

Paper

# The assessment of reinforced concrete slabs

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**Synopsis**

In the design of reinforced concrete slabs the Wood-Armer equations are used extensively. However, their direct application to assessment can result in a conservative estimate of structural capacity. Equations based on the same fundamental principles are derived which provide a more precise measure of the ability of a given slab to withstand an imposed field of moments. Application of these equations will lead, in many cases, to an improved assessment for bridges previously analysed using the Wood-Armer equations and found to require a load restriction.

**Introduction**

The Wood-Armer equations were derived for the design of reinforced concrete slabs subject to complex loadings. The equations ensure that the capacity of a slab is not exceeded in flexure by an imposed loading, whilst minimising the total amount of reinforcement required. However, the use of these equations for assessment leads to a conservative estimate of structural capacity in all cases where steel is not distributed optimally. The optimality condition is a constraint in design problems that is not relevant to assessment problems, and its use can lead to adequate structures being condemned as unsafe.

The present analysis is based on the same fundamental principles as those set out by Hillerborg<sup>1</sup>, which were extended by Wood<sup>2</sup> and Armer<sup>3</sup> in the derivation of the Wood-Armer equations, but it assumes that the reinforcement arrangement is already known. The methodology provides a systematic approach to assess whether a reinforced concrete slab has sufficient capacity to withstand an imposed loading, quantified by determination of the factor of safety on that loading.

**Loading and capacity field equations**

To maintain consistency with the Wood-Armer derivations the axis system used by Wood has been adopted and is shown in Fig 1. As a number of different conventions can be used to define bending moments it is worth emphasising that, in the following analysis, the applied bending moment  $M_x$  is about an axis perpendicular to the x-axis, so that it gives rise to stresses in the x-direction. The same convention is adopted for moments of resistance which are denoted by  $M^*$ . Thus, steel parallel to the x-axis contributes primarily to the capacity term  $M_x^*$ .

It will be observed that the convention used for moments of resistance differs slightly from that used by Wood, since the present method is concerned with analysis rather than design. Here,  $M_x^*$  is the total moment of resistance of the slab about an axis perpendicular to the x-axis, including any contribution made by reinforcement at a skew angle to the x-axis. Wood, on the other hand, used  $M_x^*$  to denote the moment of resistance needed from reinforcement parallel to the x-axis alone. For orthogonal reinforcement both conventions yield the same numerical values for  $M_x^*$  and  $M_y^*$ .

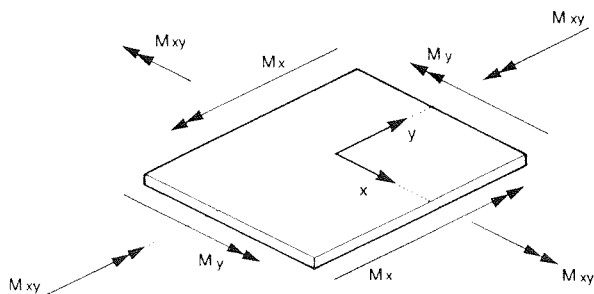


Fig 1. Notation for bending moments (positive as shown)

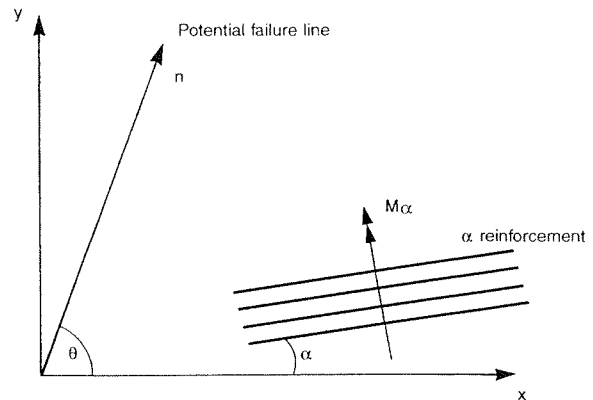


Fig 2. Relationship between the x-, n- and alpha-axes

For simplicity, moments are represented by the triad ( $M_x, M_y, M_{xy}$ ). An asterisked triad ( $M_x^*, M_y^*, M_{xy}^*$ ) will indicate moment capacity.

As defined in Fig 1, hogging moments are positive, and thus require steel primarily in the top face. Steel will be needed in the bottom face to resist negative moments. Analogous equations can be derived for other sign conventions, for both the flexural moments  $M_x$  and  $M_y$  and the twisting moment  $M_{xy}$ .

All moments in the analysis which follows will be expressed as moments/unit length, so will have units of force. It will be assumed that all sections are significantly underreinforced, so steel in the bottom face of the slab will affect only the sagging moment of resistance and will have no influence on the hogging moment of resistance, since it is adding to a compressive strength that is already more than adequate.

