Shear Analysis of Concrete with Brittle Reinforcement

Tim Stratford[1] and Chris Burgoyne[2]

Abstract: The design of steel-reinforced concrete relies on lower-bound plasticity theory, which allows an equilibrium-state to be postulated without considering compatibility. This is of particular benefit in shear design, due to the complexity of shear-transfer, where simplified models such as the truss analogy are used. Lower-bound plasticity theory, however, relies on stress-redistribution. If brittle reinforcement [such as fiber-reinforced-plastic (FRP)] is used in concrete, lower-bound plasticity theory cannot be applied. This paper studies how compatibility, equilibrium, and the material constitutive laws can be combined to establish the actual conditions within an FRP-reinforced beam subjected to shear. A crack-based analysis is proposed to model shear failure in a beam with brittle reinforcement. The analysis is used to illustrate the importance of satisfying compatibility requirements, and the results are contrasted with the current shear design proposals for FRP-reinforced concrete.

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Lower-Bound Plasticity Theory

Understanding shear in concrete has always challenged researchers. Today's state-of-the-art has evolved from the large number of tests that have been conducted on steel-reinforced concrete beams (Regan 1993; Collins et al. 1996).

A detailed description of how a reinforced concrete beam carries shear is now available (Kotsovos and Pavlović 1999). This description of the shear carrying mechanisms in a beam, however, is not sufficient (on its own) to predict the shear-capacity of a beam. Shear design is instead based on simplistic models for equilibrium conditions within the beam. For example, Fig. 1 shows the truss analogy with a fixed strut angle (Mörsch 1909), or variable strut angle (Nielsen et al. 1978), compression-field theory (Collins et al. 1996), and the compressive force-path method (Kotsovos and Pavlović 1999). Each shear model assumes a different equilibrium-state within the beam; none is based on the actual stress distribution. Despite this, all the theories have been used safely to design steel-reinforced concrete. They rely on the *lower-bound* (or safe load) *theorem of plasticity*:

''If any stress distribution throughout the structure can be found which is everywhere in equilibrium internally and balances certain external loads and at the same time does not violate the yield condition, those loads will be carried safely by the structure'' (Calladine 1969).

[1]Lecturer, Institute for Infrastructure and Environment, The Univ. of Edinburgh, Crew Building, The King's Buildings, Edinburgh, EH9 3JN, UK. E-mail: tim.stratford@ed.ac.uk

[2]Reader, Dept. of Engineering, Univ. of Cambridge, Trumpington Street, Cambridge, CB2 1PZ, UK.

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The word ''any'' in this definition is most important, since it means that the designer does not need to know the actual stress distribution, and in many cases this is difficult to determine.

Lower-bound plasticity theory is relied upon wherever a simplification is made during structural analysis. For example, a plane-section analysis might be used that assumes perfect bond, or an idealized material constitutive law adopted. It is lower-bound plasticity theory that allows safe design based on these postulated equilibrium-states.

Stress-Redistribution

Fig. 2 summarizes how lower-bound plasticity theory allows a postulated equilibrium state to be used in design. An equilibrium state is postulated that carries the externally applied loads, while ensuring that the material from which the structure is made does not fail at any point. The postulated equilibrium state is inevitably based upon simplifying assumptions, and thus it does not also satisfy compatibility.

A structure, however, does not know how it was designed, and it must fit together. Hence, the actual stress distribution at the working load may not match the postulated equilibrium-state.

If the structure is ductile, internal *stress-redistribution* can occur. Stress-redistribution allows the structure to carry the load specified in design, by means of an internal stress-distribution that also satisfies compatibility. Stress-redistribution, and hence ductility, are vital if lower-bound plasticity theory is relied upon in design (as in the shear models of Fig. 1).

There has been considerable recent research into using *fiber-reinforced-plastics* (FRPs) as concrete reinforcement. FRP reinforcement is not ductile (although it may have a large strain capacity), and thus cannot contribute to stress-redistribution (Burgoyne 1997). Similarly, concrete is a brittle material, and behaves in a quasi-ductile manner only under triaxial confinement (Kotsovos and Pavlović 1999).

Large-scale stress-redistribution cannot occur in a FRP-reinforced concrete beam, or in any structure with brittle reinforcement. Without stress-redistribution, lower-bound plasticity

