Bicycle Wheel as Prestressed Structure

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Abstract: Bicycle wheels achieve their structural efficiency by making use of prestressing in three ways. Tests show that the bottom spokes carry virtually all the load by compressive forces, which reduce the tensile prestress set up in the spokes when the wheel was made. The test results are compared with an analysis that considers the spokes as a disk that can carry force in one direction only. This is shown to give good agreement, as does an analysis that considers the rim as a straight beam on an elastic foundation. The behavior of the wheel with an inflated tire is also considered, and it is shown that good comparisons with theory are obtained if the reaction from the road is assumed to be distributed over a specific length of the rim. Prestressing is shown to be important also in the mechanism by which the various forces are transmitted through the tire from the road to the rim.

Introduction

The bicycle wheel is one of the most efficient structural elements in use today. It combines lightness with exceptional load carrying capacity, giving, in certain circumstances, strength-to-weight ratios of up to 400:1 (Archer 1956). Not only does the wheel have the ability to resist vertical loads, but it must also withstand braking, traction, and side loads, while providing for the rider the only element of suspension to give some comfort. All this has to be provided in an element which rotates almost effortlessly.

The structure of the modern bicycle wheel was not designed by one person, but evolved by the incorporation of many different ideas. The basic design has not changed significantly in 100 years; it probably therefore represents a design that cannot be greatly improved on with existing technology. At the same time, its behavior is not clearly understood; perhaps that does not matter, since it works. However, it is instructive to the structural engineer to understand how one of the most efficient structures actually behaves. In the present paper, it will be shown that prestressing is the key to the structural behavior, not once, but in three distinct ways.

This paper will describe the existing knowledge of the way the wheel works and will present the results of detailed tests on a strain-gauged wheel. Comparisons will be made between the tests and both classical and modern analyses. Finally, the paper will consider the three contributions prestressing makes to the behavior of the wheel. It will be shown that prestressing was the key element in three of the most critical inventions that produced the efficient bicycle wheel in use today.

Development of Wheel

Up to the middle of the nineteenth century, most wheels as used in carriages and wagons were based on compression. An outer metal tire surrounded the wooden rim, which was joined to the spokes by simple

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mortise and tenon joints. The tire was heated before being placed over the wheel, and thus contracted as it cooled, inducing a compressive prestress in the rim and the spokes. This prestress served to take up the slack that was inevitable in the joints between timber components, rather than altering significantly the distribution of force within the wheel. Such wheels, with radial spokes, would work almost exclusively by the lowest spoke carrying the axle loads in compression. Attempts to reduce the weight of the wheel by reducing the thickness of the spoke would lead to failure by buckling.

It is against this background that bicycles were developed. The earliest wheels were radially spoked in one plane and not prestressed, so the spokes had to resist significant compressive and bending forces. In 1870, Starley introduced the Ariel bicycle, which had spokes that were essentially radial, but that could be tensioned by rotating the hub relative to the rim. The spokes were thus all inclined in one direction, which would not have maintained symmetry in the fore and aft direction; accelerating and braking would have had different effects on the stiffness of the wheel, which must have been noticeable to the rider. This wheel also introduced the idea of two sets of spokes inclined out of the plane of the wheel which is essential for lateral stability. In 1874, Starley introduced the tangentially spoked tensioned wheel with half the spokes in each layer inclined in the forward direction, and half in the rearward direction; this geometry is essentially the same as that in use today. This is the first use of prestress.

The pneumatic tire for bicycles (the second use of prestress) was introduced by Dunlop in 1888, and followed an earlier, but commercially unsuccessful, invention by Thompson in 1845. Dunlop's tire had three layers; an inner rubber tube that maintained the air pressure; a fabric pocket that enclosed both the rim and the tube and that resisted the forces developed by the air pressure; and finally, an outer rubber cover that protected the fabric from abrasion. The forces developed in such a tire will be discussed later, but the main problem with the original Dunlop tire was the difficulty of mending punctures, since the airtight tube was contained within the fabric pocket that also enclosed the rim. It was only when Welch and Bartlett, both in 1890, developed successful systems that allowed the tire to lie wholly on the outer edge of the rim that the pneumatic tire became practical. The patents for these two developments (the third use of prestress) were subsequently obtained by Dunlop and the improvements incorporated in his tires (Dunlop 1924).