The flexural load effects at a point in a plane slab due to an imposed loading are fully defined by the moment triad ( $M_x, M_y, M_{xy}$ ). The bending moment  $M_n$ , about any other axis (see Fig 2), can be derived solely by equilibrium, giving:

$$M_n = M_x \cos^2 \theta + M_y \sin^2 \theta - 2M_{xy} \sin \theta \cos \theta \quad \dots(1)$$

For a single layer of reinforcement at an angle of  $\alpha$ , as shown in Fig 2, the moment of resistance about the normal to the n-axis,  $M_n^*$ , calculated by applying Johansen's stepped criterion of yield<sup>4</sup>, is given by

$$M_n^* = M_{\alpha}^* \cos^2(\theta - \alpha) \quad \dots(2)$$

The  $\cos^2$  function accounts for the effective increase in steel spacing across a skew hinge and the reduced component of steel stress acting perpendicular to the hinge. This equation has been verified experimentally (Morley<sup>5</sup>).

Eqn. (2) may be rewritten as

$$M_n^* = M_x^* \cos^2 \theta + M_y^* \sin^2 \theta - 2M_{xy}^* \sin \theta \cos \theta \quad \dots(3)$$

where

$$M_x^* = M_{\alpha}^* \cos^2 \alpha \quad \dots(3a)$$

$$M_y^* = M_{\alpha}^* \sin^2 \alpha \quad \dots(3b)$$

$$M_{xy}^* = -M_{\alpha}^* \cos \alpha \sin \alpha \quad \dots(3c)$$

It is a reasonable approximation to assume that multiple layers of reinforcement with different orientation act independently, although this is not strictly the case since the interaction of skewed layers of steel slightly alters the neutral axis depth.

When a slab contains several layers of reinforcement it is convenient to adopt a generalised form of the capacity field equation based on eqn. (3). The moment of resistance about the n-axis due to  $m$  layers of reinforcement at angles  $\alpha_1, \alpha_2, \dots, \alpha_m$  to the x-axis is therefore given by:

$$M_n^* = M_x^* \cos^2 \theta + M_y^* \sin^2 \theta - 2M_{xy}^* \sin \theta \cos \theta \quad \dots(4)$$

where

$$M_x^* = \sum_i (M_{\alpha_i}^* \cos^2 \alpha_i) \quad \dots(4a)$$

$$M_y^* = \sum_i (M_{\alpha_i}^* \sin^2 \alpha_i) \quad \dots(4b)$$

$$M_{xy}^* = -\sum_i (M_{\alpha_i}^* \sin \alpha_i \cos \alpha_i) \quad \dots(4c)$$

and

$\alpha_i$  is the angle between the 'i' layer of reinforcement and the x axis; and  $M_{\alpha_i}^*$  is the moment of resistance of the 'i' layer of reinforcement about an axis perpendicular to its direction, neglecting all other layers of reinforcement

**Comparison of loading and capacity field equations**

A slab will have adequate capacity in flexure provided the moment capacity defined by eqn. (4) exceeds the loading moment defined by eqn. (1) for all values of  $\theta$ . This condition applies to both the design and assessment of slabs. However, in design there is generally a secondary condition that the total amount of reinforcement should be minimised for economy. Wood showed that reinforcement could be optimised by constraining the loading and capacity curves to just touch at a particular value of  $\theta$ .

The capacity condition is most readily illustrated graphically. In Fig 3 the loading fields at a point due to two load cases are plotted and may be compared with the corresponding capacity field curve. Both load cases have  $M_x = 25$  kNm/m and  $M_y = 35$  kNm/m, which are less than the resistance moments, which have been taken as  $M_x^* = 30$  kNm/m and  $M_y^* = 60$  kNm/m. Comparison of the loading and resistance moments might suggest that the slab capacity is acceptable. However, this does not take account of the  $M_{xy}$  term, which is 10 kNm/m in the first case and -40 kNm/m in the second. The resistance moment  $M_{xy}^*$  has been taken as zero, which would be typical if steel were placed only in the x- and y-directions.

Load case 1 gives a moment which is everywhere adequate, but the second case exceeds the capacity over quite a large range of values of  $\theta$ . Furthermore, it causes negative moments for some values of  $\theta$ , so that steel in the opposite face must be checked. Fig 3 highlights the dangers of trying to infer whether a slab has sufficient capacity to withstand an imposed loading by comparing the moment of resistance and the bending moment about the reinforcement directions alone; instead, the resistance moment must be checked for all values of  $\theta$ .

**Development of equations for determining the factor of safety**

The approach that follows is based on the method set out by Hillerbourg<sup>1</sup> to determine equations governing reinforcement requirements in elastic

design, extended to multiple layers of skew reinforcement and modified to calculate the factor of safety on an applied field of moments. A similar approach was also used by Kemp<sup>6</sup> to determine the yield criterion for an orthogonally reinforced slab.

The derivation is applicable to slabs under the action of bending and twisting moments and does not consider the effects of membrane forces. As usual, the effects of shear forces and transverse stresses are neglected.

