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Statistical variability in the strength and failure strain of aramid and polyester yarns

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The scatter in the failure strain, load and stress of high-tenacity polyester and aramid yarns is studied experimentally. From the data, the failure strains of polyester and aramid yarns can be fitted to a two-parameter Weibull distribution. However, the log–log dependence of the strain on the gauge length is best represented by the Watson–Smith modification. Whereas the strengths of polyester yarns are best described by the two-parameter Weibull distribution, those of aramid yarns are best represented by the Gumbel distribution. The effect of strain rate on the strength distribution of aramid yarns is also examined. The strength of aramid yarns decreases slightly with an increase in the strain rate. This is contrary to theoretical predictions but in line with other test data.

Nomenclature

- H Cumulative distribution function
- H_k Sample cumulative distribution function
- l Length
- ℓ Load
- m Number of sub-bundles
- $\mathcal{N}$ Number of yarns
- R Strain rate
- α Constant associated with exponential-law breakdown rule
- α_0 Constant Weibull scale parameter
- β Location parameter for Gumbel distribution
- γ Euler's constant
- δ Scale parameter for Gumbel distribution
- η Constant associated with power-law breakdown rule
- κ Shape parameter associated with breakdown rules
- λ Breakdown rule
- μ_c Mean strength of a sub-bundle
- ν Length constant associated with Weibull distribution
- ρ Weibull shape parameter
- σ Failure stress
- σ_c Standard deviation of strength for a sub-bundle
- ϕ Constant associated with power-law breakdown rule
- ψ Constant associated with exponential-law breakdown rule

1. Introduction

Polymers and brittle materials used in many engineering applications exhibit a large scatter in their mechanical properties. As a consequence, the value of the mean as a design parameter is lost and the exact material properties are difficult to predict.

The need to understand the variability in the mechanical properties has received considerable attention. The scatter is attributed to the fact that brittle materials contain flaws which weaken them [1]. The study of fibre strength is often made with the assumption that a fibre consists of an aggregate of links; the fibre therefore fails with the failure of the weakest link [2–5]. This can also be formulated in terms of the statistics of extremes [2, 6]. From the weakest-link concept, if a fibre is imagined to consist of links of n small volume elements, where each element has a probability of failure H_1 under a specific load, then the probability of failure for the whole fibre, H_v, is given by $H_v = 1 - (1 - H_1)^n$.

The conventional Weibull distribution [3] assumes a constant fibre diameter and gives the cumulative distribution function of strength, z, as

$$H_1(z) = 1 - \exp\left[-l\left(\frac{z}{a}\right)^{\omega}\right] \quad (1)$$

where a and ω are material constants and l is the fibre length. The term $(z/a)^{\omega}$ was chosen for mathematical convenience [3]; in general any monotonically increasing positive function would suffice.

The conventional Weibull form has proved to be useful, although not very satisfactory, and engineers have used it to explain the strength behaviour of materials which are believed to exhibit flaws [7–10].

The basic weakness of the conventional Weibull-/weakest-link model is its inability to deal satisfactorily with the size effect [7, 11–14]. This has led to the introduction of other variants such as the multimodal and the volume versions [15–18].

Watson and Smith [13] tried to reconcile experimental values with the Weibull/weakest-link model and gave an explanation which is based on the random variation in the fibre diameter. By using a limit theorem they modified the dependence of the strength on the length by raising the length l in Equation 1 by a power. The physical significance of the power is unclear, but the modification does resolve the length effect for long fibres. Although the modification is based on diameter variability, it does not address the transverse size effect. Knoff [19] also introduced a modification into the Weibull model to deal with the length effect. His model works for fibres at short lengths, but for long fibres it becomes equivalent to the conventional Weibull model.

The strength of polymeric fibres are also known to depend on the strain or extension rate during tensile tests, but the response differs from one fibre to another. Aramid fibres are reported to be relatively insensitive to the strain rate [16, 20], while high-strength polyethylene fibres exhibit a pronounced strain-rate effect [16]. Carbon fibres show different behaviour at low and high strain rates. At a low strain rate the strength of carbon fibres is reported to increase slowly with the strain rate whereas a strong decrease is observed at high strain rates [20].

Normally, the stochastic concept for the failure of single fibres is used to predict the effect of the strain rate on the strength of fibres. The two models used, based on the power-law and the exponential-law breakdown rules of fibres, are not easily differentiated. Whereas the former gives the relation $\sigma = R^{1/(1+\eta)}$ where σ is the failure stress, R the strain rate and η a positive material constant, the latter gives $\sigma = \ln R$. In both cases the strength is expected to increase with strain rate. η is known to be large for many fibres and the dependence of strength on strain rate turns out to be mild.

The theory has failed to resolve the dependence of the strength on the strain rate for some materials. As pointed out by Wagner *et al.* [20], this may be attributed to the fact that different mechanisms of failure may dominate at different strain rates.

A yarn is usually described as a group of fibres which are assembled with or without twist. Parallel-lay ropes are normally made from twisted aramid or polyester yarns and are used in many civil engineering projects [21–23]. The behaviour of the ropes is complex and requires analysis from the constituent elements. Although yarns are made from fibres, and data from fibres are available [7, 12, 17, 24], there are inconsistencies in the yarn properties. These inconsistencies may be due to the inherent scatter in the fibre characteristics combined with the sensitivity of the fibres to various measuring techniques. Experimental data on the mechanical behaviour of raw aramid and polyester yarns are lacking and there is a need for such data. In this paper the short-term properties of Kevlar 49 aramid and high-tenacity polyester yarns are investigated.

Parafil ropes are made by Linear Composites Ltd; they contain a core of parallel yarns, with a slight degree of twist, within a non-structural polymeric sheath. A variety of fibres can be used, but those of interest here are the Type A ropes, which have polyester yarns in the core, and Type G ropes which have Kevlar 49 aramid yarns. The fibres tested here were all taken from the cores of such ropes, unless stated otherwise.

2. Experimental procedure

Kevlar 49 aramid and high-tenacity polyester yarns were removed from parallel-lay ropes to determine the statistical distribution of their short-term properties. Each yarn thus came from a different spool. The failure strain was considered to be the strain at which the load dropped significantly during a test. This could be dependent on the frequency with which the logging program sampled the force and the manner in which tests were carried out; however, a drop of 50% of the force between two successive strain readings proved to be a reliable criterion.

A Howden tensile testing machine was used for the experiments and was operated under strain control. The yarns were clamped by pneumatically operated jaws and the nominal gauge length was found by measuring the distance between the clamps of the jaws with a ruler. The data were recorded directly by a Schlumberger Solatron SI3531D Orion data acquisition system.

Yarns from a 5 tonne Parafil Type A (polyester) rope and two different batches of 6 tonne Type G (Kevlar 49) rope were carefully separated from the rope sheath with minimum disturbance. This was done to make sure that the original twists of the yarns were maintained. The yarns from the two batches of the Type G rope were labelled as batch 1 and 2. Each yarn was assigned a number and a list of random integers was generated to select the specimens for the tests.