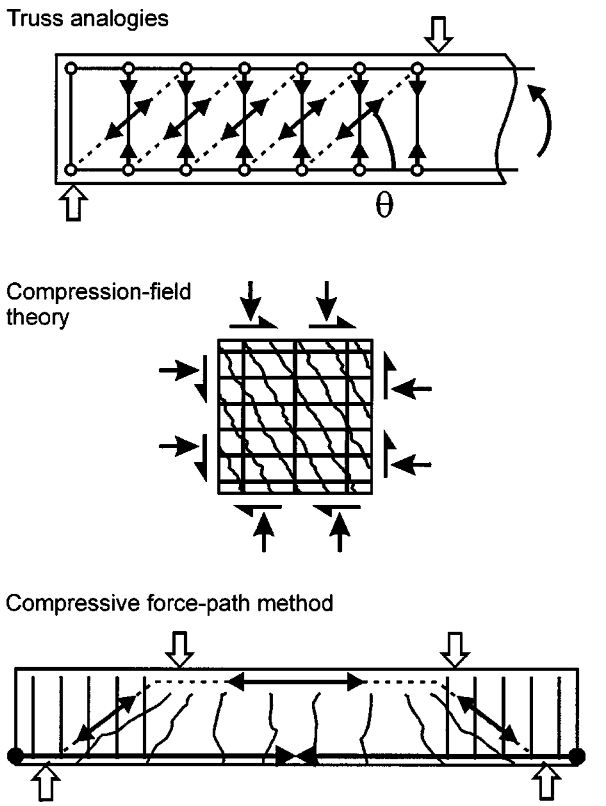

Fig. 1. Simplified models of equilibrium in shear-span of reinforced concrete beam

theory cannot be applied, and we cannot postulate an equilibrium-state without also satisfying compatibility requirements.

Shear in a Beam Without Shear Reinforcement

Analysis of an FRP-reinforced concrete beam must be based on the actual stress-state. This stress-state must satisfy both compatibility and equilibrium, which are linked by the material constitutive laws. Thus, a detailed understanding is required of the mechanisms by which shear load is carried through a beam.

Steel-reinforced concrete beams *without shear reinforcement* often fail in a brittle manner. Like FRP-reinforced concrete,

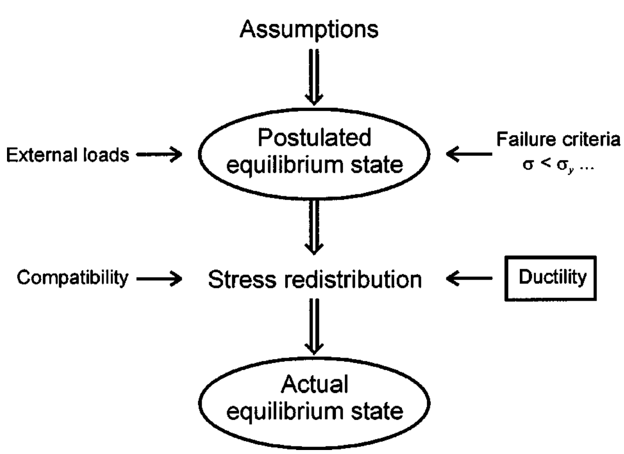

Fig. 2. Stress redistribution and lower bound plasticity theory

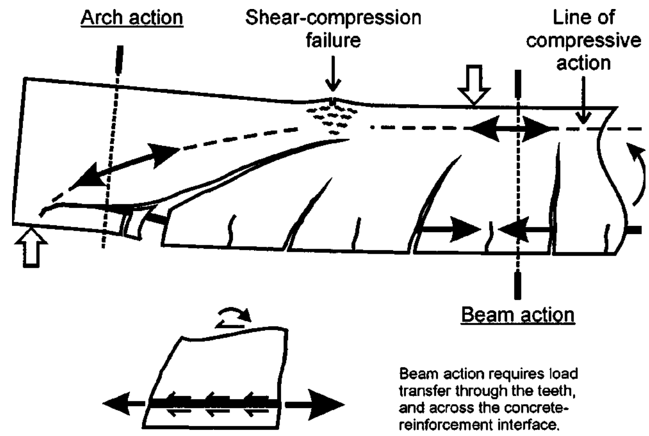

Fig. 3. Shear in beam with no shear reinforcement

lower-bound plasticity theory cannot be applied (Reineck 1991). Many researchers have examined equilibrium and compatibility in beams without shear reinforcement, resulting in a reasonably detailed picture of the internal load-carrying mechanisms (Regan 1993). This is of great help when examining shear in FRP-reinforced beams.

This paper seeks to establish equilibrium and compatibility conditions only in general terms. More details of models for steel-reinforced beams without stirrups can be found in the literature (Regan 1993; Kotsovos and Pavlović 1999). For brevity, beams with shear reinforcement are termed "*with stirrups*," while those without shear reinforcement are termed "*without stirrups*."

Shear Transfer Mechanisms

Fig. 3 is an overview of the shear mechanisms acting in a beam without stirrups. The details of these mechanisms will be discussed in subsequent sections. (To simplify discussion, only a four-point, simply supported beam is considered here).

The moment carried by a beam can be represented by an internal force couple between the compression-zone concrete and flexural reinforcement actions. For equilibrium in a shear-span, the moment must vary along the beam according to $V = \mathrm{d}M/\mathrm{d}x$. A change in moment (thus shear transfer along the shear-span), can be by one of two mechanisms (Fig. 3)

- Variation in the magnitude of the internal actions, and
- Variation in the lever-arm between the actions.