The aim is to formulate a systematic approach to determine whether a slab has sufficient capacity to withstand an imposed loading, as quantified by the factor of safety on the loading, which will be denoted by  $\gamma$ . In graphical terms, the factor of safety is the maximum factor which may be applied to the loading curve so that it just touches the capacity curve.

The reserve of strength is everywhere given by  $M_n^* - \gamma M_n$ . In the limit, the capacity and loading curves must touch, but not cross. From eqns. (1) and (4), therefore, there will be a particular value of  $\theta (= \theta_0)$  which must satisfy the two conditions:

$$M_n^* - \gamma M_n = (M_x^* - \gamma M_x) \cos^2 \theta_0 + (M_y^* - \gamma M_y) \sin^2 \theta_0 - 2(M_{xy}^* - \gamma M_{xy}) \cos \theta_0 \sin \theta_0 = 0 \quad \dots(5)$$

and

$$\frac{d(M_n^* - \gamma M_n)}{d\theta} = 0 \quad \dots(6)$$

If eqn.(5) is divided by  $\cos^2 \theta_0$  and  $k$  defined as  $\tan \theta_0$ , it follows that :

$$\gamma = \frac{M_x^* - 2kM_{xy}^* + k^2 M_y^*}{M_x - 2kM_{xy} + k^2 M_y} \quad \dots(7)$$

and from eqn. (6),

$$k = \frac{\gamma M_{xy} - M_{xy}^*}{\gamma M_y - M_y^*} \quad \dots(8)$$

If eqn. (8) is substituted into eqn. (7) and the result rearranged, a quadratic in  $\gamma$  results:

$$(M_x M_y - M_{xy}^2) \gamma^2 + (2M_{xy} M_{xy}^* - M_x^* M_y - M_x M_y^*) \gamma + (M_x M_y^* - M_{xy}^2) = 0 \quad \dots(9)$$

It should be noted that, for the case where  $\gamma = 1$ , eqn. (9) is an alternative formulation of the yield criterion given by Morley<sup>7</sup>.

Since eqn. (9) is a quadratic, there are two solutions for  $\gamma$ , both of which can be shown to be real.

There are two sets of resistance moments  $M_x^*$ ,  $M_y^*$  and  $M_{xy}^*$ , one for positive moments and the other for negative moments, corresponding to steel in the different faces of the slab. These need to be considered separately, although, as is demonstrated below, the same equations can be used for both cases.

**Assessment for positive reinforcement**

It is now possible to consider the range of possible solutions of eqn. (9) for positive reinforcement. Since it is a quadratic, there will be two solutions,  $\gamma_1$  and  $\gamma_2$ ; only one of these can correspond to the critical load factor. A way must be found to distinguish which is relevant.

The reserve capacity of the section is everywhere given by  $M_n^* - \gamma M_n$ . At the critical value of  $\theta (= \theta_0)$ , this will be zero. For the critical solution, it will be also be a minimum, indicating that the factored applied load is everywhere less than the capacity.

Thus the factor of safety  $\gamma$  must satisfy

$$\frac{d^2(M_n^* - \gamma M_n)}{d\theta^2} \geq 0 \quad \dots(10)$$

whence

$$1 - \gamma (M_x / M_x^*) \geq 0 \quad \dots(11)$$

Table 1 gives all five possible combinations of  $\gamma$  and  $1 - \gamma(M_x/M_x^*)$  that can arise, denoted as cases 1-5. It shows which of the two values of  $\gamma$  is the critical load factor. There are three basic forms of solution that can occur, depending on the form of the applied moment field. The significance of

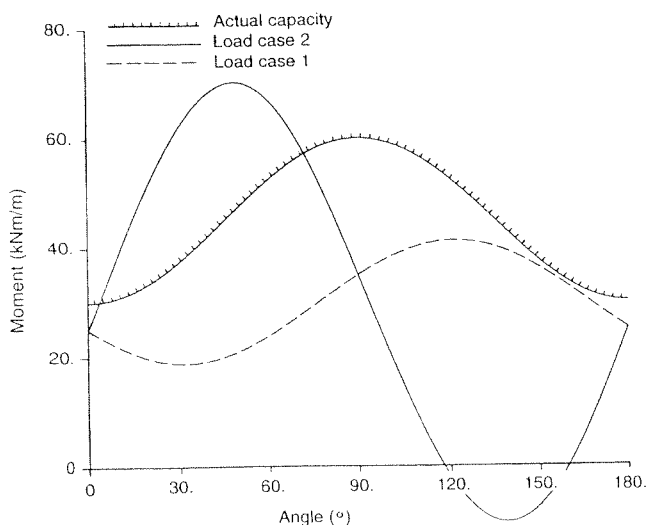


Fig 3. The variation in moment of resistance and applied bending moment for a reinforced concrete slab under two load cases

