the external applied stress. This relationship has been applied to such materials as electrical insulation, ball and roller bearings, flash lamps, and simple metal fatigue due to mechanical loading, etc. (Nelson 1990; Meeker and Escobar 1998).

The time to failure t_i of a specimen under an applied stress x can be expressed as

$$t_i = \frac{N}{x^{\gamma}} \tag{7}$$

where N and γ=parameters characteristic of the product.

A stress factor can again be introduced, but this time takes the form $X=\ln(x)$, and as before, the Weibull scale parameter, α, can be related to stress by replacing t_i with α; taking logs of Eq. (7) gives Eq. (6) again.

Exponential Relationship

The exponential relationship is widely used for electronic components, especially in dielectrics (Nelson 1990). The exponential relationship is

$$t_i = e^{(\gamma_0 - \gamma_1 x)} \tag{8}$$

where γ_0 and γ_1=parameters characteristic of the product.

The Weibull scale parameter, α, can be fitted to the exponential variations with stress by replacing t_i with α in Eq. (8) and making the simple substitution that the stress factor $X=x$. Taking natural logs again yields Eq. (6).

Summary of Variations

Thus, by making different substitutions for the stress factor X, as shown in Table 2, all three of the life-stress relationships can be expressed in the same way [Eq. (6)]. There are four variations for the scale parameter (α) that can be considered in each life-stress relationship; these are summarized in Table 3.

Table 3. Possible Variations of Scale Parameter

Statistical variation	Variations of scale parameter
Linear (L)	$\ln(\alpha)=B_0+B_1X$
Exponential linear (EL)	$\ln(\alpha)=e^{(B_0+B_1X)}$
Quadratic (Q)	$\ln(\alpha)=B_0+B_1X+B_2X^2$
Exponential quadratic (EQ)	$\ln(\alpha)=e^{(B_0+B_1X+B_2X^2)}$

Variations of Shape Parameter

It is unreasonable to expect the shape parameter not to vary with applied stress, although there may not be a simple physical model for a particular form of that variation. In such cases, all possible variations should be considered and their suitability should be checked (Glaser et al. 1984; Nelson 1990; Meeker and Escobar 1998; Davies et al. 1999).

In this analysis, the following variations are considered. For example, the variations of the Weibull shape parameter (β) with stress factor, X, are

- Linear: $\beta = A_0 + A_1 X$
- Exponential linear: $\beta = e^{(A_0 + A_1 X)}$
- Quadratic: $\beta = A_0 + A_1 X + A_2 X^2$
- Exponential quadratic: $\beta = e^{(A_0 + A_1 X + A_2 X^2)}$

where A_0, A_1, and A_2=coefficients.

Invariance of the shape parameter ($\beta = A_0$) with stress factor is also considered.

In the lognormal distribution, the log mean μ_e takes the place of the parameter $\ln \alpha$ and the standard deviation σ_e takes the place of the shape parameter β. All the forms of variation that are applied to the Weibull case have also been applied to the lognormal case.

Different Models

The term "model" is used to describe a distribution with a possible variation of the scale and the shape parameters with stress factor, X. All the models can be placed into six categories:

1. Inverse power-Weibull,
2. Inverse power-lognormal,
3. Arrhenius-Weibull,
4. Arrhenius-lognormal,
5. Exponential-Weibull, and
6. Exponential-lognormal.

In each category, there are 20 possible variations (four with the scale $\times$ five with the shape), as described previously. Thus, 120 different models are considered in this analysis.

To clarify these ideas in detail a particular model is considered as an example; the inverse power-Weibull category using a linear variation of the scale and the shape parameters. *Inverse power* refers to the life-stress model used, while *Weibull* represents the statistical distribution assumed.