Beam Action

Beam action describes shear transfer by changes in the magnitude of the compression-zone concrete and flexural reinforcement actions, with a constant lever-arm, requiring load-transfer between the two forces (Kotsovos and Pavlović 1999).

In a cracked beam, load-transfer from the flexural reinforcement to the compression-zone occurs through the "teeth" of concrete between cracks, requiring bond between the concrete and reinforcement. Bending and failure of this concrete is studied by tooth models (Regan 1993).

Arch Action

Arch action occurs in the uncracked concrete near the end of a beam, where load is carried from the compression-zone to the support by a compressive strut. The vertical component of this strut transfers shear to the support, while the constant horizontal component is reacted by the tensile flexural reinforcement. Both beam action and arch action can act in the same region.

Equilibrium and compatibility near the end of a beam and across a single shear crack are studied by shear-compression theories (Regan 1993). Recent shear-compression models (implemented by finite-element analysis) incorporate details of the reinforcement-concrete bond, tension-softening mechanisms across the crack, and detailed analysis of the compression-zone concrete (Gustafsson and Hillerborg 1988). Shear-compression analyses have also been applied to FRP-reinforced concrete (Sato et al. 1995; Kamiharako et al. 1999).

Compatibility in the Shear-Span

Development of the tooth models and shear-compression models for steel-reinforced concrete has necessitated an examination of compatibility requirements in the shear-span of a beam.

Crack Propagation

Compatibility in the shear-span is dominated by the growth of inclined cracks through the concrete (Kotsovos and Pavlović 1999). The cracks determine how the arch and beam mechanisms carry shear load (Fig. 3), and are a fundamental part of shear failure. Crack propagation must be considered in conjunction with compatibility of each of the components of a beam.

Two distinct modes of shear failure are observed, which describe the manner in which the compression-zone concrete fails

- Shear-compression failure, and
- Diagonal-tension failure.

Shear-Compression Failure

The integrity of the compression-zone concrete relies upon triaxial confinement. If this confinement is lost, the concrete can dilate, and microcracks form in the compression-zone concrete, parallel to the top-fiber of the beam (Kotsovos and Pavlović 1999). These microcracks coalesce, resulting in shear-compression failure of the compression-zone concrete, often described as "crushing."

The degree of confinement, and hence the strain-capacity of the compression-zone, depends upon the triaxial stress-state within the compression-zone. The triaxial stress-state, however, is difficult to model (Kotsovos and Pavlović 1999; Stratford and Burgoyne 2002). Confinement in the compression-zone is reduced by shear action, but it is increased by the presence of shear reinforcement, and under a point of load application.

Diagonal-Tension Failure

The concrete immediately in front of a crack is subjected to a tension field that causes the crack to propagate diagonally into the beam. If shear-compression failure is avoided, the crack propagates along the shear-span towards the point at which load is applied. Load cannot be transferred between the compression-zone concrete and flexural reinforcement across the crack, so that beam action is not possible. An unstable diagonal-tension failure follows, which splits the beam into two pieces (Kotsovos and Pavlović 1999).

Compatibility of the Flexural Reinforcement

Failure of the compression-zone concrete is rarely the sole cause of shear failure. It is also necessary to consider compatibility of the flexural reinforcement with the concrete where it crosses a crack (Fig. 4). At the base of a shear crack, the local crack opening has both axial and shear components (with respect to the reinforcement). Compatibility of the reinforcement across a crack is achieved by a combination of

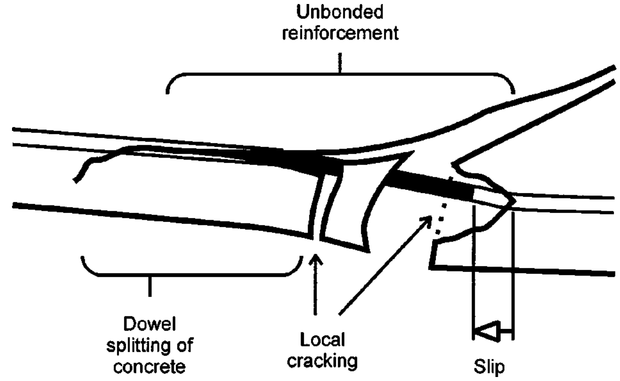

Fig. 4. Compatibility of flexural reinforcement with crack opening

- Stretching of the unbonded reinforcement, and
- Slip of the bonded reinforcement relative to the concrete (Stratford and Burgoyne 2002).

With steel reinforcement, the slip is usually assumed to be negligible compared with plastic stretching.

With FRP reinforcement, both elastic stretching and slip are important. For a given crack opening, the force in the flexural reinforcement depends upon the bond characteristics of the reinforcement, the stiffness of the reinforcement, and the unbonded length over which the reinforcement can stretch.