TABLE 1 – Interpretation of solutions for  $\gamma$

Case no.	$\gamma_1$	$\gamma_2$	$1 - \gamma_1 \left( \frac{M_y}{M_y^*} \right)$ or <sup>1</sup> $1 - \gamma_1 \left( \frac{M_{y\ live}}{M_{y^*} - M_{y\ dead}} \right)$	$1 - \gamma_2 \left( \frac{M_y}{M_y^*} \right)$ or <sup>1</sup> $1 - \gamma_2 \left( \frac{M_{y\ live}}{M_{y^*} - M_{y\ dead}} \right)$	Nature of applied or live loading moment <sup>2</sup>	Safety factor on live loading
Case 1	> 0	> 0	> 0	< 0	Same as capacity field	$\gamma_1$
Case 2	> 0	> 0	< 0	> 0	Same as capacity field	$\gamma_2$
Case 3	> 0	< 0	> 0	> 0	Mixed moment field	$\gamma_1$
Case 4	< 0	> 0	> 0	> 0	Mixed moment field	$\gamma_2$
Case 5	< 0	< 0	Any value	Any value	Opposite to capacity field	N/A <sup>3</sup>

NOTES:

- (1) The upper expression should be used for the interpretation of  $\gamma$  from eqn. (9) and the lower expression for the interpretation of  $\gamma$  from eqn. (17).
- (2) The applied moment field from eqn. (9), or live loading moment field from eqn. (17), is either positive (i.e. both principal moments are positive), negative or mixed.
- (3) As the applied or live loading field is the opposite sense to the capacity field it is not possible to calculate a safety factor on the applied loading.

these is best illustrated graphically, and the following three cases are examined in Figs 4(a), 4(b) and 4(c), respectively:

- purely positive moment field (i.e. both principal moments are positive)
- mixed moment field (i.e. the principal moments have opposite signs)
- purely negative moment field (i.e. both principal moments are negative)

From Fig 4(a) it can be seen that, for a purely positive applied moment field, the solutions for  $\gamma$  correspond to two cases which satisfy the conditions that the curves touch at a single value of  $\theta$ . However, only one value of  $\gamma$  results in the capacity exceeding the loading for all  $\theta$ . For the curve satisfying this requirement, the critical value of  $\theta$  corresponds to a minimum in the function  $M_n^* - \gamma M_n$ , and will therefore satisfy eqn. (11).

For a mixed moment field (see Fig 4(b)), both load curves lie below the capacity curve. However, one solution for  $\gamma$  will always be negative and therefore unacceptable.

If the applied moment field is purely negative, as in Fig 4(c), both solutions for  $\gamma$  will be negative. In this case, no positive reinforcement is required.

Assessment for negative reinforcement

The same loading case will also have to be checked against the negative moment capacity; the same argument can be used as that for positive reinforcement. However, the critical value of  $\theta$ , now corresponds to a maximum of the function  $M_n^* - \gamma M_n$  and, therefore, the acceptable solution for  $\gamma$  satisfies the equation:

$$\frac{d^2(M_n^* - \gamma M_n)}{d\theta^2} \leq 0 \quad \dots(12)$$

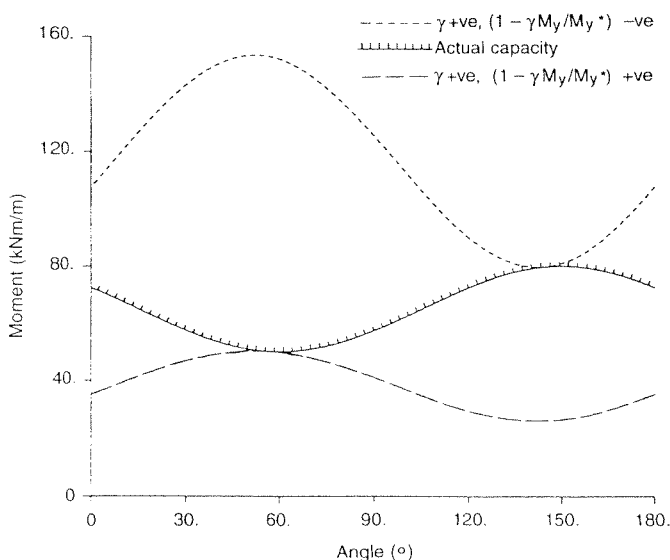


Fig 4(a). Solutions for  $\gamma$  from eqn. (9) with a positive applied field of moments

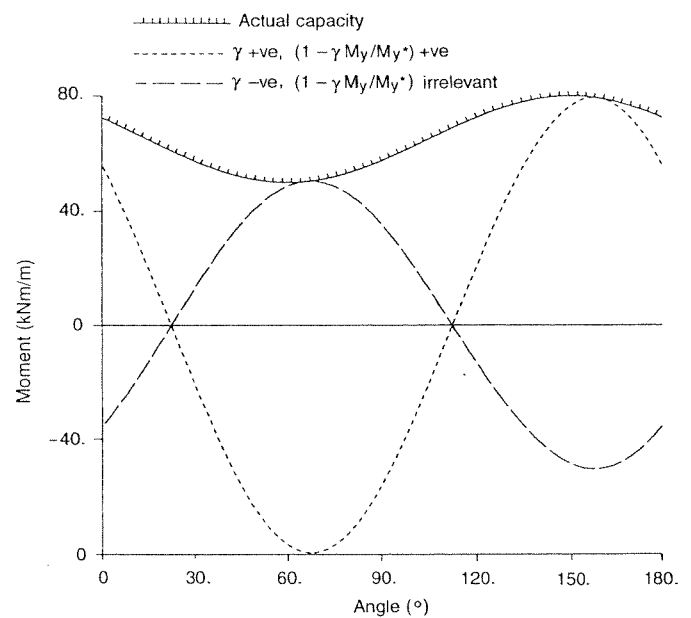


Fig 4(b). Solutions for  $\gamma$  from eqn. (9) with a mixed applied field of moments