The fraction failed up to a specific time can be expressed by the cumulative distribution function. Rearranging the terms of Eq. (1) gives

$$t = \alpha\{-\ln[1 - F(t)]\}^{1/\beta} \tag{9}$$

Taking the natural log of Eq. (9) yields

$$\ln(t) = \ln(\alpha) + \frac{1}{\beta}\ln\{-\ln[1 - F(t)]\} \tag{10}$$

By assuming that $\ln(\alpha)$ and β vary linearly with the stress factor X in Eq. (10), a general relationship can be obtained

$$\ln(t) = B_0 + B_1 X + \frac{1}{A_0 + A_1 X}\ln\{-\ln[1 - F(t)]\} \tag{11}$$

where B_0, B_1, A_0, and A_1=model coefficients; and $F(t)$ represents the fraction which failed up to time t.

It will be convenient to use the single variable θ to describe the set of model coefficients (B_0, B_1, A_0, and A_1).

Once the model coefficients have been determined as described below, Eq. (11) can be used to obtain the percentile lines of a life-stress distribution by substituting values for $F(t)$ (say, 5, 50, or 95%).

Determination of Model Coefficients

Maximum likelihood theory is used to estimate the model coefficients; it is versatile, as it can be used to handle both observed (failed) and censored (nonfailed) data (Nelson 1990). For completeness, handling of both failed and singly censored data is described here. However, because the test set of data contained only completed tests, the analysis has been carried out only for failed data. As an example these are described for the inverse power-Weibull model, but similar techniques have been used for all the other models.

The likelihood of a data point is expressed using the probability density function of the Weibull distribution; the scale and the shape parameters are assumed to vary linearly with stress factor, X. Likelihood is the probability of failure at a data point.

It is assumed that the data (time to failure of each specimen) are statistically independent; therefore, the product of the likelihood of each data point gives the sample likelihood.

If the time to failure of a rope specimen is t_i, then its likelihood is given by the Weibull probability density function

$$f(t_i) = \left(\frac{\beta}{\alpha^{\beta}}\right) t_i^{\beta - 1} e^{-(t_i/\alpha)^{\beta}} \tag{12}$$

If the specimen is singly censored, its likelihood is

$$1 - F(t_i) \tag{13}$$

where $F(t_i)$=cumulative distribution function of the Weibull distribution.

The sample likelihood can be expressed as the product of two terms

$$L = \begin{cases} \prod_{i=1}^{n-m} f(t_i) & t_i, \text{ Failure data} \\ \times & \\ \prod_{i=1}^{m} 1 - F(t_i) & t_i, \text{ Singly censored data} \end{cases} \tag{14}$$

where n=total number of data points in the sample, of which m values are singly censored.

Similarly, the sample log likelihood (ψ) can be determined by taking the natural log of Eq. (14)

$$\psi = \sum_{i=1}^{n-m} \ln[f(t_i)] + \sum_{i=1}^{m} \ln[1 - F(t_i)] \tag{15}$$

Substituting the probability density function, $f(t_i)$ and the cumulative distribution function, $F(t_i)$ of the Weibull distribution

$$\psi = \sum_{i=1}^{n-m} [\ln(\beta) - \exp(z_i) + z_i - \ln(z_i)] + \sum_{i=1}^{m} [-\exp(z_i)] \tag{16}$$

where $z_i = [\ln(t_i) - \ln(\alpha_i)]\beta$.

In this particular example, the shape and the scale parameters, α and β, are both assumed to vary linearly with applied stress. Thus, four model coefficients have to be estimated (B_0, B_1, A_0, and A_1).

The maximum likelihood estimates $\hat{B}_0, \hat{B}_1, \hat{A}_0$, and $\hat{A}_1$ (denoted by $\hat{\theta}$), are the coefficient values that maximize the sample log likelihood given by Eq. (16). The corresponding maximum log likelihood value is denoted by $\hat{\psi}_{\hat{\theta}}$. These estimators can be obtained by iterative numerical optimization. This theory is used in several statistical software packages. In this analysis the package JMP has been used to determine the model coefficients and the corresponding maximum log likelihood values.

Choice between Various Models

The choice between the various models is based on an assessment of the model parameters. These parameters can be assessed by graphical or analytical methods. Graphical methods provide useful information but are subjective. Analytical methods are better, especially when several models appear to give a good fit to the data. Four such methods are available to investigate the best model:

- The likelihood ratio (LR) test,
- The likelihood ratio (LR) confidence limit check,
- The Kullback-Leibler discrepancy criterion (KLD), and
- The Akaike information criterion (AIC).