Theoretical Analysis of the Wheel

To assist understanding of how the wheel worked, Sharp (1977), in his original 1896 work, produced an analysis, based on the polygon of forces, relating spoke tension to rim compression, but this was largely limited to the case of the unloaded wheel. A more thorough understanding of the properties of spoked wheels was made by Pippard and Francis (1931, 1932) and Pippard and White (1932). This work was primarily aimed at producing an understanding of light wheels for aircraft. Pippard could not analyze the behavior of individual spokes; at the suggestion of Southwell, he idealized the spokes as being disks with the property that stress could be carried only in one direction. He was then able to produce continuous functions for the spoke tensions, which can be compared with the discrete values measured in experiments.

Pippard and his team analyzed radially and nonradially spoked wheels under in-plane loads, and nonradially spoked wheels under out-of-plane loads. It is believed that experiments were carried out to verify the theoretical predictions, but these appear to have been reported directly to the sponsor, the Royal Air Force (RAF) and not to have been published.

No significant further advances were made prior to the introduction of computer programs that made feasible the large number of calculations needed to accurately model the highly redundant structure of the wheel. A commonly held idea was that the axle load hung off the top spokes, with an increased tension there to mirror the decreased tension in the bottom spokes, with the implication that the side spokes played no part in carrying the load (Roy 1983).

However, the most comprehensive recent book on the behavior of bicycles (Whitt and Wilson 1985), which refers to the work by Pippard, concluded:

> "Under load, a spoked wheel takes up not an oval shape, as is often stated, but an approximately circular shape with a flattened portion in the vicinity of road contact. . . . In the spoked wheel, the increased compressive stress in the rim increases the tension of all spokes except those in the (slightly) flattened region, where the spoke tension naturally decreases. The load on the axle is taken, then, by the combined effect of the increased spoke tension at the top of the wheel and the decreased tension in the region of contact. All other spokes have approximately equal tension and balance each other."

Two other studies are worthy of note. Brandt (1981) analyzed a wheel using a finite-element analysis, and Ivey (1985) loaded a complete bicycle through the seat in order to determine the stresses in a wheel. Both gave results that are broadly in agreement with the principles quoted previously, but neither result was compared with any other work. The finite-element analysis looked only at a single point load applied to one point on the rim (thus ignoring any load-spreading from the tire) and assumed that the rim was made from straight elements between spoke positions, while the test was carried out on a bicycle with inflated tires and there is considerable scatter in the results.

Description of Wheel Analyzed and Tested

The standard wheel used in modern lightweight adult bicycles is typically 630 mm (nominal) in diameter and takes a narrow (32 mm nominal) tire. The cross section is either formed from folded steel tube or an aluminum alloy extrusion, as in the wheel tested here. To determine the properties of the section used in these tests, Fig. 1 was produced by taking an enlarged photograph of a polished cross section of the wheel, and digitizing the perimeter of the section, from which the section properties could be determined.

The spokes, normally 36 in number, are arranged as two truncated cones, one to the left, the other to the right. Half the spokes in each cone are inclined forward, the rest backward (Fig. 2). The spoking arrangement is such that each spoke crosses two others in the same cone; one intersection occurs close to the hub, but the two spokes do not touch since they pass through the hub in different directions. At the other intersection, the spokes come into contact, thus causing some bending in the spokes.

The angle of inclination of the spoke to the radius is quite small. An individual spoke, attached to the outer rim immediately above the axle, will be attached to the hub at a point offset by 60° from the vertical (Fig. 3).