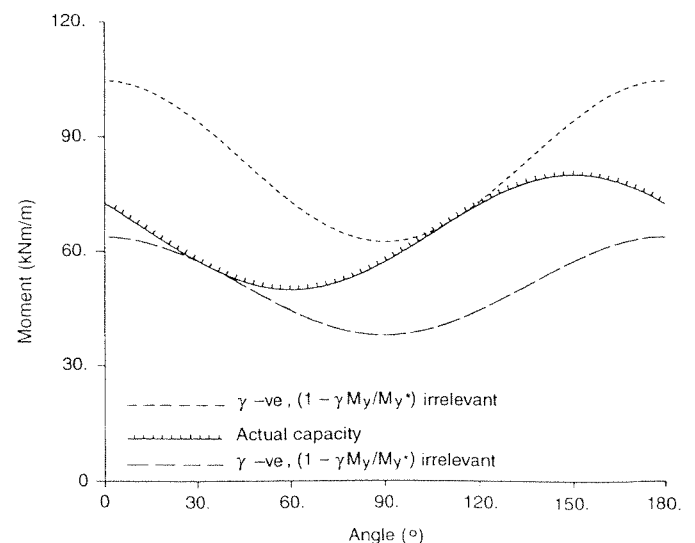


Fig 4(c). Solutions for  $\gamma$  from eqn. (9) with a negative applied field of moment

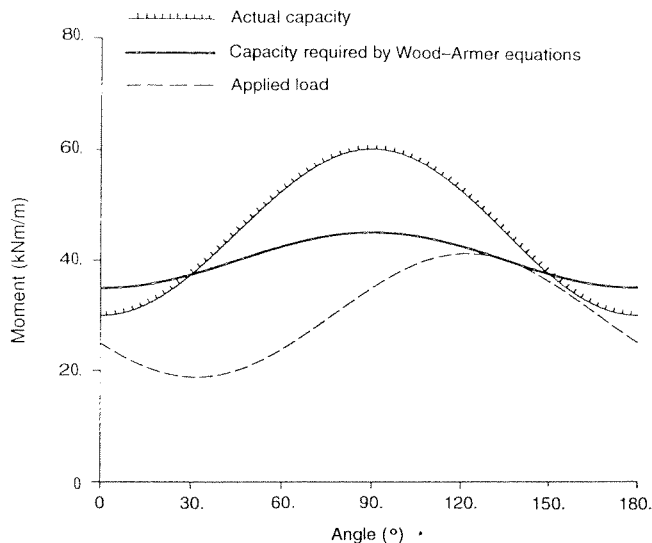


Fig 5. Resistance moment field calculated using the Wood-Armer equations

After rearrangement, and taking account of the fact that  $M_y^*$  is negative, it follows that:

$$1 - \gamma(M_y / M_y^*) \geq 0 \quad \dots(11 \text{ bis})$$

It is convenient that this criterion is identical with that for positive reinforcement (see eqn. (11)), so Table 1 can be used unchanged, and the principles illustrated in Figs 4(a)-4(c) still apply.

**Special cases**

It can be shown that, for values of  $M_x^*$ ,  $M_y^*$  and  $M_{xy}^*$  where,

$$M_x^* M_y^* - M_{xy}^{*2} \geq 0 \quad \dots(13)$$

all solutions for  $\gamma$  will be real. Eqn. (13) expresses the requirement that the capacity curve must not cross between the hogging and sagging regions, and will always be satisfied if  $M_x^*$ ,  $M_y^*$  and  $M_{xy}^*$  are calculated using eqns. 4 (a), 4 (b), and 4 (c).

When a slab contains only a single direction of reinforcement, eqn. (13) is satisfied identically, and it is interesting to note that if, in addition, the loading field is purely of the same sign as the capacity field, the only admissible solution for  $\gamma$  is zero.

Eqn. (9) holds for all values of  $k$ , including  $\theta_n = 0^\circ$  and  $\theta_n = 90^\circ$ ; these cases occur when  $M_x M_{xy}^* = M_x^* M_{xy}$  and  $M_y M_{xy}^* = M_y^* M_{xy}$ , respectively. If both these conditions hold,  $\theta_n$  is undefined, and eqn. (9) becomes a perfect square yielding a single solution for  $\gamma$ . In this case,

$$\frac{M_x}{M_x^*} = \frac{M_y}{M_y^*} = \frac{M_{xy}}{M_{xy}^*} \quad \dots(14)$$

which corresponds to the load curve being an exact multiple of the capacity curve.