Likelihood Ratio Test

The LR test (Lawless 1982) is used when two models have to be compared and one is a special case of the other (Nelson 1990) and can only be used to choose between models with a different number of coefficients. Hypothesis tests can be used when the choice of the best model is based on the difference between the maximum likelihood values.

The model with the higher number of model coefficients is referred to as the general model; the constrained model has fewer coefficients. As an example, consider two models from the inverse power-Weibull category:

- General model: scale parameter linear, shape parameter quadratic; and
- Constrained model: scale parameter linear, shape parameter linear.

Five model coefficients are used to describe the general model, whereas in the constrained model four are used. Therefore, the general model has one degree of freedom in comparison to the constrained model.

The LR test assumes that p and p' are the number of model coefficients in the general and the constrained models, respectively, and that $p > p'$. The LR test statistic Λ is then defined (Lawless 1982; Nelson 1990) by

$$\Lambda = \frac{\text{Sample likelihood of the constrained model}, L_c}{\text{Sample likelihood of the general model}, L_g}$$

A function T is then defined which can be related to the probability density functions of the two models

$$T = -2\ln(\Lambda) \tag{17}$$

whence

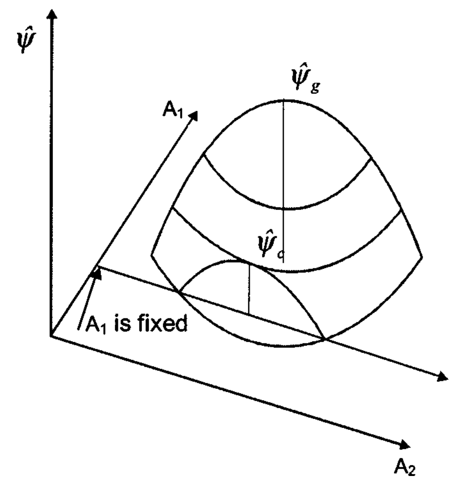

Fig. 2. Maximum log likelihood values of general and constrained models

$$T = 2(\hat{\psi}_g - \hat{\psi}_c) \tag{18}$$

where $\hat{\psi}_g$ and $\hat{\psi}_c$=maximum log likelihood values of the general and constrained-models, respectively. These can be computed numerically as described earlier.

The value $(\hat{\psi}_g - \hat{\psi}_c)$ can be represented graphically (Fig. 2). Suppose the general model has two coefficients, A_1, A_2, and its maximum log likelihood is denoted by $\hat{\psi}_g$. The plane $(A_1–A_2)$ represents the model space for the general model. The constrained model has fewer coefficients. A line in the plane $(A_1–A_2)$ represents the model space for the constrained model (Fig. 2), and its maximum log likelihood is denoted by $\hat{\psi}_c$. If the difference between $\hat{\psi}_g$ and $\hat{\psi}_c$ is less than some critical value t_α, the constrained model appears to give a good fit to data; otherwise, the general model fits the data better.

Nelson (1990) states that T can be approximated to a chi-square distribution (χ^2) if the number of failure data is large. α is then the upper confidence limit of the chi-square distribution (95 or 98%) and t_α is given by

$$t_\alpha \approx \chi^2(1-\alpha, p-p') \tag{19}$$

There is no firm rule available to define the number of failure data required to allow use of the chi-square distribution; therefore, Nelson recommends the use of a higher value for α (say, 98%) if there is a small number of observed (failed) data in the sample.

The LR test can be summarized as follows:

- If $T \leqslant t_\alpha$, accept the constrained model; and
- If $T > t_\alpha$, accept the general model.

This test is only useful when the constrained model is a subset of the general model. In the present study, a broader technique is essential, as many models are available, and the objective of this analysis is to find the best from the full range of candidate models.

Likelihood Ratio Confidence Limit Check

It is assumed here that the parameters in a model can vary linearly, quadratically, or in other ways. A confidence limit check on the likelihood ratio (LR) can be used to see whether the assumed variations fit closely to the data (Nelson 1990).