Reinforcement-Concrete Bond

It is known that with steel reinforcement, the strength of the reinforcement-concrete bond is a governing factor in shear failure (Kani 1964; Bažant and Kazemi 1991). If the bond is weak, the reinforcement can pull out from the surrounding concrete, usually towards the center of a beam (Kotsovos and Pavlović 1999). This destroys beam action, which relies on load transfer across the reinforcement-concrete interface.

Unbonded Length of Reinforcement

The unbonded length of reinforcement between the two surfaces of a crack is important with brittle reinforcement. For a given crack width, an increase in the unbonded length results in a reduction in the reinforcement strain, and hence the load carried by the reinforcement. To re-establish equilibrium of the beam section, the crack must propagate further into the compression-zone of the beam.

In some cases, it may not be possible to re-establish equilibrium. The crack propagation will be unstable, resulting in either shear-compression or diagonal-tension failure of the compression-zone concrete, and thus failure of the beam.

The unbonded length of reinforcement is increased by local cracking of the concrete (Fig. 4). Local cracking describes failure of the surface concrete around the reinforcement, caused by load transfer across the reinforcement-concrete interface (Kim and White 1991; Stratford and Burgoyne 2002). The length of reinforcement that becomes unbonded from the concrete can be large compared with the crack width.

Dowel-Splitting

It has been suggested that the load carried by dowel (shearing) action of the reinforcement across a crack is negligible in steel-reinforced beams (Kotsovos and Pavlović 1999). With FRP reinforcement (which has a low transverse stiffness) an even smaller load will be carried by dowel action (Kanematsu et al. 1993).

Although the load carried is negligible, dowel action can cause longitudinal cracking of the concrete along the flexural reinforcement (Kotsovos and Pavlović 1999; Sakai et al. 1999). Dowel-splitting results in a sudden increase in the unbonded length of reinforcement. As described in the preceding section, an increase in the unbonded length of flexural reinforcement can lead to unstable crack propagation into the compression-zone, resulting in failure of the beam (Stratford 2000).

Dowel Rupture

Dowel rupture is a further important mode of failure due to dowel action, which describes rupture of reinforcement under combined shear and tensile actions before its pure tensile strength is achieved. Dowel rupture does not occur with steel reinforcement. The low transverse strength of FRPs makes them susceptible to dowel rupture (Maruyama et al. 1989; Kanematsu et al. 1993; Naaman and Park 1997; Bank and Ozel 1999).

Predicting the Shear Failure Load

Ideally, the shear-capacity of a beam could be predicted by detailed examination of the shear transfer mechanisms, crack propagation, and failure of the beam components. Further research is, however, required before this is possible (Stratford 2000).

The ‘‘state-of-the-art’’ for predicting the *capacity* of a beam without stirrups is Kani’s ‘‘shear valley’’ (Kani 1964; Kotsovos and Pavlović 1999). The original (1964) ‘‘shear valley’’ concept has been refined by many researchers (Bažant and Kim 1984; Krauthammer and Hall 1982; Ahmad and Lue 1987; Krauthammer et al. 1987; Russo et al. 1991), but remains empirical. It is based on tests using steel-reinforced concrete, and thus cannot be directly applied to FRP-reinforced concrete. The ‘‘shear-valley’’ only predicts *failure*, and does not describe *compatibility*, which must be considered when brittle shear reinforcement is used.

The shear-capacity of a beam without stirrups is also found empirically in design codes (BS8110 1985; Eurocode 2 1992; ACI 1999). The code equations are based on the load at which the first shear crack forms, which can be significantly lower than the ultimate load (particularly for short shear-spans) (Kotsovos and Pavlović 1999).

Shear in a Beam with Shear Reinforcement

Shear reinforcement is used to ensure that a beam fails in flexure. As in beams without stirrups, equilibrium and compatibility must be satisfied by examination of arch and beam actions, crack propagation, and component failure. The addition of shear reinforcement affects the mechanisms by which shear is carried by a beam in a number of ways

- Shear reinforcement carries tensile actions across cracks,
- Shear reinforcement confines the compression-zone concrete, and thus increases its shear-capacity,
- Shear reinforcement encloses the flexural reinforcement and can prevent dowel-splitting of the concrete. Dowel-rupture of FRP reinforcement is, however, promoted,
- For a given applied load, equilibrium of a cracked section with stirrups requires a shorter crack length, but larger crack width, than one without stirrups. The shape of the crack will also differ, and