**Comparison with the Wood-Armer equations**

In the analysis given above, the load and capacity equations touch (for the critical value of  $\gamma$ ) at  $\theta$ , which can take any value. In design, where the capacity is not yet known, Wood showed that the minimum amount of orthogonal reinforcement is required if  $\theta$  is either  $45^\circ$  or  $135^\circ$ . The effect of this can be shown by considering the first load case from Fig 3, the moment triad (25, 35, 10), which has already been shown to be adequately resisted by the capacity (30, 60, 0)\*.

The Wood-Armer equations lead to design capacities or 'reinforcement moments' of 35 kNm/m and 45 kNm/m for reinforcement parallel to the x-axis and y-axis, respectively. The total amount of reinforcement (which Wood assumes to be proportional to the sum of the design capacities) is higher for the actual reinforcement (= 90) than for the Wood-Armer equations (= 80), but this is an irrelevant consideration when checking the adequacy of the section.

The variations in actual capacity, applied moment and the capacity field generated by the Wood-Armer equations are shown in Fig 5.

**Example 1**

The procedure for assessing a slab is shown as a flowchart in Fig 7.

Suppose that a slab is being checked which has the reinforcement arrangement shown in Fig 6, with the moment of resistance of the reinforcement parallel to the x-axis alone equal to 100 kNm/m and that of the skew reinforcement alone equal to 35 kNm/m. It follows from eqns. (4a),(4b) and (4c), that

$$\begin{aligned} M_x^* &= 100.\cos^2 0 + 35.\cos^2 70 &= 104.09 \text{ kNm/m} \\ M_y^* &= 100.\sin^2 0 + 35.\sin^2 70 &= 30.91 \text{ kNm/m} \\ M_{xy}^* &= -(100.\sin 0.\cos 0 + 35.\sin 70.\cos 70) &= -11.25 \text{ kNm/m} \end{aligned}$$

and hence, using the resistance triad notation,

$$M^* = (104.09, 30.91, -11.25) \text{ kNm/m.}$$

Suppose, further, that the applied moment field, as determined by a suitable analysis method such as finite elements, comprises

$$M = (35, 15, 10) \text{ kNm/m.}$$

Then, from the solution of eqn. (9),

$$\begin{aligned} \gamma_1 &= 5.402 \\ \gamma_2 &= 1.346 \end{aligned}$$

so that

$$\begin{aligned} 1 - \gamma_1(M_x/M_x^*) &= -1.622 \\ 1 - \gamma_2(M_x/M_x^*) &= 0.347 \end{aligned}$$