For example, assume that the Weibull scale parameter is fitted to a quadratic function

$$\ln(\alpha) = B_0 + B_1 X + B_2 X^2 \tag{20}$$

where B_0, B_1, and B_2=model coefficients.

A check is required to see whether the quadratic variation appears to give a good fit to the data. This can be done by assessing the approximate confidence limits of the model coefficient, B_2 (95% has been used in examples quoted hereafter). If the confidence limits include zero, it can be concluded there is no quadratic variation of the scale parameter; otherwise, the quadratic variation is accepted. This test is used to select the possible variations of the model parameters.

Kullback-Leibler Discrepancy

The complexity of the model increases when a larger number of parameters are present, and this can lead to overfitting the data. However, by introducing appropriate penalty forms, it is possible to compare models with a different number of parameters. Davison (2001) has outlined several types of penalty forms that can be used to penalize the redundant parameters in a model.

In contrast to the likelihood ratio test, the Kullback-Leibler discrepancy method can be used to choose between two models that have the same number of model coefficients or where one model is not a special case of the other. Using this method, the number of parameters present in a model can be sufficiently penalized so that it can be compared with other candidate models.

The operating model denotes the best model, and its probability density function is denoted by $f_0(t,X)$; this is unknown at the start of the process and indeed never needs to be defined. The lack of fit between the operating model and each of the candidate models is then compared and is termed the *expected discrepancy*. The method seeks the model with the smallest expected discrepancy.

The properties of the operating model are unknown, so it is not easy to define the expected discrepancy (Linhart and Zucchini 1986). Therefore, several empirical discrepancies are introduced based on asymptotic criteria. The Kullback-Leibler discrepancy (KLD) is one of these and is given by

$$\Delta[f_o(t,X) - f_\theta(t,X)] = \frac{\hat{\psi}_{\hat{\theta}}}{n} + \frac{\mathrm{tr}(\Omega_{rs}^{-1} A_{rs})}{n} \tag{21}$$

where

$$\Omega_{rs} = \left\{ -\frac{1}{n}\sum_{i=1}^{n} \frac{\partial^2 \ln[f_{\hat{\theta}}(t_i,X_i)]}{\partial\theta_r\,\partial\theta_s} : r,s = 1,2,\ldots\ldots,p \right\}$$

$$A_{rs} = \left\{ \frac{1}{n}\sum_{i=1}^{n} \left(\frac{\partial \ln[f_{\hat{\theta}}(t_i,X_i)]}{\partial\theta_r}\right)\left(\frac{\partial \ln[f_{\hat{\theta}}(t_i,X_i)]}{\partial\theta_s}\right) : r,s = 1,2,\ldots\ .\ p \right\}$$

θ_r and θ_s=model coefficients; n=number of data points; and p=number of model coefficients.

For each model $\hat{\psi}_{\hat{\theta}}/n + [\mathrm{tr}(\Omega_{rs}^{-1}A_{rs})]/n$ can be computed. The one that has the smallest expected discrepancy is the model which best fits the data.

An approximation to the preceding criterion is referred to as the Akaike information criterion (AIC), where the second term of the right-hand side of Eq. (21) is replaced by p/n. This eliminates the laborious calculations to determine the $[\mathrm{tr}(\Omega_{rs}^{-1}A_{rs})]/n$. The analysis is carried out here using both methods, and the results are compared.

Selection Strategy for Choice of Model

Step 1: The JMP statistical package was used to estimate the model coefficients, and the corresponding maximum log likelihood estimates for each of the 120 candidate models.

Step 2: A model was discarded if the confidence limits of one of the model coefficients includes zero, which implies that the model does not fit the data.

Step 3: For the remaining models, the minimum expected discrepancy of a model was calculated using the Kullback-Leibler discrepancy method. The value $\hat{\psi}_{\hat{\theta}}/n + [\mathrm{tr}(\Omega_{rs}^{-1}A_{rs})]/n$ is calculated for each candidate model.