Each specimen was weighed with an Oertling chemical balance which is capable of measuring to 0.001 g and the lengths of the yarns were measured with a metre rule. This allowed the nominal linear density to be obtained. Ten aramid yarns were weighed and put into a Townson and Mercer electronic oven at a temperature of 120 °C for half an hour to remove the moisture absorbed by the yarns. They were then conditioned for 16 h at 55% relative humidity and 24 °C in a Tenney humidity cabinet. The samples were weighed again and the difference in the two weights was used to adjust the value for the linear density of all the Kevlar 49 aramid yarns. This conditioning was in accordance with the manufacturer's specification [25]. The conditioning was not carried out for the polyester yarns.

A separate study of the effect of the jaws on the gauge length of the specimens was carried out. This is reported elsewhere [26].

Tensile tests at four different effective gauge lengths of 676, 566, 466 and 370 mm for batch 1; 633, 523, 423 and 327 mm for batch 2 of Kevlar 49 aramid yarns, and 497, 397, 297 and 197 mm for polyester yarns were performed. Loads were applied at a strain rate of 10% min^{-1} for all the tests involving the aramid yarns except the tests at 676 mm gauge length, where a strain rate of 6.3% min^{-1} was used. Tests involving the polyester yarns were carried out at a strain rate of 35% min^{-1}.

Two batches of non-twisted Kevlar 49 aramid yarns, taken directly from the manufacturer's spools, were tested at five different strain rates to study the effect of strain rate on the yarns at an effective gauge length of 560 mm.

3. Analysis of yarn data

The failure forces, stresses and strains of the yarns were fitted to normal, log-normal, Weibull and Gumbel distributions and the best distribution was chosen for each case. Full details of the various distributions can be found elsewhere [27]. It was conjectured, however, that the linear densities would follow the normal distribution so the linear densities were fitted to the normal distribution only. The Kolmogorov–Smirnov (K–S) test was used to select the best fit for the yarns, and a computer program based on the maximum likelihood [27] method was written to determine the parameters for the Weibull and Gumbel distributions.

The goodness-of-fit was used to determine the best distributions for the yarn characteristics because a non-parametric test was desired. Non-parametric tests have the advantage of being independent of the forms or the values of the parameters in the distributions. The K–S test was chosen instead of the well known χ^2 test because the latter requires the grouping of data which might result in a loss of information. The K–S test also detects smaller deviations in the cumulative distribution which the χ^2 test might miss [28].

In the analysis, a significance level of 5% was chosen. For the sake of consistency, apart from the distributions rejected at this level, the distribution which was found to be valid for test data of a particular yarn at all gauge lengths and with the smallest D [28] value was selected to represent each yarn characteristic.

To estimate the cumulative distribution of a sample, the data for n values were ordered and ranked from the smallest to the largest value. The sample cumulative distribution function (CDF), H_k, was then estimated from its rank k by the median position method which is given by

$$H_k = \frac{k - 0.3}{n + 0.4} \quad (2)$$

As pointed out by Asloun *et al.* [29] and Nelson [27], the different approaches for determining the CDF do not affect the general shape of the probability plots which are derived from H_k.

3.1. Effect of strain rate on the failure load of yarns

The distribution of the lifetimes for many synthetic fibres and yarns can be represented by the Weibull distribution. In accordance with Coleman [30] the distribution function for the time to failure of a yarn under load $\ell(t)$, $t \geq 0$ can be assumed to be of the form

$$H(t) = 1 - \exp\left(-\mathscr{P}\left\{\int_0^t \lambda[\ell(t)]\mathrm{d}t\right\}^{\kappa}\right) \quad (3)$$

where $\lambda[\ell(t)]$ is the breakdown rule, κ is the shape parameter (a positive constant) for the lifetime, and $\mathscr{P}$ is a non-dimensional parameter representing the volume of the yarns; for simplicity $\mathscr{P}$ can be assumed to be unity. The two commonly assumed functional forms of λ are the exponential-law breakdown rule

$$\lambda(x) = \alpha\exp(\psi x) \quad (4)$$

and the power-law breakdown rule

$$\lambda(x) = \phi x^{\eta} \quad (5)$$

where α, ϕ, ψ and η are positive constants [31, 32]. For the short-term strength the linearly increasing load history $\ell(t) = Rt$, $t \geq 0$ is considered where R is the loading rate.

The cumulative distribution function for the break load ℓ follows the Weibull distribution under the power-law breakdown rule [31], since

$$H(\ell) = 1 - \exp\left[-\left(\frac{\ell}{a}\right)^{\omega}\right] \quad \ell \geq 0 \quad (6)$$

where

$$a = \left(\frac{R(\eta + 1)}{\phi}\right)^{1/(\eta+1)} \quad (7)$$

is the scale parameter and $\omega = \kappa(\eta + 1)$ is the shape parameter. The mean breaking load becomes [31]

$$E[\ell] = \left(\frac{R(\eta + 1)}{\phi}\right)^{1/(\eta+1)}\Gamma\left(1 + \frac{1}{\kappa(\theta + 1)}\right) \quad (8)$$

The effect of the loading rate on the breaking load is therefore reflected by the factor $R^{1/(\eta+1)}$. A log–log graph of the mean failure load against the strain rate would be a straight line and the mean failure load would increase with an increase in strain rate since η is a positive constant. This dependence is mild if η is large.

Under the exponential-law breakdown rule, the resulting distribution function for the short-term failure load is not the Weibull distribution but rather a double exponential distribution with the mean

$$E[\ell] = \frac{1}{\psi}\left[\ln\left(\frac{\psi R}{\alpha}\right) - \frac{\gamma}{\kappa}\right] \quad (9)$$

where $\gamma = 0.0577\ 2156$ is Euler's constant [30]. The mean failure load increases with the strain rate, and a graph of the mean failure load against the logarithm of the strain rate would be a straight line.

4. Results and discussion

4.1. Distributions for the yarn characteristics

A summary of the basic statistics of the tests for the yarns is shown in Table I. Tables II to V show the parameters obtained when the data were fitted to the Weibull, Gumbel, Gaussian and log-normal distributions, respectively. In Table VI the various

TABLE I Statistical data for tensile tests of aramid (KR) and polyester (PR) yarns

Test	Gauge length (mm)	no. of yarns	Breaking stress			Breaking strain			Breaking force			Linear density		
			Mean (MPa)	SE (MPa)	CV (%)	Mean (%)	SE (%)	CV (%)	Mean (N)	SE (N)	CV (%)	Mean (dtex)	SE (dtex)	CV (%)
KRI45	676.2	146	2477.8	18.259	8.9	1.78	0.0100	6.8	422.2	3.149	9.0	2470.3	2.134	1.0
KRI34	566.2	100	2461.4	22.414	9.1	1.72	0.0133	7.7	419.4	3.941	9.4	2470.0	2.988	1.2
KRI24	466.2	100	2467.5	24.608	10.0	1.71	0.0130	7.6	419.4	4.196	10.0	2463.6	2.850	1.2
KRI14	370.2	100	2520.0	22.194	8.8	1.70	0.0111	6.5	429.3	3.869	9.0	2469.7	2.748	1.1
KRII45	633.4	149	2299.1	18.892	10.3	1.87	0.0125	8.2	394.4	3.248	10.1	2487.7	1.904	0.9
KRII34	523.4	99	2270.5	22.510	9.9	1.80	0.0143	7.9	388.9	3.861	9.9	2483.7	3.039	1.2
KRII24	423.4	100	2384.4	16.636	7.0	1.83	0.0103	5.6	407.0	2.934	7.2	2475.3	2.773	1.1
KRII14	327.4	100	2417.3	17.959	7.4	1.89	0.0113	6.0	414.4	3.194	7.7	2485.3	2.447	1.0
PR50	497.4	200	920.5	3.725	5.7	11.08	0.0389	4.7	75.0	0.291	5.5	1124.3	1.641	2.1
PR40	397.4	200	879.7	4.042	6.5	11.07	0.0661	8.4	71.7	0.316	6.6	1125.3	1.566	2.0
PR30	297.4	198	887.2	4.803	7.6	11.56	0.0582	7.1	72.2	0.398	7.8	1123.0	1.536	1.9
PR20	197.4	200	909.0	4.525	7.0	11.54	0.0555	6.8	73.7	0.383	7.3	1118.3	1.639	2.1