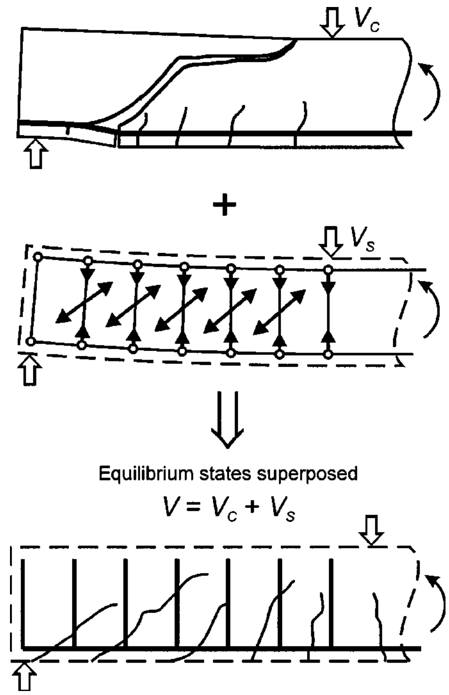

Fig. 5. Superposition of ‘‘concrete’’ and ‘‘stirrup contributions’’ using 45° truss analogy

- Concrete softening mechanisms are less effective across a wider crack; if the surfaces of a crack are completely separate, aggregate interlock cannot occur (Kotsovos and Pavlović 1999).

Shear transfer in beams with stirrups has not been examined in so much detail as that in beams without stirrups. In steel-reinforced concrete, researchers have been able to take advantage of stress-redistribution (afforded by the yielding stirrups), and apply lower-bound plasticity theory.

Truss Analogies

The truss analogies are most commonly used in design. The assumed internal equilibrium-state comprises tensile shear reinforcement and inclined compressive struts of concrete.

The original *Mörsch truss analogy* (Regan 1993; Mörsch 1909) uses a 45° strut angle and predicts failure when the shear reinforcement yields. The *modified truss analogy* (Nielsen et al. 1978; Eurocode 2 1992) establishes an optimal lower-bound for the shear-capacity by varying the compressive strut angle to give reinforcement yield and web concrete failure simultaneously (and hence uses plasticity theory explicitly).

The truss mechanism is not observed experimentally: The assumed compressive struts would have to cross curved cracks in the shear-span, even though the crack surfaces are completely separate. Furthermore, the truss analogies are sectional design methods: The shear capacity is calculated on a critical vertical section, whereas in reality failure occurs along a single crack (Kotsovos and Pavlović 1999). Both truss analogies rely on stress-redistribution from the postulated fully developed plastic truss, to the actual equilibrium-state.

Superposition of the “Concrete” and “Stirrup Contributions”

Superposition of the ‘‘concrete contribution’’ and ‘‘stirrup contribution’’ is an underlying assumption in most shear capacity analyses. For example, the ''*stirrup contribution*'' predicted using the 45° truss analogy (V_S) is often combined with a ''*concrete contribution*'' (V_C), to give the total shear capacity (Fig. 5) (BS8110 1985; Eurocode 2 1992; ACI 1999):

$$V = V_C + V_S$$

The ''stirrup contribution'' assumes continuous curvature along the shear-span. The ''concrete contribution'' is the shear-capacity of an equivalent beam without stirrups, in which the curvature at failure is concentrated at a single critical crack. Thus, the equilibrium-state postulated by superposition is not compatible, and requires stress redistribution. There is no consideration of how the stirrups are embedded in the surrounding concrete.

With brittle reinforcement, stress redistribution cannot occur, and the ''concrete'' and ''stirrup contributions'' cannot be superposed. This is illustrated by dowel-rupture of brittle flexural reinforcement: The truss analogy does not consider compatibility of the reinforcement across a crack, and hence cannot predict dowel-rupture.

Compressive-Force-Path Method

The compressive-force-path method (Kotsovos and Pavlović 1999) is based on a more realistic assessment of the capacity of a beam without stirrups than currently used in the codes (but remains empirical). Shear reinforcement is placed to prevent propagation of the critical shear crack, and is assumed to yield. The net shear-capacity is found by superposition, thus relying on stress-redistribution.

Compression-Field Theory

Compression-field theory is based on the biaxial response of square elements of steel-reinforced concrete. The original constitutive relationships were derived analytically, but these have been replaced by more realistic empirical equations (Vecchio and Collins 1986). A small number of tests have been carried out to establish equivalent constitutive equations for FRP-reinforced elements (Kanakubo and Shindo 1997; Sato and Fujii 1999). A different constitutive relationship is likely to be needed for each type of FRP, due to the considerable variation in reinforcement properties.

If the element is considered in isolation, the use of empirical constitutive relationships avoids assumptions about the internal equilibrium-state. If the element is part of a beam, however, simplifications must be made that rely on stress-redistribution. For example, a uniform shear stress is assumed through the depth of the beam (Collins et al. 1996). Furthermore, compression-field theory is a sectional design method (like the truss analogy), and shear is not a sectional failure.

Shear Design with Brittle Reinforcement

The danger of using lower-bound plasticity theory for shear design with brittle reinforcement has been noted in the literature (Burgoyne 1997; Mostofinejad and Razaqpur 1997). Despite this, the proposed shear design clauses (Canadian Standards Association 1996; Machida 1997; ACI 2001) for FRP-reinforced concrete reflect their steel-reinforced origins, and are based on truss analogies.