Step 4: As an alternative, the AIC method was used to seek the minimum expected discrepancy. The value $(\hat{\psi}_{\hat{\theta}}/n) + (p/n)$ is calculated for each candidate model.

Results

The results for the KLD calculations are summarized in Table 4, and those for the AIC are given in Table 5. The numerical calculations for all models converged, but many were discarded, because the confidence limits for one of the coefficients included zero; only 36 remain from the original 120 for this reason.

From Table 4, the best model (exponential-Weibull; linear scale variation; exponential-linear shape variation) and the worst model (inverse power-lognormal; exponential-linear scale variation; constant shape variation) were found, according to their Kullback-Liebler Discrepancy values. Plots of mean applied stress versus log time to failure together with 5 and 95% confidence limits for the best and worst models are shown in Figs. 3 and 4, respectively, for comparison. Eq. (22) gives the 5% percentile line of the best model

$$\ln(t) = 34 - 0.25X + \frac{\ln[-\ln(1-0.5)]}{e^{(6.02-0.08X)}} \tag{22}$$

where X=applied stress (% ABL); and t=time in seconds.

A variation of the scale and the shape parameters with stress is apparent in the best model, giving apparently tight bounds on the predictions at low stress levels (albeit on a log time scale), whereas in the worst model the shape parameter is constant, giving a much broader set of confidence limits.

Confidence limits (obtained from the 95th and 5th percentile lines) of the best and the worst models at 60% ABL are summarized in Table 6. Points A and B are the confidence limits along the logarithmic time axis. Points C and D are the confidence limits of the load axis (Figs. 3 and 4). Clearly the difference between the confidence limits of the worst model is about two decades on the logarithmic time axis. This difference between the models is very significant, especially at low stress levels (below 50% ABL). The confidence range of the load is about 10% ABL.

The worst model gives longer predicted stress-rupture lifetimes, which would be desirable if true, but gives larger uncertainty, which would mean larger factors of safety would be required. In contrast, the model that is statistically best gives shorter lifetimes but less apparent uncertainty.

Table 4. Results of Kullback-Leibler Discrepancy Calculation

Model category	L, C	L, EL	L, L	EL, C	EL, EL	EL, L
	Candidate models (variation of scale and shape parameters)					
1. Inverse power-Weibull	2.3385	2.7438	2.3462	2.2087	2.7816	2.3167
	0.0754	0.0885	0.0757	0.0712	0.0897	0.0747
	15.1906	15.1037	15.1113	15.2009	15.1103	15.1168
2. Inverse power-lognormal	2.5147	2.8534	3.3102	2.4317	2.9015	3.3975
	0.0811	0.0920	0.1068	0.0784	0.0936	0.1096
	15.2005	15.1266	15.1333	15.2191	15.1365	15.1464
3. Arrhenius-Weibull	D	D	D	2.1848	2.9116	2.3167
				0.0705	0.0939	0.0747
				15.2102	15.1148	15.1181
4. Arrhenius-lognormal	2.5111	2.9326	3.4165	D	D	D
	0.0810	0.0946	0.1102			
	15.2110	15.1317	15.1424			
5. Exponential-Weibull	2.3615	2.6561	2.3597	2.2350	2.6791	2.3220
	0.0762	0.0857	0.0761	0.0721	0.0864	0.0747
	15.1817	**15.1005**	15.1099	15.1918	15.1067	15.1155
6. Exponential-lognormal	2.5221	2.7970	D	2.4379	2.8340	3.3023
	0.0814	0.0902		0.0786	0.0914	0.1065
	15.1914	15.1235		15.2077	15.1321	15.1379

Note: For each combination of category and candidate models three numbers are shown: $\mathrm{tr}(\Omega_{rs}^{-1}A_{rs})$=trace term; $[\mathrm{tr}(\Omega_{rs}^{-1}A_{rs})]/n$=penalty term; and $\hat{\psi}_{\hat{\theta}}/n+[\mathrm{tr}(\Omega_{rs}^{-1}A_{rs})]/n$=Kullback-Leibler discrepancy. D=discarded because confidence limits for one of the coefficients included zero. The model which has the minimum discrepancy is in bold text.