TABLE II Parameters of conventional Weibull distribution defined by the cumulative distribution function $H(x) = 1 - \exp[-(x/\alpha)^{\rho}]$ for aramid (KR) and polyester (PR) yarns

Test	No. of yarns	Failure stress		Failure strain		Failure force	
		α (MPa)	ρ	α (%)	ρ	α (N)	ρ
KRI45	146	2566.9	16.40	1.83	20.445	437.6	15.95
KRI34	100	2554.3	15.24	1.78	16.696	435.7	14.60
KRI24	100	2563.4	15.35	1.77	17.372	435.9	14.87
KRI14	100	2608.5	16.89	1.75	20.366	444.9	15.95
KRII45	149	2393.4	14.05	1.94	15.548	410.7	13.77
KRII34	99	2365.6	13.31	1.87	14.685	405.4	12.94
KRII24	100	2451.2	21.28	1.83	23.050	419.0	19.25
KRII14	100	2491.6	18.77	1.94	21.161	427.7	17.70
PR50	200	944.7	20.10	11.30	28.024	76.9	20.49
PR40	200	904.7	18.59	11.50	12.909	73.7	18.05
PR30	198	916.5	16.31	11.93	15.749	74.6	15.65
PR20	200	937.5	16.71	11.92	13.908	76.1	15.31

TABLE III Parameters of Gumbel distribution defined by the cumulative distribution function $H(x) = 1 - \exp\{-\exp[(x - \beta)/\delta]\}$ for aramid (KR) and polyester (PR) yarns

Test	No. of yarns	Failure stress		Failure strain		Failure force	
		δ (MPa)	β (MPa)	δ (%)	β (%)	δ (N)	β (N)
KRI45	146	148.95	2572.20	0.0871	1.836	26.23	438.60
KRI34	100	160.30	2560.27	0.1039	1.781	28.57	436.84
KRI24	100	157.45	2569.39	0.0993	1.769	27.79	436.99
KRI14	100	147.28	2613.51	0.0840	1.751	26.80	445.83
KRII45	149	162.31	2399.91	0.1219	1.942	28.58	411.88
KRII34	99	170.62	2372.72	0.1267	1.872	30.29	406.64
KRII24	100	111.22	2454.24	0.0805	1.873	21.29	419.65
KRII14	100	128.61	2495.46	0.0914	1.945	23.46	428.47
PR50	200	46.66	945.91	0.3962	11.312	3.74	76.96
PR40	200	48.53	906.04	0.8915	11.533	4.14	73.81
PR30	198	55.48	918.30	0.7429	11.952	4.71	74.80
PR20	200	55.68	939.29	0.8907	11.946	4.96	76.28

distributions are compared by using the Kolmogorov–Smirnov D-statistics.

Whilst the Gumbel distribution best describes the failure stress and force at the gauge lengths considered for the aramid yarns, the Weibull distribution is found to be the best distribution to represent the failure strain. For the polyester yarns the failure force is best described by the normal distribution, but the failure

TABLE IV Mean μ and standard deviation σ for aramid (KR) and polyester (PR) yarn data when fitted to the Gaussian distribution

Test	No. of yarns	Failure stress		Failure strain		Failure force	
		μ (MPa)	σ (MPa)	μ (%)	σ (%)	μ (N)	σ (N)
KRI45	146	2477.8	220.62	1.78	0.121	422.2	38.05
KRI34	100	2461.4	224.14	1.72	0.133	419.4	39.41
KRI24	100	2467.5	246.08	1.71	0.130	419.4	41.96
KRI14	100	2520.0	221.94	1.70	0.111	429.3	38.69
KRII45	149	2299.1	230.61	1.87	0.153	394.4	39.65
KRII34	99	2270.5	223.97	1.80	0.142	388.9	38.42
KRII24	100	2384.4	166.36	1.83	0.103	407.0	29.34
KRII14	100	2417.3	179.59	1.89	0.113	414.4	31.94
PR50	200	920.5	52.68	11.08	0.522	75.0	4.11
PR40	200	879.7	57.16	11.07	0.935	71.7	4.47
PR30	198	887.2	67.59	11.56	0.819	72.2	5.60
PR20	200	909.0	63.99	11.54	0.785	73.7	5.41

TABLE V Mean μ and standard deviation σ for aramid (KR) and polyester (PR) yarn data when fitted to the log-normal distribution

Test	No. of yarns	Failure stress		Failure strain		Failure force	
		μ (MPa)	μ (MPa)	μ (%)	σ (%)	μ (N)	σ (N)
KRI45	146	7.811	0.0956	0.576	0.0713	6.041	0.0968
KRI34	100	7.704	0.0967	0.540	0.0806	6.034	0.1000
KRI24	100	7.805	0.1089	0.534	0.0794	6.033	0.1088
KRI14	100	7.828	0.0949	0.529	0.0679	6.058	0.0969
KRII45	149	7.735	0.1075	0.624	0.0849	5.972	0.1075
KRII34	99	7.723	0.1037	0.587	0.0806	5.958	0.1034
KRII24	100	7.774	0.0742	0.601	0.0585	6.006	0.0764
KRII14	100	7.788	0.0782	0.637	0.0619	6.034	0.0811
PR50	200	6.823	0.0581	2.404	0.0484	4.316	0.0555
PR40	200	6.777	0.0673	2.401	0.0856	4.271	0.0642
PR30	198	6.785	0.0793	2.445	0.0729	4.276	0.0806
PR20	200	6.810	0.0723	2.444	0.0676	4.297	0.0750

stress is best represented by the Weibull distribution. The failure strain of the polyester yarns is, however, best represented by the Weibull distribution.

The conjecture that the linear densities follow the Gaussian distribution has been vindicated by the fact that the linear densities of both aramid and polyester yarns are well represented by the normal distribution and pass the K–S test at all the gauge lengths considered.

4.1.1. *Kevlar 49 aramid yarns*

4.1.1.1. Failure stress and breaking force. Figs 1 and 2 show examples of the Gumbel plots for the failure stress and force. The plots are reasonably straight, confirming the K–S tests to be valid. It is worth noting that from the results of the K–S tests shown in Table VI, the Weibull distribution could also be used to model the force and the stress behaviour of the yarns at the 5% significance level for all the gauge lengths considered, except that of batch 2 at a gauge length of 633 mm (KRII45). The results for the Weibull distribution are close to those of the Gumbel distribution, but the Gumbel distribution is superior as seen from its lower D values. This, coupled with the fact that other distributions are usually not considered, could be the reason why the Weibull distribution has been used extensively to describe the strength behaviour of materials. The results obtained here, however, agree with the argument of Smith and Phoenix [33] that yarns with mild bonding or friction between fibres may follow the Gumbel distribution.