The writers have described an analytical investigation of compatibility requirements in the region of a shear crack (Stratford and Burgoyne 2002). This crack-based analysis examines the propagation of the crack through the concrete beam.

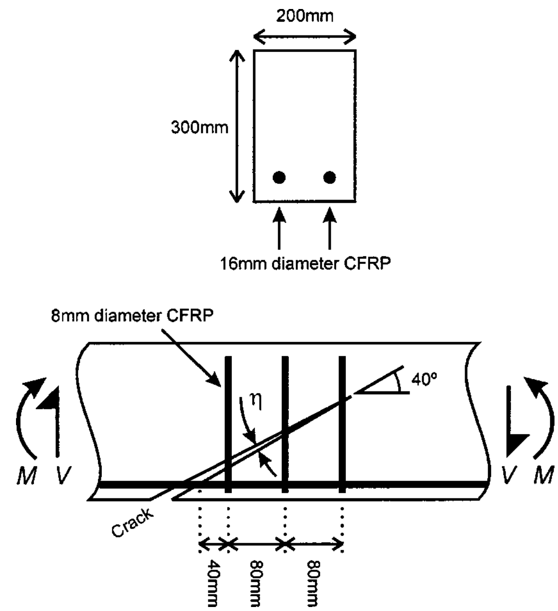

Fig. 6. Geometry of single shear crack example, showing shear reinforcement arrangement

Compatibility conditions across the cracked section are described by the horizontal and vertical projected lengths of the crack, and the crack opening angle (η in Fig. 6). The flexural reinforcement, shear reinforcement, and concrete must be compatible with the crack geometry (as described above). Constitutive laws describing stretching of the unbonded reinforcement, pull out of the reinforcement from the surrounding concrete and the response of the compression-zone concrete, allow equilibrium conditions across the cracked section to be determined.

The cracked section must be in equilibrium with the externally applied loading. The crack-based analysis determines the variation in compatibility variables that satisfies equilibrium as the crack propagates into the beam. The analysis determines the load-deflection response, and hence capacity of the cracked section.

The crack-based analysis can be extended to study multiple, curved cracks (Stratford 2000), however, additional research is required before the crack-based analysis can be used to give quantitative predictions of the shear capacity of a beam. Despite this, a single, straight shear crack model can be used to illustrate the consequences of using brittle reinforcement. An example crack-based analysis is used here to identify specific concerns with the current design proposals. [Full details of the crack-based model, and the assumptions involved in the present analysis can be found in Stratford and Burgoyne (2002)].

Action of Brittle Shear Reinforcement

Fig. 6 shows the geometry of the specimen considered in the example. A straight shear crack is analyzed, angled at 40° to the beam axis. Carbon-fiber-reinforced-plastic (CFRP) reinforcement is used for both the flexural and shear reinforcement, with three CFRP stirrups crossing the crack.

Fig. 7 gives moment-deflection responses predicted using the crack-based analysis. The moment has been normalized by the moment-capacity of the beam without shear reinforcement. The deflection is expressed by the crack opening angle. The responses