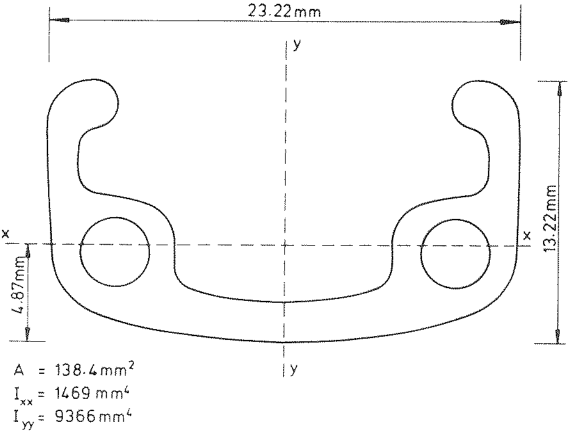

FIG. 1. Cross Section Alloy Rim Used in Tests

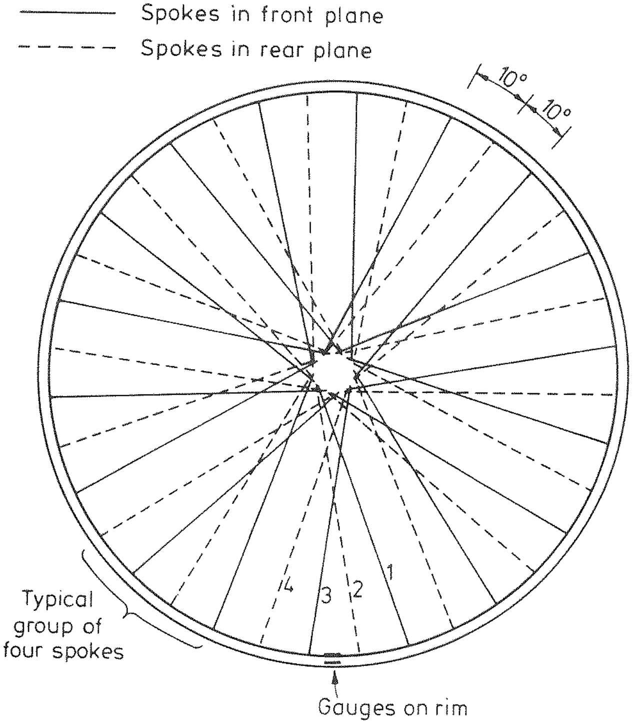

FIG. 2. Spoking Arrangement of Wheel Showing Numbered Spokes Used in Tests and Position of Strain Gauges on Rim

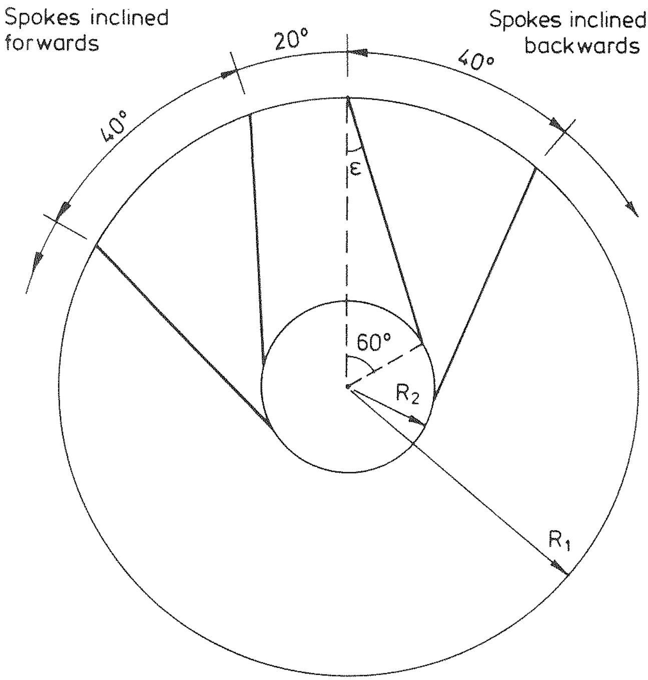

FIG. 3. Exaggerated View of Wheel Geometry

For typical values of the hub and rim radius on an adult's cycle, this leads to an inclination angle ε of about 2.5°. Smaller wheels generally have higher values of ε.

For all front wheels, the out-of-plane angles of the spokes on both sides of the wheel are equal, which ensures equality of the forces in the two sets of spokes. However, for back wheels that have to accommodate the block of a set of derailleur gears, one set of spokes has to be much steeper than the other (Brandt 1981). Thus, to keep the rim in equilibrium, the set of spokes on the gear side has to be under much higher tension than on the other side. This leads to greater susceptibility to deflect under the action of lateral loads, which can lead to a snap-through buckling failure.

Wheel Tests

To obtain data to compare with the theories, tests were carried out on a wheel, both with and without the tire. The wheel was supported in a frame by a pivoted arm and loaded by a screw jack acting through a load cell and tie rods to apply a vertical load through the axle (Fig. 4), as in the real structure. The wheel rested on a flat steel plate.

The wheel was a standard production model, although it was one of the higher quality wheels that are adjusted by hand, rather than by machine. Spoke tensions are adjusted during manufacture to make the wheel true, both in the radial and out-of-plane directions; variations in the shape of the unstressed rim thus manifest themselves as variations in spoke tensions,