As far as the authors can determine, no one has used the Gumbel distribution to model either stress or force behaviour. The Weibull distribution has been used extensively to describe the strength of many materials (e.g. [7, 24]), although there are still a lot of unanswered questions as regards the length effect of the distribution. Phoenix and Wu [34] and Schwartz *et al.* [16], among others, used the Weibull distribution to model Kevlar 49/epoxy strands. Apart from the fact that they dealt with composite yarns, no investigation was made into the possibilities of other distributions fitting their data. Chambers [35] also used the Weibull distribution to model the failure stress of Kevlar 49 yarns. He did not consider the possibility of other distributions describing the yarn stress behaviour.

A Gumbel distribution can be described by the cumulative distribution function H as

$$H(x) = 1 - \exp\{-\exp[(x - \beta)/\delta]\} \qquad -\infty < x < \infty \tag{10}$$

TABLE VI Maximum absolute deviations for Kolmogorov–Smirnov test of aramid (KR) and polyester (PR) yarns[a]

Test	Failure stress				Failure strain			
	Normal D_n	Log-normal D_{ln}	Weibull D_w	Gumbel D_g	Normal D_n	Log-normal D_{ln}	Weibull D_w	Gumbel D_g
KRI45	*0.1152*	*0.1370*	0.0861	**0.0739**	0.1049	*0.1190*	**0.0476**	0.0527
KRI34	0.1147	0.1327	***0.0785***	0.0802	**0.0673**	0.0811	0.0701	0.0828
KRI24	*0.1776*	*0.1981*	0.1061	**0.0865**	0.0891	0.1073	0.0631	**0.0623**
KRI14	*0.1640*	*0.1843*	0.1081	**0.0936**	0.0817	0.0976	**0.0579**	0.0689
KRII45	*0.1828*	*0.1968*	0.1318	**0.1138**	0.0803	0.0984	0.0498	**0.0454**
KRII34	0.0992	0.1197	0.0820	**0.0795**	0.0644	0.0682	**0.0626**	0.0682
KRII24	*0.1435*	*0.1618*	0.0816	**0.0741**	0.1008	0.1068	**0.0933**	0.1004
KRII14	0.1082	0.1236	0.0545	**0.0494**	0.0734	0.0843	**0.0824**	0.0851
PR50	**0.0395**	0.0513	0.0580	0.0669	*0.1019*	*0.1103*	**0.0638**	0.0648
PR40	0.0772	0.0673	**0.0440**	0.0509	**0.0267**	0.0300	0.0682	0.0831
PR30	0.0686	0.0851	**0.0242**	0.0284	0.0460	**0.0405**	0.0878	0.0953
PR20	0.0607	0.0683	**0.0466**	0.0566	0.0453	**0.0367**	0.0913	*0.1114*

[a] Underlined values show lowest D values (best fits) and values in italics show distributions which failed the test at 5% limit.

TABLE VI (*continued*)

Test	Failure force				Linear density	5% limit
	Normal D_n	Log-normal D_{ln}	Weibull D_w	Gumbel D_g	Normal D_n	D_5
KRI45	*0.1139*	*0.1354*	0.0765	**0.0615**	0.0458	0.1126
KRI34	0.0987	0.1162	0.0603	**0.0563**	0.0782	0.1360
KRI24	*0.1693*	*0.1932*	0.0948	**0.0841**	0.0822	0.1360
KRI14	*0.1651*	*0.1877*	0.0903	**0.0770**	0.1103	0.1360
KRII45	*0.1600*	*0.1793*	0.1087	**0.0911**	0.0589	0.1114
KRII34	0.1072	0.1264	0.0779	**0.0736**	0.0888	0.1367
KRII24	0.1223	*0.1414*	0.0807	**0.0693**	0.0859	0.1360
KRII14	0.1050	0.1231	0.0513	**0.0453**	0.0820	0.1360
PR50	0.0458	0.0496	*0.0978*	*0.1065*	0.0450	0.0962
PR40	**0.0663**	0.0803	0.0712	0.0801	0.0485	0.0962
PR30	**0.0577**	0.0747	**0.0285**	0.0353	0.0414	0.0967
PR20	**0.0462**	0.0615	0.0649	0.0762	0.0439	0.0962

where β is the location parameter, which may take values from $-\infty$ to ∞, and δ is the scale parameter which determines the spread of the distribution. The mean $E[x]$ and the standard deviation $SD[x]$ are given [26] by

$$E[x] = \beta - 0.5772\delta \quad SD[x] = 0.4082\pi\delta \quad (11)$$

From Equation 11 the mean and the variance of the Gumbel distribution can be obtained from the maximum likelihood parameters β and δ shown in Table III. These values were used to estimate the population means and the standard deviations for the various gauge lengths and the results are presented in Table VII. A comparison of these estimates with those estimated by ordinary statistics (Table VII) reveals that there are differences between them, albeit very small.

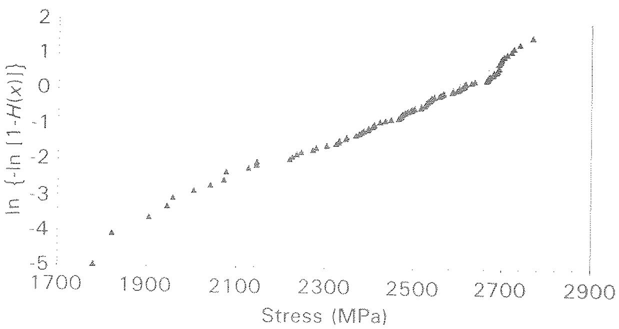

Figure 1 Typical Gumbel plot for aramid yarns (stress data): (▲) data points for KRI34, ( ) max. likelihood estimate. Sample size = 100.

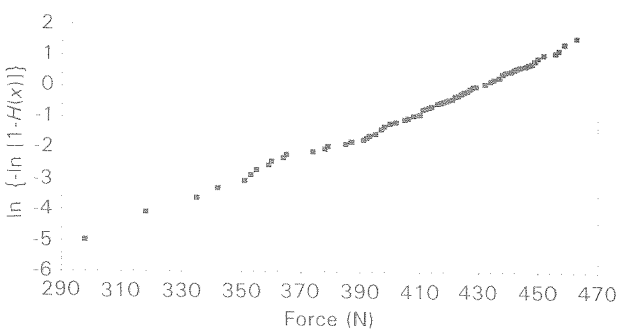

Figure 2 Typical Gumbel plot for aramid yarns (force data): (■) data points for KRII14, ( ) max. likelihood estimate. Sample size = 100.

TABLE VII Comparison of mean and variance for the stress of aramid yarns obtained from normal statistics and Gumbel parameters

Tests	Values from normal analysis		Values from Equation 11	
	Mean, μ (MPa)	Standard deviation, σ (MPa)	Mean, μ (MPa)	Standard deviation, σ (MPa)
KRI45	2477.8	220.62	2486.2	191.04
KRI34	2461.4	224.14	2467.7	205.59
KRI24	2467.5	246.08	2478.5	201.94
KRI14	2520.0	221.94	2528.5	188.89
KRII45	2299.1	230.61	2306.2	208.17
KRII34	2270.5	223.97	2274.2	218.83
KRII24	2384.4	166.36	2390.0	142.65
KRII14	2417.3	179.59	2421.2	164.95

Whilst the means obtained from Equation 11 are higher, the standard deviations are lower than those from the normal distribution. The maximum relative difference for the means is less than 0.5%, which can be considered to be negligible. However, the maximum relative difference for the standard deviations is about 22% which is quite high.