An important factor to note is that there is only a small difference in the KLD values for completely different models. For example, the range of discrepancy values in Tables 4 and 5 is less than 1%. There is no way of judging whether a 1% difference in the KLD values is significant. Each model is being compared with a notional operating model, whose parameters are not known and cannot be determined; this means that the values of the KLD (and similarly those of the AIC) are not in themselves important. However, the rank order of those discriminators is a valid means of determining which is the best model.

Table 7 shows a comparison between the various models, ranked in ascending order of their Kullback-Liebler discrepancy. The table shows the two values of most interest to engineering designers; (1) the mean load that would be expected to cause failure after 100 years; and (2) the value with only a 5% chance of causing failure in the same time. It is these values, and their difference (shown in the table as "Mean–5%"), which is likely to form the basis of clauses in codes of practice that limit the permanent stress that will be applied to prestressing tendons. Note that three of the models gave results where the mean load was less than the 5% load; these models have shape factors that vary linearly with stress, and at the stress levels to cause failure after 100 years, the shape factors are negative. They have thus been discounted here, despite passing the earlier validity checks.

Study of Table 7 shows that, in general, the "Mean–5%" values increase as the KLD values increase. The best results are

Table 5. Results of Akaike Information Criterion Calculation

Model category	L, C	L, EL	L, L	EL, C	EL, EL	EL, L
	Candidate models (variation of scale and shape parameters)					
1. Inverse power-Weibull	468.57	465.471	466.105	469.02	465.637	466.305
	15.2119	15.1442	15.1646	15.2265	15.1496	15.1711
2. Inverse power-lognormal	468.70	466.07	465.823	469.36	466.33	466.141
	15.2161	15.1635	15.1556	15.2374	15.1719	15.1658
3. Arrhenius-Weibull	D	D	D	469.33	465.646	466.344
				15.2365	15.1499	15.1724
4. Arrhenius-lognormal	469.03	466.15	465.999	D	D	D
	15.2268	15.1661	15.1613			
5. Exponential-Weibull	468.27	465.46	466.047	468.71	465.63	466.259
	15.2023	**15.1439**	15.1628	15.2165	15.1494	15.1697
6. Exponential-lognormal	468.41	466.03	D	469.00	466.26	465.972
	15.12068	15.1623		15.2258	15.1697	15.1604

Note: For each combination of category and candidate models two numbers are shown: $\hat{\psi}_{\hat{\theta}}$=maximum log likelihood estimation; and $\hat{\psi}_{\hat{\theta}}/n+p/n$=Akaike information criterion (AIC), where n=number of data points. D=discarded because confidence limits for one of the coefficients includes zero. L=linear; EL=exponential linear; C=constant. Example: L, EL (scale parameter varies linearly with stress; shape parameter varies exponential linear with stress). The model which has the minimum discrepancy is in bold text.