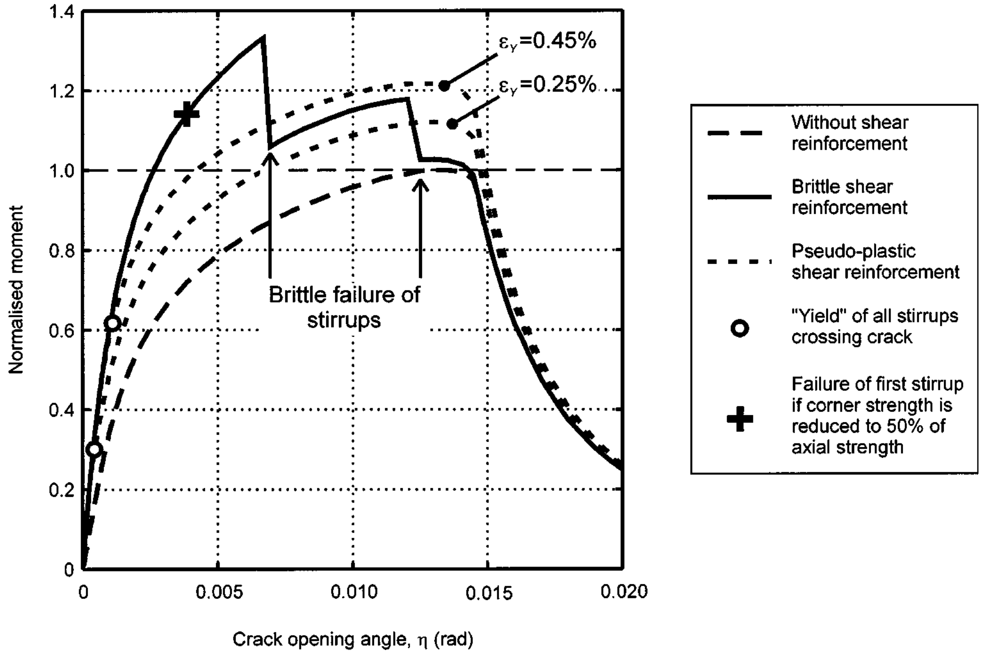

Fig. 7. Moment-deflection responses predicted by single shear crack model

of beams with brittle shear reinforcement, and without shear reinforcement are plotted. (The remaining lines, for "pseudoplastic" shear reinforcement, will be described in a subsequent section).

As expected, adding shear reinforcement increases the shear-capacity of the beam. Failure, however, is brittle, by rupture of the stirrup nearest the base of the crack (Fig. 6). Failure of the second stirrup from the base of the crack follows, giving a second (lower) peak in the moment-deflection curve. The strain in the third stirrup does not reach its rupture capacity before failure of the compression-zone concrete occurs. This stirrup is close to the crack tip, and the load that it carries does not make a significant contribution to the moment carried across the cracked section (and hence the net load carried by the beam). The remaining response is thus similar to that for a beam without shear reinforcement.

Like the flexural reinforcement, the shear reinforcement must be compatible with the local crack opening. The stirrup strain increases along the crack with the crack width, as shown experimentally by Zhao et al. 1995. The distribution of axial strain in the shear reinforcement predicted by the crack-based analysis, just before failure of the first stirrup, is shown in Fig. 8 (in which the strain is normalized by the strain in the first stirrup).

The distribution of stirrup strain along the crack depends upon the stirrup bond characteristics, and stretching of the unbonded length of reinforcement (which is increased by local concrete failure). The crack geometry and the position of the stirrups relative to the base of the crack will also affect the stirrup strain.

Current Proposals for Shear Design with Fiber-Reinforced Polymer Reinforcement

"Concrete Contribution"

The current shear design proposals for FRP-reinforced concrete [Canadian Standards Association 1996; Machida 1997; ACI 2001, and described in Guadagnini et al. (1999)] take the "concrete contribution" for steel-reinforcement and modify it by the ratio of the stiffness of FRP to steel. The stiffness of the reinforcement certainly affects the shear-capacity of the beam, but it is only one of the parameters that changes when steel reinforcement is substituted with FRP. It has not been established that it is the most important parameter.

Furthermore, the "concrete contribution" suggested in the FRP design proposals has been validated predominantly by tests on beams *without* shear reinforcement. In a beam *with* shear reinforcement, the load carried by the concrete at failure is governed by compatibility of the cracked concrete with the shear reinforcement. For example, in Fig. 7, the crack opening angle at failure of the beam with brittle stirrups is $\eta \approx 0.007$, compared

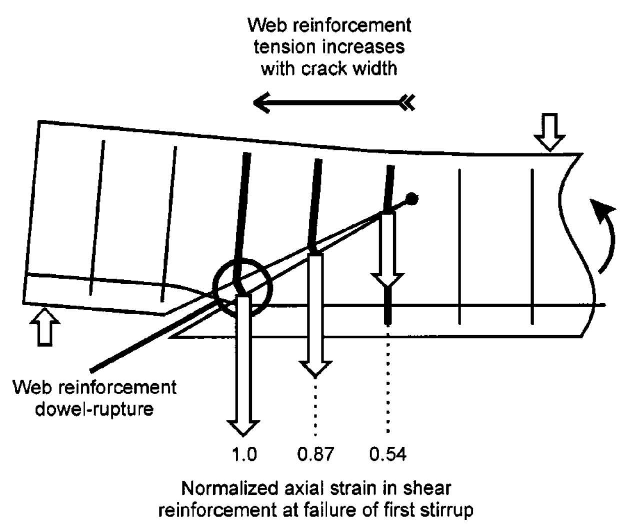

Fig. 8. Variation in shear reinforcement strain and concrete-reinforcement slip along shear crack, just before failure of first stirrup

with $\eta \approx 0.013$ for a beam without shear reinforcement. The load carried by a beam without shear reinforcement for a crack opening of $\eta \approx 0.007$ is only 85% of its shear-capacity.

It is convenient conceptually to split the shear capacity of a beam into a ''concrete contribution'' and ''stirrup contribution,'' but with brittle reinforcement the two mechanisms cannot be treated in isolation; they must be compatible. The code proposals do not recognize this.

''Stirrup Contribution''

Shear reinforcement must be effective at small crack openings, to restrain crack propagation. It must not, however, fail at a large crack opening. With steel reinforcement, lower-bound plasticity theory allows us to assume that all the reinforcement yields along a crack, and both criteria can be satisfied.