4.1.1.2. Failure strain. Figs 3 and 4 show examples of the Weibull plots for the strain data. The results are in accordance with those of Steenbakkers and Wagner [36] who used the Weibull distribution to model the failure strain of Kevlar 149 aramid fibres. The Gumbel distribution could also be used to describe the strain behaviour. In fact, it passes the K–S test at the 5% level for all the gauge lengths considered, but the D values are generally higher than those for the Weibull distribution. This indicates the superiority of the Weibull distribution in representing the strain behaviour. The normal and log-normal distributions also pass the K–S test at the level considered for all the gauge lengths dealt with; however, for the same reason of higher D values, these distributions were not chosen to represent the failure strain.

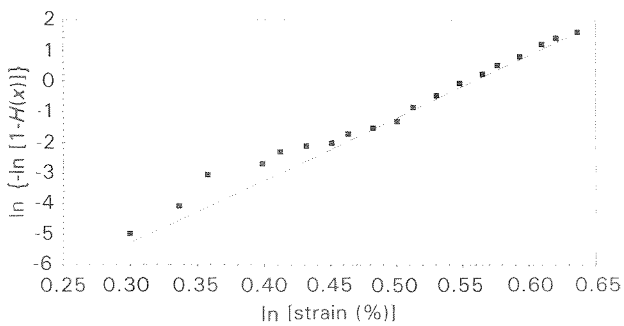

Figure 3 Typical Weibull plot for aramid yarns (strain data): (■) data points for KRI14, (– – –) max. likelihood estimate. Sample size = 100.

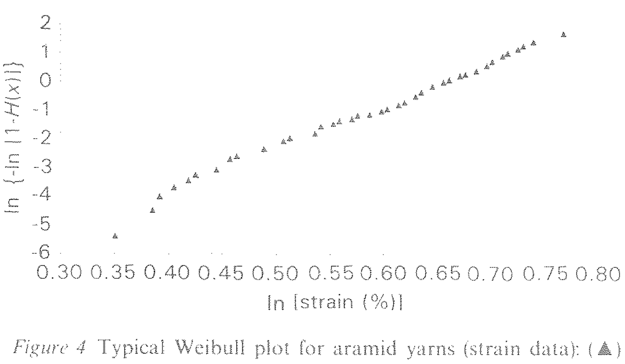

Figure 4 Typical Weibull plot for aramid yarns (strain data): (▲) data points for KRII45, (– – –) max. likelihood estimate. Sample size = 149.

4.1.2. Polyester yarns

4.1.2.1. Failure stress and breaking force. Whilst the failure force is best described by the normal distribution, the failure stress is best represented by the Weibull distribution. The Weibull and the log–normal distributions could also be used to describe the force behaviour; both distributions pass the K–S test at the 5% level, but since the aim of the experiment is to select the best distribution for the yarn characteristics, the yarns are not represented by these two distributions. The Gumbel distribution also adequately represents the failure stress; it passes the K–S test at the 5% level for all the gauge lengths considered, but the Weibull distribution is superior. Figs 5 and 6 show examples of the normal and Weibull plots of the failure load and stress, respectively.

Although the Weibull distribution has been used by many authors to represent the failure stress of many of materials, apart from Peirce [2] and Frenkel and Kontorova [4], the normal distribution has not been seen, in the literature, to model the failure load of

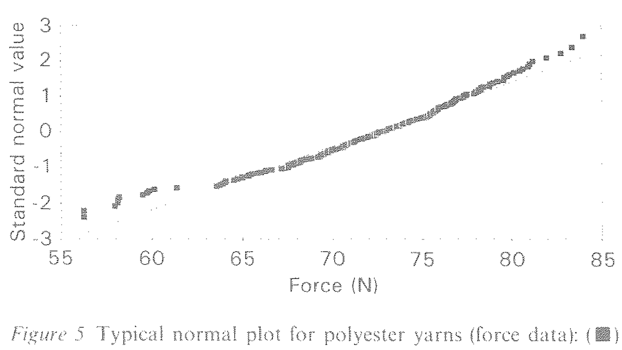

Figure 5 Typical normal plot for polyester yarns (force data): (■) data points for PR30, (– – –) max. likelihood estimate. Sample size = 198.

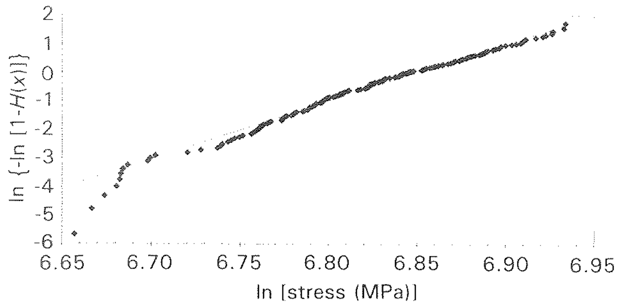

Figure 6 Typical Weibull plot for polyester yarns (stress data): (◆) data points for PR50, (– – –) max. likelihood estimate. Sample size = 200.

materials. However, from Daniels [37] the failure stress of a bundle of fibres, under certain circumstances, follows the normal distribution. Daniels' work was related to stress, but the cross-sectional area of the fibres was assumed to be constant and therefore stress and force were equivalent.

4.1.2.2. Failure strain. A typical Weibull plot of the failure strain is presented in Fig. 7. In fact the normal distribution also adequately represents the strain for tests at low gauge lengths (below 497 mm). The K–S values for the normal distribution are lower than those for the Weibull distribution for all the tests except test PR50. This suggests that the normal distribution fits the data better at the lower gauge lengths; unfortunately the normal distribution fails at the 5% level for test PR50 and cannot be selected as the best distribution for the strain data.

4.2. Gauge length and yarn distributions
4.2.1. Failure stress

Fig. 8 shows the mean stress against the gauge length for both batches of Kevlar 49 aramid yarn. There is a slight drop in the strength with increasing test length. However, in view of the large scatter in bundle strength it is not easy to make a definitive judgement about the way the yarn strength decreases with increasing test length merely by using the experimental data.

If it is assumed that the strength of the yarns follows the Gumbel distribution because of the series-parallel

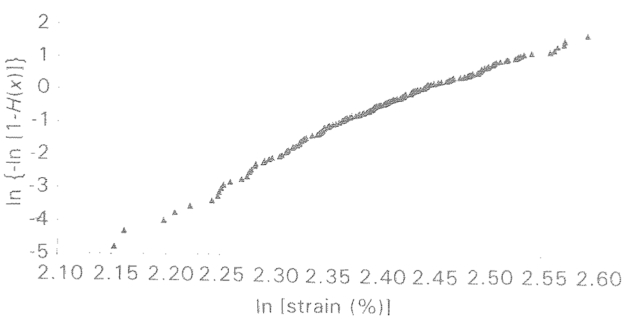

Figure 7 Typical Weibull plot for polyester yarns (strain data): (▲) data points for PR40, (– – –) max. likelihood estimate. Sample size = 200.