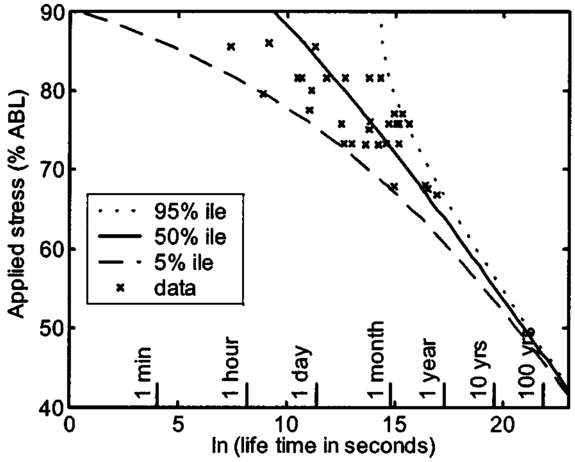

Fig. 3. Lifetime distribution of best model

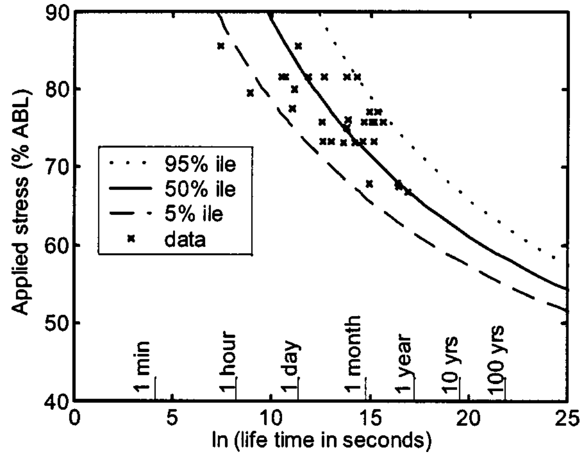

Fig. 4. Lifetime distribution of worst model

given by the Weibull distribution, and the worst results all have constant values of the shape parameter. This is perhaps not surprising, because the calculations are being carried out on log (times to failure) and it seems unlikely that there will be the same spread, when expressed as a number of decades, for loads that can be sustained for hundreds of years, as for loads that cause failure after a few seconds. Another factor that may be of note is that the Arrhenius models either failed the confidence limit check (and so are omitted from Table 4) or lie in the middle of the table. This may indicate that the mechanism of creep rupture is not best described using the modified version of the Arrhenius equation, or at least not as one that could be described by a single activation energy.

The final factor that emerges from Table 7 is that no one model emerges as a clear "best" model. Many extrapolations can be made, using standard statistical techniques, with models that give apparently good fits to the data, yet which produce predictions that vary quite significantly. The best 10 models (which are all Weibull models) give 5% values that range from 45.4% ABL to 55.8% ABL, and which give "Mean–5%" spreads that vary from 0.3% ABL to 3.8% ABL.

The reason for this is the lack of data with times-to-break beyond one year. In the absence of such data, which is very expensive to obtain, or in the absence of any accelerated testing results that can validly be used to fill the gap between 1 year and 100 years, the lack of certainty is bound to be of concern to engineers who have to certify such structures for a long time. Further accelerated testing, as described in (Alwis 2003) is essential.

Conclusion

Techniques have been evaluated for the statistical assessment of alternative models for the stress-rupture of aramid ropes, using a set of existing data as a basis for calculation. The techniques would allow the inclusion of censored data if that were available.

Arrhenius, inverse-power, and exponential life models were considered using the Weibull and lognormal statistical distributions; the scale and shape parameters following were assumed to vary linearly, quadraticly, and exponentially with stress.

The maximum likelihood method was used to determine the optimum parameters, and the 120 different models were compared using the likelihood ratio confidence limit check, and the Kullback-Leibler discrepancy criterion (KLD) or the Akaike information criterion (AIC).

It was found that the difference between the best and worst models varied by only 1% using the KLD or AIC methods.

Using these methods it was possible to predict the confidence limits of the models to determine the long-term stress-rupture behavior of aramid ropes.

It was found that the exponential model, using the Weibull distribution, with the scale parameter varying linearly and the shape parameter varying exponentially-linearly, gave the best fit to the data set. However, this is not regarded as the final word on the subject, because more tests should be carried out to obtain stress-rupture data that can then be incorporated into the current data set using the techniques described here.

No attempt has been made to provide a physical explanation as to why a particular model best fits the data.

This paper was intended only to evaluate the statistical techniques that could be used when extrapolating stress-rupture data

Table 6. Confidence Limits in Both Stress and Time from Mean Time to Failure of Rope Loaded by 60% ABL for Best and Worst Models

	Best model (exponential-Weibull, scale: L; shape: EL)		Worst model (inverse power-lognormal, scale: EL; shape: C)	
Percentile lines	Stress (% ABL)	Time (years)	Stress (% ABL)	Time (years)
5%	57.6	1.26	56.4	2.43
50%	60.0	2.79	60.0	31.66
95%	61.3	4.37	64.4	411.85

Note: L=linear; EL=exponential linear; and C=constant.