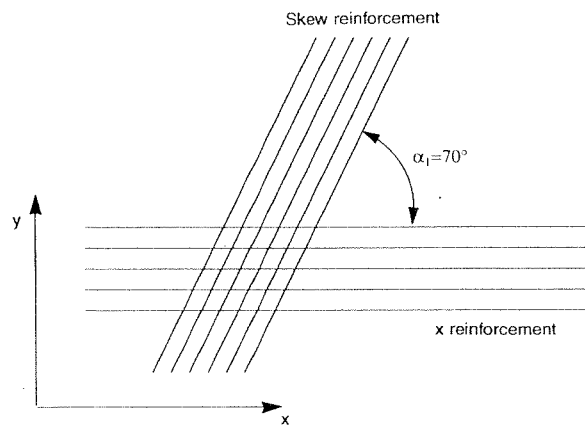


Fig 6. Skew reinforcement

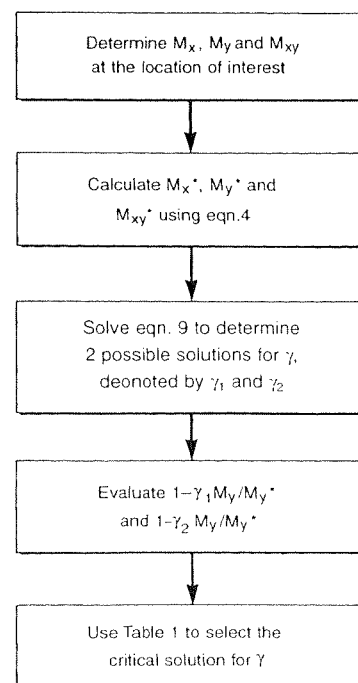


Fig 7. Procedure for assessment under a single applied moment field

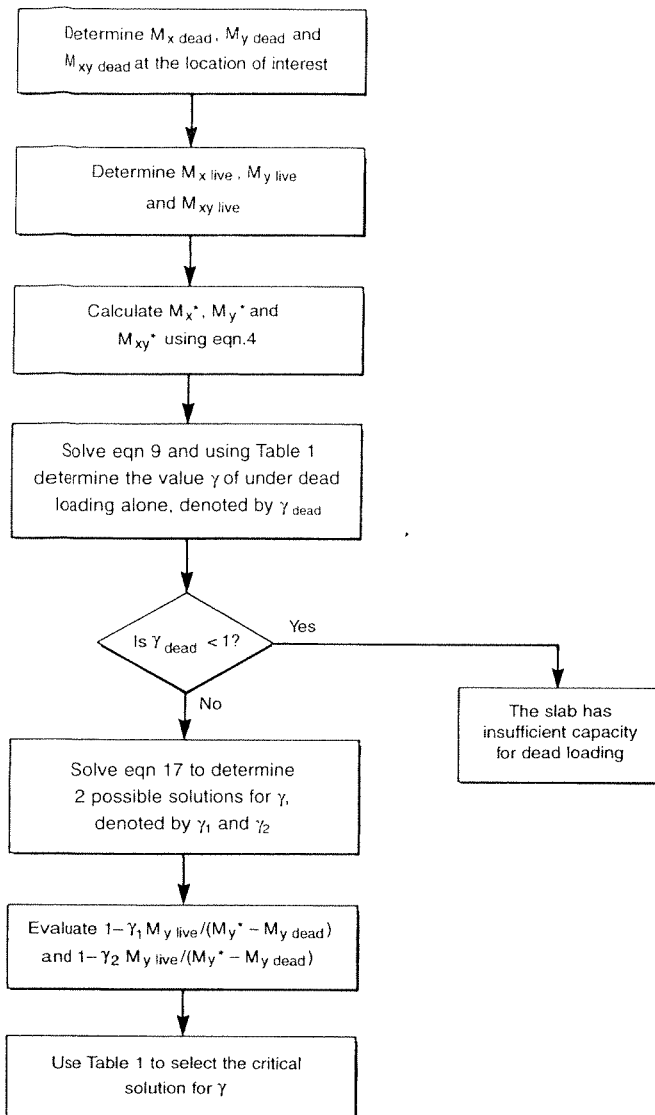


Fig 8. Procedure for assessment under combined dead and live loading

From Table 1, this can be identified as case 2, so the safety factor on the applied loading is  $\gamma_2 (=1.346)$ .

For the same applied moment field, the reinforcement moments calculated using the Wood-Armer equations are 60.7 kNm/m for the reinforcement parallel to the x-axis and 33.4 kNm/m for the skew reinforcement.

If the value of the skew reinforcement moment calculated using the Wood-Armer equations is compared with the actual moment of resistance of the skew reinforcement alone, the resulting factor of safety is 35/33.4 (= 1.04). Thus an improvement in the assessed capacity of approximately 30% is achieved through the use of the present approach in this case.

**Assessment under dead and live loading**

When a slab is assessed to determine whether it has sufficient capacity to withstand some additional loading or when a slab is subjected to a combination of dead and live loading, it is often more informative to calculate the factor of safety on the live (or additional) loading after the full dead (or permanent) loading has been applied. This assessment requires two stages – the first to ensure that the structure can withstand the dead load and, if it passes that test, a second analysis to see how much live load can be carried.

The first analysis can be undertaken by the method given above, but a modification is required for the second analysis. In this case, the dead load moments have to be taken into account. This can be done by subtracting the dead load moments from the load capacity, to give the load capacity available for live load moments.

$$\text{Thus, } M_n^* \text{ live} = M_n^* - M_n \text{ dead}$$

Although this is the principle of the revised analysis, it is convenient not to have to calculate the live load capacities directly. Instead, an approach

similar to that used above can be developed. It then follows that, at the critical angle  $\theta_0$ ,

$$M_n^* - M_n \text{ dead} - \gamma M_n \text{ live} = (M_x^* - M_x \text{ dead} - \gamma M_x \text{ live}) \cos^2 \theta_0 + (M_y^* - M_y \text{ dead} - \gamma M_y \text{ live}) \sin^2 \theta_0 - 2(M_{xy}^* - M_{xy} \text{ dead} - \gamma M_{xy} \text{ live}) \cos \theta_0 \sin \theta_0 = 0 \quad \dots(15)$$

and

$$\frac{d(M_n^* - M_n \text{ dead} - \gamma M_n \text{ live})}{d\theta} = 0 \quad \dots(16)$$

where  $M_x \text{ dead}$ ,  $M_y \text{ dead}$  and  $M_{xy} \text{ dead}$  define the dead or permanent loading field and  $M_x \text{ live}$ ,  $M_y \text{ live}$  and  $M_{xy} \text{ live}$  define the live or additional loading field.