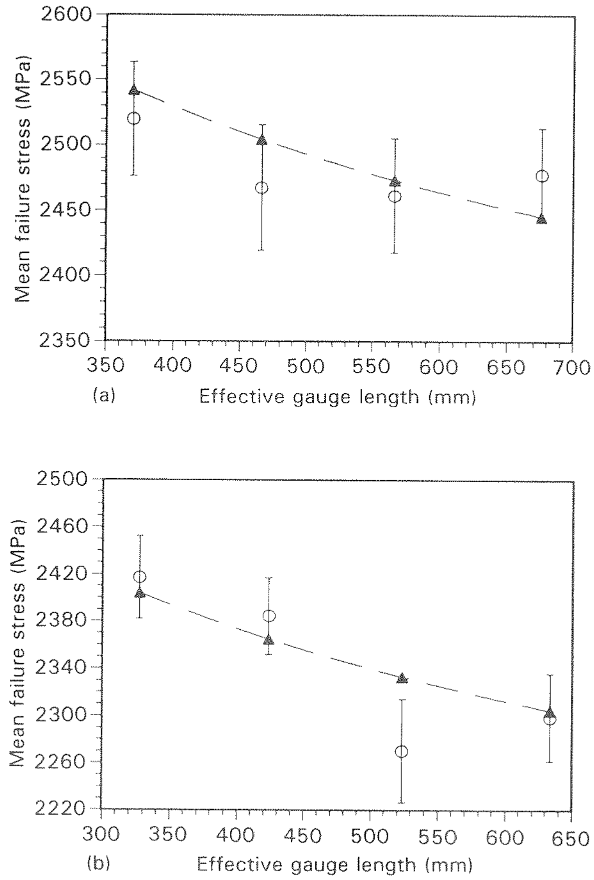

Figure 8 Gauge length effect on the strength of aramid yarns: (a) batch 1, (b) batch 2. (○) Experimental, (▲) theoretical; bars are 95% confidence intervals.

model [33], then the parameters of the Gumbel distribution defined by Equation 10 can be expressed by

$$\delta = \frac{\sigma_c}{(2\mathcal{N}\ln m)^{1/2}} \qquad (12)$$

and

$$\beta = \mu_c + \frac{\sigma_c}{2\mathcal{N}^{1/2}}\left(\frac{\ln(\ln m) + \ln(4\pi) - 4\ln m}{(2\ln m)^{1/2}}\right) \qquad (13)$$

where μ_c and σ_c represent the mean and the standard deviation of each sub-bundle, respectively. By using Equations 12 and 13 and other properties of the Gumbel distribution, the mean μ_c and the standard deviation σ_c of the sub-bundles can be calculated. The relationship between the test length and the yarn strength can then be obtained.

To solve for μ_c and σ_c the number of the fibres per yarn $\mathcal{N}$ and the number of sub-bundles m should be known before Equations 12 and 13 can be used. There are 1000 filaments per yarn but m is unknown, so another equation is needed. This can be obtained by using the following property of the Gumbel distribution [6]:

$$h_c(2\mu - \beta) = h_c(\beta) \qquad (14)$$

where $h_c(\cdot)$ is the density function of the sub-bundle, β is the parameter of the Gumbel distribution, and μ is the corresponding mean strength of the yarn or bundle at the bundle length under consideration.

With the mean and the standard deviation of the yarn at the required length known, Equation 11 is used to obtain δ and β. Equations 12, 13 and 14 can then be solved simultaneously to obtain μ_c, σ_c and m.

The above method was employed to obtain $\mu_c = 4472.9$ and 4156.3 MPa for batches 1 and 2 of the aramid yarns, respectively. Standard deviations $\sigma_c = 18\,544.04$ and 17 121.42 MPa were also obtained for batches 1 and 2, and the lengths of the sub-bundles (specimen length/m) were evaluated as 0.344 and 0.371 mm, respectively. The yarns have about 2.2 turns per inch (1 turn in about 10 mm). One would thus expect that the effect of a broken yarn would be limited to a length of the order of a few millimetres, and because of the twist, it is probable that the yarn is not really behaving as a parallel bundle even within that length. The values for the means and the standard deviations are very high, the mean strengths being higher than any filament tests reported (≈ 3500 MPa). These high mean strengths (4200 MPa), extremely high standard deviations and very short sub-bundle lengths probably mean that the series-parallel model is not correct.

The values obtained above are used to obtain the graphical relationships between the yarn strength and test length which are shown in Fig. 9 as the theoretical results. With μ_c and σ_c determined, the relationship between length and strength is obtained from Equations 13 and 11. The theoretical values fit reasonably well with the experimental results. The yarn strength depends on the test length but the change is small and can be masked by the scatter in the yarn strength.

Fig. 9 shows the log–log graph of the mean failure stress against the gauge length of the polyester yarns. The plot of the data does not give a straight line as predicted by the Weibull distribution; by assuming a linear relationship a correlation coefficient of 0.275 was obtained. A t-test of the hypothesis of a zero slope or a zero correlation coefficient gave a t-value of 0.41 which corresponds to a significance greater than 20% ($t_2 = 1.89$), so the hypothesis of no correlation was accepted. Thus the failure stress of the polyester yarns does not depend on the test length. Watson and Smith [13] suggested a modification to the Weibull distribution so that the cumulative distribution function H of the Weibull distribution could be represented by

$$H(x) = 1 - \exp\left[- l^{\nu}\left(\frac{x}{\alpha_0}\right)^{\rho}\right] \quad x > 0 \qquad (15)$$

where l, ρ and ν refer to dimensionless parameters representing the length, the shape parameter and a constant, respectively, and α_0 is a constant scale parameter. If the constant ν is equal to or close to zero then the failure stress will be independent of length, and if $\nu = 1$ the conventional Weibull distribution is correct.

The parameters ν, α_0 and ρ were determined by the maximum likelihood method to be 0.06, 926.5 MPa and 16.94, respectively. The small value of ν confirms that the strength is independent of the length. Since the t-test above also accepts the hypothesis of no correlation between length and stress, it is reasonable to take $\nu = 0$, which modifies the maximum likelihood estimates of α_0 and ρ to $\alpha_0 = 925.85$ MPa and $\rho = 17.928$. These are taken as the parameters of the distribution.

4.2.2. Failure strain

Fig.10 shows the log–log graph of the failure strain versus the gauge length for both batches of aramid yarn. The data do not fit a straight line as would be predicted by the Weibull distribution. Whilst the failure strain for batch 1 increases slightly with an increase in length, that of batch 2 first decreases and then increases as the length increases. Without the contributions of the tests at higher gauge lengths, the log–log curves for both batches would be straight lines. A t-test of the hypothesis of a zero slope gives t-values of 2.57 and 0.52 for batches 1 and 2, respectively. At a 10% significance level ($t_2 = 2.94$) the hypothesis of zero slope is accepted. Thus it is concluded that the failure strain is independent of the length. By using the maximum likelihood method, a value of $\nu = 0.000\,01$ is obtained for both batches. This further confirms that the failure strains of both batches are independent of test length, and the modification of the Weibull scale parameter given by Watson and Smith [13] is considered to be the most appropriate with $\nu = 0$.