In contrast, the strain in FRP shear reinforcement varies along a crack (Fig. 6). The shear reinforcement capacity (the ''stirrup contribution'') is the shear carried by the stirrups across a crack just before the first stirrup fails. As discussed above, the distribution of stirrup strain along a crack is determined by compatibility requirements. Compatibility of the shear reinforcement with the surrounding concrete depends upon the bond characteristics of the reinforcement (the bond-stress—slip curve). Different FRP bars are manufactured with different surface finishes, and have different bond characteristics. If two beams have shear reinforcement with the same ultimate strain-capacity but different bond characteristics, the load carried by the stirrups will differ.

The code proposals for FRP reinforcement assume an artificial stirrup yield strain (the ''allowable strain'') for use in the truss analogy (Guadagnini et al. 1999). Thus, for shear design, the brittle FRP reinforcement is modeled by an imaginary pseudo-plastic FRP reinforcement, which is elastoplastic with a yield strain equal to the allowable strain.

The crack-based analysis can be used to examine the effect of assuming pseudo-plastic FRP reinforcement (Stratford and Burgoyne 2002). Fig. 7 includes two such analyses, for stirrup ''yield strains'' of $\varepsilon_Y = 0.25\%$ [suggested by the *Eurocrete* project (Clarke and O'Regan 1995)], and $\varepsilon_Y = 0.45\%$ [proposed in ''the Sheffield approach'' (Guadagnin et al. 1999)].

The shear-capacity predicted using pseudoplastic FRP reinforcement is lower than with brittle reinforcement. The pseudoplastic FRP reinforcement analysis, however, is not necessarily conservative. The brittle reinforcement analysis predicts stirrup failure at the crack (where the stirrup's strength is reduced by the combination of tensile and shear actions). It is also possible for failure to occur at the corner of a stirrup. If the corner strength of a stirrup is reduced to 50% of the straight stirrup strength (Machida 1997), the brittle-reinforced beam fails at a normalized moment of 1.15 (''+'' in Fig. 7), which is lower than the pseudoplastic FRP prediction for $\varepsilon_Y = 0.45\%$.

The pseudoplastic FRP analysis does not predict individual stirrup failure events. Furthermore, the crack-opening angle at failure of a pseudo-plastic FRP reinforced beam is much greater than that with brittle reinforcement.

The original intention of the ''allowable strain'' concept (Clarke and O'Regan 1995) was to limit the stirrup strain so that the crack width at failure was similar to that in steel-reinforced concrete, thus allowing the full ''concrete contribution'' to be developed. The crack-based analysis, however, shows that the ''allowable strain'' of the stirrups is reached at a crack opening angle of $\eta \approx 0.001$ (''●'' in Fig. 7), whereas the shear-capacity of a beam without shear reinforcement requires $\eta \approx 0.013$.

The ''allowable strain'' concept does not consider compatibility of the shear reinforcement with the cracked concrete, which the crack-based analysis shows is essential for shear design. There is no reason to suppose that a uniform limiting strain can be applied to find the net shear carried across a crack.

Summary

We Must Recognize When We Rely on Lower-Bound Plasticity Theory

The importance of lower-bound plasticity theory is rarely recognized. Both designers and researchers must recognize that the ''safety-net'' of lower-bound plasticity theory does not exist with brittle FRP reinforcement.

Whenever an assumption or simplification is made during analysis, a postulated equilibrium state is being used. With steel reinforcement, we are used to making assumptions about equilibrium conditions in a concrete beam, such as the assumption that the ''stirrup'' and ''concrete'' contributions can be superimposed in shear analysis. With brittle reinforcement, such assumptions are not safe. Large-scale stress redistribution (required, for example, by the truss analogies) is not possible. Small-scale stress redistribution *may* be possible, but assumptions will always be necessary in shear design (for example, plane sections, or simplification of the material constitutive law), and these assumptions will require small-scale stress redistribution.

A Rational Approach to Shear Design with Brittle Reinforcement

The current proposals for shear design with FRP reinforcement have been adopted in the absence of a more rational analysis. These proposals rely on stress-redistribution, which cannot occur in an FRP-reinforced beam.

A realistic model for shear in brittle-reinforced concrete must be based on a fundamental examination of equilibrium, compatibility, and the material constitutive laws in a beam. The modern understanding of shear in steel-reinforced concrete beams *without stirrups* is based on a very similar approach, and the techniques developed for those beams can be extended to analyze beams with FRP reinforcement.

Crack-based modeling (Stratford 2000; Stratford and Burgoyne 2002) is a more valid approach to analysis than current design proposals, since it considers compatibility requirements in detail. Previous research on FRP reinforcement (such as bond characteristics and dowel-rupture) is incorporated into the model. While further research is required to calibrate and verify the model (Stratford and Burgoyne 2002), it has been used in this paper to highlight the implications of using brittle reinforcement in a concrete beam.

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