As before, these conditions can be rearranged to give a quadratic in  $\gamma$

$$\left\{ M_x \text{ live} M_y \text{ live} - (M_{xy} \text{ live})^2 \right\} \gamma^2 + \left\{ 2M_{xy} \text{ live} (M_{xy}^* - M_{xy} \text{ dead}) - M_y \text{ live} (M_x^* - M_x \text{ dead}) - M_x \text{ live} (M_y^* - M_y \text{ dead}) \right\} \gamma + \left\{ (M_x^* - M_x \text{ dead}) (M_y^* - M_y \text{ dead}) - (M_{xy}^* - M_{xy} \text{ dead})^2 \right\} = 0 \quad \dots(17)$$

which has two solutions  $\gamma_1$  and  $\gamma_2$ . The criterion for selecting the correct value for  $\gamma$  is similar to that for a single applied moment field and is governed by the equation,

$$1 - \gamma (M_y \text{ live} / (M_y^* - M_y \text{ dead})) \geq 0 \quad \dots(18)$$

Table 1 may be used to identify the required value of  $\gamma$ , and the procedure for assessing a slab for live loading is shown in Fig 8.

**Example 2**

Suppose that the loads applied to the slab in example 1 represented the dead loading, so that

$$M_{\text{dead}} = (35, 15, 10) \text{ kNm/m}$$

and that the live loads, also determined by a suitable (but here unspecified) analysis technique, are

$$M_{\text{live}} = (6, 4, 5) \text{ kNm/m.}$$

It was shown in the first example that the safety factor was greater than one; the slab therefore has some capacity available for live loads.

From the solution of eqn. (17), it follows that

$$\gamma_1 = -585.4$$

$$\gamma_2 = 1.106$$

so that,

$$1 - \gamma_1 (M_y \text{ live} / (M_y^* - M_y \text{ dead})) = 148.22$$

$$1 - \gamma_2 (M_y \text{ live} / (M_y^* - M_y \text{ dead})) = 0.722$$

From Table 1, this can be identified as case 4, so the safety factor on the applied loading  $\gamma_2 (= 1.106)$ . The slab therefore has sufficient capacity to withstand the combined live and dead loading.

For the same applied moment fields, the reinforcement moments calculated for the skew reinforcement using the Wood-Armer equations are 33.4 kNm/m for the dead load alone and an additional 11.4 kNm/m when the live load is added. If these values are compared with the actual skew reinforcement capacity, the resulting factor of safety on live loading is (35-33.4)/11.4 (= 0.14), which is clearly inadequate. Thus, whilst the use of the Wood-Armer equations suggests that the slab only has sufficient capacity to withstand 14% of the live loading in combination with the dead loading, the present analysis demonstrates that the slab can withstand the full combined loading. There would be no need to take remedial action for this slab.

**Conclusions**

The Wood-Armer equations, originally derived for design purposes, provide a conservative assessment of the capacity of a reinforced concrete slab because of their use of an optimality condition. However, by adopting the

present alternative methodology based on the same fundamental principles, a more accurate assessment of the structural capacity under an imposed field of moments can be achieved. In most cases, this approach will lead to a higher assessed capacity for bridges previously analysed using the Wood-Armer equations and found to require a load restriction.

#### Acknowledgements

The authors wish to acknowledge the contribution of Miss R. Lingwood (Department of Engineering, University of Cambridge) in developing succinct forms of the assessment equations and the assistance of Rust Consulting Ltd, in particular Mr T.J.C. Christie.

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# The Institution of Structural Engineers Benevolent Fund



A company limited by guarantee number 3087463  
Registered charity number 1049171

#### Notice is hereby given

that the first Annual General Meeting of members will be held at 11 Upper Belgrave Street, London SW1X 8BH on Thursday 23 May 1996 at 6:15 pm (or immediately following the conclusion of the Annual General Meeting of the Institution of Structural Engineers at 6:00 pm).

The agenda is given below.

By order of the Trustees  
H S KITCHING, FCIS, FInstAM, MIMgt  
Secretary  
7 May 1996

#### Agenda

The Chairman of the Board of Trustees to take the Chair.

1. To read the notice convening the meeting.
2. To receive the Trustees' review of activities and unaudited financial statements 1995.
3. To appoint Auditors for the ensuing year. (The Trustees recommend the reappointment of Wheawill & Sudworth, chartered accountants and registered auditors, at a fee to be agreed with them by the Trustees.)
4. To determine the number of Trustees. (The Trustees recommend that this is fixed at 9.)
5. To appoint Trustees. (In accordance with Article 9.1, all the Trustees shall retire from office at the first Annual General Meeting: Professor P J Dowling, Mr S G Evans and Dr H P J Taylor are not seeking reappointment; Mr J M Allen and Mr B W

Cooper are willing to be appointed; the Trustees accordingly recommend the appointment of

Mr John Michael Allen  
Ms Carol Elaine Bailey  
Professor Leslie Arthur Clark  
Professor Anthony Ralph Cusens  
Mr Bryan Walton Cooper  
Mr Aderemi Oladipupo Ogundehin  
Dr John Maxwell Roberts  
Mr Brian Simpson  
Mr Jack Arthur Waller.)

*Copies of the Trustees' review of activities and unaudited financial statements 1995 may be obtained on application to the Secretary, the Institution of Structural Engineers Benevolent Fund, 11 Upper Belgrave Street, London SW1X 8BH.*