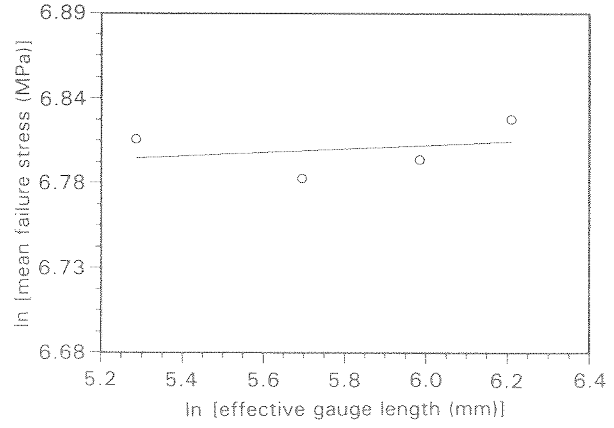

Figure 9 Gauge length effect on the strength of polyester yarns: (○) experimental, (——) $y = 6.7428 + 0.010955\,x$; $r = 0.275$.

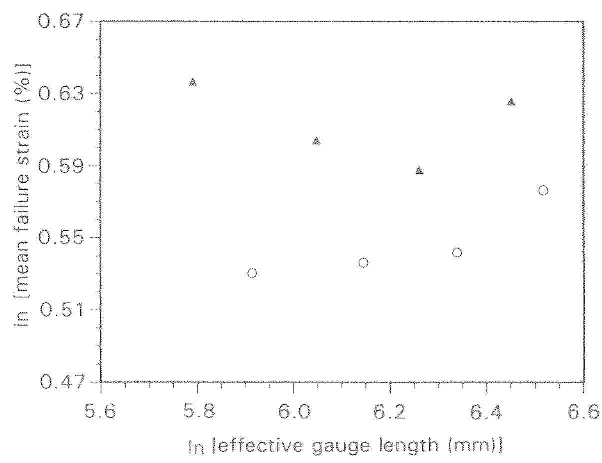

Figure 10 Effect of gauge length on the failure strain of aramid yarns: (○) batch 1, (▲) batch 2.

TABLE VIII Experimental results for aramid yarns: failure load, strain and strain rate

Test	Strain Rate, R (%/min)	ln R	Failure load, L (N)	Failure strain, e (%)	ln e
TB1-1	2.0	0.6931	238.8	2.073	0.7300
TB1-2	5.0	1.6094	234.6	2.049	0.7174
TB1-3	8.0	2.0794	240.9	2.093	0.7386
TB1-4	10.0	2.3026	243.2	2.105	0.7443
TB1-5	15.0	2.7081	238.6	2.084	0.7343
TB1-6	20.0	2.9957	232.8	2.024	0.7051
TB2-1	2.0	0.6931	223.8	1.957	0.6714
TB2-2	5.0	1.6094	222.6	1.927	0.6560
TB2-3	8.0	2.0794	220.8	1.932	0.6586
TB2-4	10.0	2.3026	221.5	1.949	0.6673
TB2-5	15.0	2.7081	221.3	1.915	0.6497
TB2-6	20.0	2.9957	217.8	1.897	0.6403

In Fig. 11 the log–log graph of the failure strain versus the gauge length of the polyester yarns is presented. By fitting a straight line through the points, a correlation coefficient of 0.861 is obtained. The failure strain decreases as the length increases. From the slope of the straight line in Fig. 11 the Weibull shape parameter ρ can be calculated. A value of $\rho = 19.065$ is obtained from the graph but the average value of ρ from the data in Table II is 17.647. The difference can be accounted for by using a maximum likelihood method based on Equation 15, giving values of $\nu = 1.082$, $\alpha_0 = 11.63$ and $\rho = 15.302$. The slope of the line in Fig. 11 should be equal to the ratio of ν to ρ. This ratio gives a value of $\nu/\rho = 0.07$, whilst the slope of the graph is 0.05. Although there is a difference between these two values, the maximum likelihood value is accepted because of the wide scatter in the plots and indeed another acceptable line can be drawn to give a slope with the value of 0.07. The modification by Watson and Smith is thus considered to be the most appropriate for the failure strain of the polyester yarns, with $\nu = 1.1$. The fact that strength appears to be independent of length, while strain is not, is a problem that must be solved by further testing.

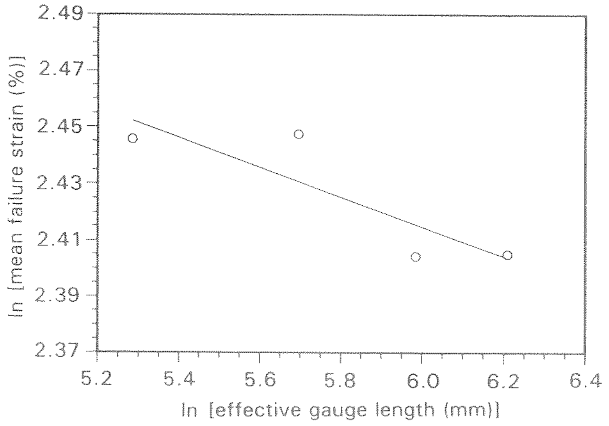

Figure 11 Effect of gauge length on the failure strain of polyester yarns: (○) experimental, ( ) $y = 2.7296 - 0.052\,453\,x$; $r = 0.862$.

4.3. Effect of strain rate on Kevlar 49 aramid yarns

A summary of the results for the failure load and strain of the aramid yarns at various strain rates is presented in Table VIII. In Fig. 12 the mean failure loads and strains are plotted against the strain rate.

Although there is a large scatter for batch 1, there is a general trend whereby the failure load and strain decrease slightly as the strain rate increases. The log–log and semi-log plots which arise out of the

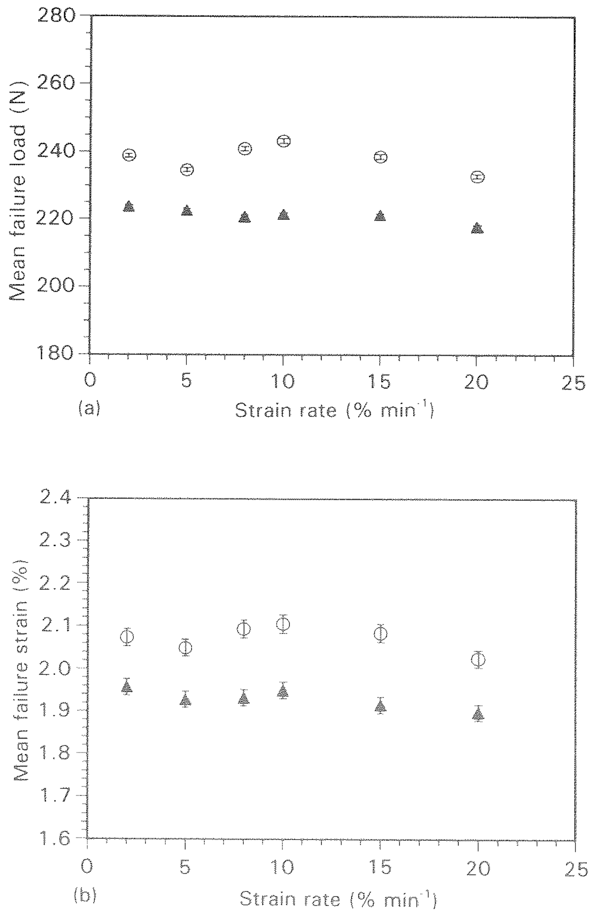

Figure 12 Effect of strain rate on (a) failure load and (b) failure strain of aramid yarns: (○) batch 1, (▲) batch 2. Bars are 95% confidence intervals.

power-law and exponential-law breakdown rules (Equations 8 and 9) also follow the same trend (Figs 13 and 14). This behaviour does not agree with the statistical theory described in Section 3.1 despite the fact that the Weibull distribution was found to be appropriate for both the failure load and strain of the yarns tested. The discrepancy may be because the failure process is not attributed to a statistical distribution of defects in the yarns, which is an implicit assumption in the theory.