Table 7. Predicted Mean and 5% Loads To Give Stress-Rupture Failure after 100 Years, for All Models, Ranked by Their Kullback-Liebler Discrepancy Values

Physical model	Statistical model	Scale parameter	Shape parameter	KLD	Mean	5%	Mean–5%
Exponential	Weibull	L	EL	15.1005	46.519	45.572	0.947
Inverse power	Weibull	L	EL	15.1037	50.755	50.239	0.516
Exponential	Weibull	EL	EL	15.1067	50.850	50.019	0.831
Exponential	Weibull	L	L	15.1099	45.446	41.622	3.824
Inverse power	Weibull	EL	EL	15.1103	53.779	53.286	0.493
Inverse power	Weibull	L	L	15.1113	49.969	47.608	2.361
Arrhenius	Weibull	EL	EL	15.1148	55.775	55.460	0.315
Exponential	Weibull	EL	L	15.1155	49.988	47.196	2.792
Inverse power	Weibull	EL	L	15.1168	53.084	51.224	1.860
Arrhenius	Weibull	EL	L	15.1181	55.165	53.810	1.355
Exponential	Lognormal	L	EL	15.1235	49.540	48.424	1.116
Inverse power	Lognormal	L	EL	15.1266	52.970	52.283	0.687
Arrhenius	Lognormal	L	EL	15.1317	55.217	54.750	0.467
Exponential	Lognormal	EL	EL	15.1321	53.930	53.043	0.887
Inverse power	Lognormal	L	L	15.1333	55.009	49.952	5.057
Inverse power	Lognormal	EL	EL	15.1365	56.092	55.504	0.588
Exponential	Lognormal	EL	L	15.1379	54.363	55.947	[a]
Arrhenius	Lognormal	L	L	15.1424	55.550	57.651	[a]
Inverse power	Lognormal	EL	L	15.1464	56.478	57.814	[a]
Exponential	Weibull	L	C	15.1817	46.649	34.500	12.149
Inverse power	Weibull	L	C	15.1906	51.202	43.456	7.746
Exponential	Lognormal	L	C	15.1914	51.743	44.251	7.492
Exponential	Weibull	EL	C	15.1918	51.666	44.114	7.552
Inverse power	Weibull	EL	C	15.2009	54.607	49.274	5.333
Inverse power	Lognormal	L	C	15.2050	55.009	49.952	5.057
Exponential	Lognormal	EL	C	15.2077	56.377	51.978	4.399
Arrhenius	Weibull	EL	C	15.2102	56.582	52.410	4.172
Arrhenius	Lognormal	L	C	15.2110	57.163	53.215	3.948
Inverse power	Lognormal	EL	C	15.2191	58.290	54.917	3.373

Note: L=linear; EL=exponential linear; and C=constant.
[a]Mean strength is less than 5% strength.

to useful structural lifetimes. It does not provide a definitive answer for the allowable prestress in a tendon in a concrete beam, and the numerical results quoted here should not be taken as the basis for such a discussion unless further test data becomes available.

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Notation

The following symbols are used in this paper:

A_0, A_1, A_2 = coefficients;
B_0, B_1, B_2 = coefficients;
$\hat{B}_0, \hat{B}_1, \hat{A}_0, \hat{A}_1$ = maximum likelihood estimates (denoted by $\hat{\theta}$);
$F(t)$ = cumulative distribution function;
$f(t)$ = probability density function;
k = Boltzmann's constant;
M, M', N = constants;
m = number of singly censored data points in sample;
n = total number of data points in sample;
p, p' = number of coefficients in general and constrained models;
$R(t)$ = reliability or survival function;
T = absolute temperature (Kelvin);
t = time;
t_i = time to failure;
X = stress factor;
x = external applied stress;
α = Weibull scale parameter;
β = Weibull shape parameter;
$\gamma, \gamma_0, \gamma_1$ = parameters;
ΔH = activation energy of reaction;
Λ = LR test statistic;
μ_e = lognormal scale parameter;
σ_e = lognormal shape parameter;
ψ = sample log likelihood;
$\hat{\psi}_g, \hat{\psi}_c$ = maximum log likelihood values of general and constrained models; and
$\hat{\psi}_{\hat{\theta}}$ = maximum log likelihood value.

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