From the linear regression results (Figs 13 and 14) it is impossible to distinguish between the power-law and the exponential-law breakdown rules, although long-term experiments done by Wu and Schwartz [38] with single fibres of Kevlar 49 indicated that the exponential version of the model might be the more appropriate.

The inability of the theory to predict the effect of strain rate on the strength of fibres has also been observed by Schwartz *et al.* [12] and Wagner *et al.* [20]. Schwartz *et al.* [12] observed that the strength of ultra-high strength polyethylene fibres generally increased with increasing strain rate, but the behaviour was bilinear. Results from Wagner *et al.* [20] showed that the log–log and semi-log plots for the mean strength and strain rate for carbon fibres had two regimes. At very low strain rates up to a rate of 0.24 min^{-1} there was a slight increase of strength with strain rate, whereas at higher rates a pronounced decrease occurred. In the same paper, they reported that Kevlar 29, Kevlar 49 and Kevlar 149 fibres were insensitive to the strain rate.

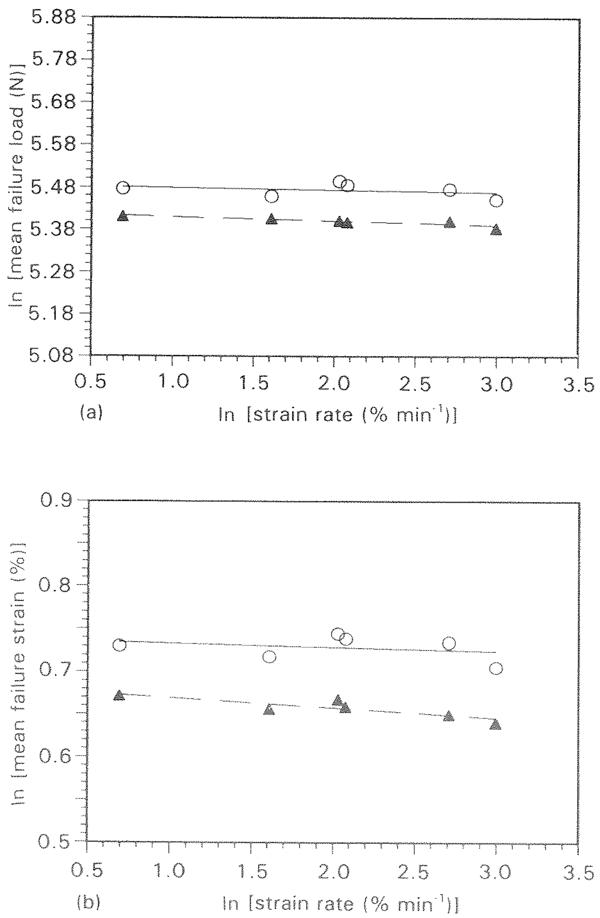

Figure 13 Effect of strain rate on aramid yarns; a power-law breakdown model. (a) Failure load. Batch 1: (○) experimental, (——) $y = 5.4832 - 5.1774 \times 10^{-3}x$; $r = 0.261$. Batch 2: (▲) experimental, (– –) $y = 5.4193 - 9.8222 \times 10^{-3}x$; $r = 0.877$. (b) Failure strain. Batch 1: (○) experimental, (——) $y = 0.73761 - 4.6192 \times 10^{-3}x$; $r = 0.260$, Batch 2: (▲) experimental, (– –) $y = 0.68116 - 1.1855 \times 10^{-2}x$; $r = 0.853$.

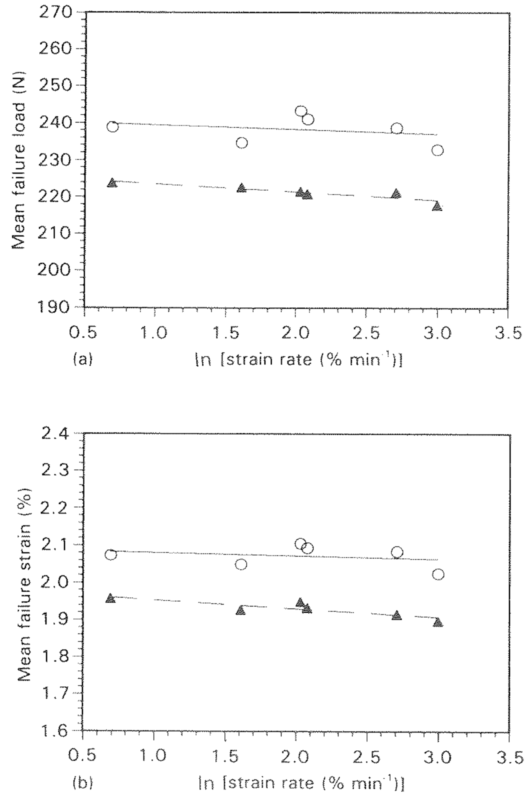

Figure 14 Effect of strain rate on aramid yarns; an exponential law breakdown model. (a) Failure load. Batch 1: (○) experimental, (——) $y = 240.62 - 1.2209\,x$; $r = 0.258$. Batch 2: (▲) experimental, (– –) $y = 225.69 - 2.1719\,x$; $r = 0.879$. (b) Failure strains. Batch 1: (○) experimental, (······) $y = 2.0887 - 8.5990 \times 10^{-3}x$; $r = 0.235$. Batch 2: (▲) experimental, (··· ··) $y = 1.9757 - 2.2870 \times 10^{-2}x$; $r = 0.853$.

5. Conclusions

A study of the variability in the mechanical properties of Kevlar 49 aramid and polyester yarns has been conducted. The major findings are as follows:

1. The failure stress of the aramid yarns is best represented by the Gumbel distribution but that of the polyester yarns is best represented by the Weibull distribution. The log–log dependence of the strength on the gauge length for the polyester yarns is not linear as predicted from the weakest-link and the Weibull models for failure. The strength of the polyester yarns is found to be independent of the gauge length, whereas that of the aramid yarns decreases slightly as the gauge length increases.

2. The failure strains of both yarns follow the Weibull distribution. However, the log–log dependence of the strain on the gauge length is not as predicted by the weakest-link and the Weibull models. The modification of Watson and Smith [13] is found to be adequate for the failure strains of both types of yarn. Whereas the failure strain of the aramid yarns is found to be independent of the gauge length, that of the

polyester is found to decrease with an increase in the gauge length.

3. The strength of the aramid yarns is found to be slightly dependent on the strain rate. The strength decreases with an increase in strain rate, which is contrary to theoretical predictions but in line with other test data.

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