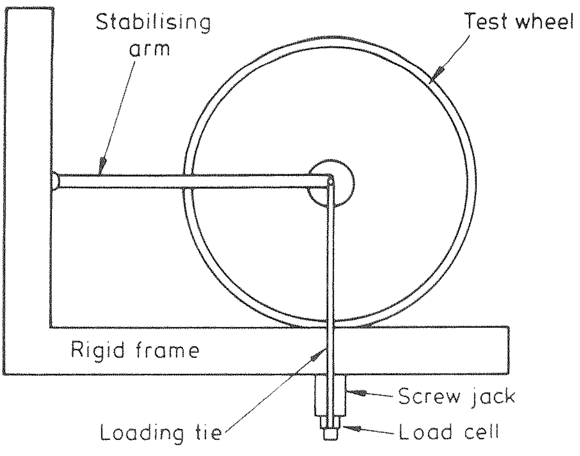

FIG. 4. Test Rig Geometry

TABLE 1. Properties of Tested Wheel

Quantity (1)	Value (2)
Outer rim radius (to centroid of rim)	309.4 mm
Inner hub radius (to center of spoke holes)	18.0 mm
Spoke diameter	2.10 mm
Area of spokes in one plane	62.34 mm^2
Elastic modulus of rim	70 kN/mm^2
Elastic modulus of spokes	210 kN/mm^2
Area of rim	138.4 mm^2
2nd moment of area of rim (for bending in plane of wheel)	1469 mm^4

which are not themselves measured directly during manufacture. Shape tolerances are of the order of ± 0.5 mm.

Six strain gauges were attached to the wheel; four were attached near the center of each spoke in a group of four spokes, while the remaining two were attached to the rim. These were placed as close as possible to the outer edge of the rim and on the inside centerline. The spokes that were gauged were believed to be typical of those in the rest of the wheel, but without strain gauging and then detensioning every spoke, it was impossible to be certain about this. There was no need to gauge every spoke, since the changes in the stress distribution around the wheel could be ascertained by repeating the loading tests with the wheel in different orientations.

The properties of the wheel tested were as shown in Table 1. The elastic modulus figures are data book values for aluminum and steel, respectively; they were not measured directly in the tests. The rim of the wheel was marked in degrees to allow the rotation to be determined. The zero mark for rotation was taken to be the point on the rim at the center of the group of four strain-gauged spokes; all angular measurements were expressed relative to this point. The spokes were located at 10° intervals around the rim, so the spokes nearest the zero mark were at $\pm 5°$.

The valve was modified to allow a pressure gauge to be attached to monitor the change in air pressure as the wheel was loaded.

Test Procedure

The test procedure adopted was to position the wheel with the chosen angular reference mark at the bottom, thus fixing the corect angular rotation. The wheel was then lifted off the base plate to obtain the zero reading for the strain gauges. This corresponded to zero load on the wheel but, due to the prestress in the spokes, did not refer to zero stress in the wheel. The wheel was then lowered to the base plate, thus subjecting it to the dead weight of the loading system. Additional load was applied by means of the screw jack, in small increments. Although attempts were made to load to a fixed value of force, it was found in practice that it was more convenient to make a relatively large number of load increments, and subsequently interpolate the results to standard load levels to compare different orientations of the wheel. Once the lower spokes became visibly detensioned, the load was removed and a check made on the residual force in the spokes.

This procedure was repeated for the wheel in many different orientations. Most attention was paid to the case where the gauged spokes were nearly in contact with the ground. Tests were made with the wheel rotated by 5° between ±15° of the reference point. Tests were also made with the reference point at +60°, +120°, and 180°.

The complete set of tests was carried out on the bare wheel (i.e., with no tire), and with the tire fitted and pressurized to 0.21, 0.42, and 0.63 N/mm^2 (30, 60, and 90 lb/sq in.).

Test Results

The data from each test were stored on data cassette on the microcomputer for subsequent processing. So that the data could be displayed as a function of the rotation of the wheel, but at a standard load, the raw data were interpolated to give results at standard loads of 500, 1,000, 1,500, and 2,000 N, by means of a parabolic blending procedure.

Bare Rim Test

Fig. 5 shows the spoke strain results when each of the gauged spokes was at the bottom. The predictions of Pippard's analysis are shown, as are the strains in the spokes due to the pretensioning; to assure that the 'trueing' of the wheel was not upset, these latter values were obtained after the main tests by detensioning the four gauged spokes to obtain a true zero strain.

It is clear that until the compressive strain cancels out the pretension strain introduced when the wheel was made, Pippard's analysis gives a good first approximation representation of the way the load is carried, at least as far as the most heavily pretensioned spokes are concerned. Spokes 1 and 4 behaved linearly up to axle loads of nearly 2 kN, while spokes 2 and 3 became slack and behaved nonlinearly at much lower load levels. When detensioned, these spokes were found to have much lower pretensions, and so lost tension at a lower wheel load.

All the test results were interpolated to the standard loads and plotted on polar plots, as in Fig. 6, which shows the results for the bare rim, under a load of 1,000 N. Individual strain readings for each spoke are shown, but as the four gauged spokes are 10° apart, the results are rotated by −15°, −5°, +5°, and +15° for the four spokes to make the results comparable.

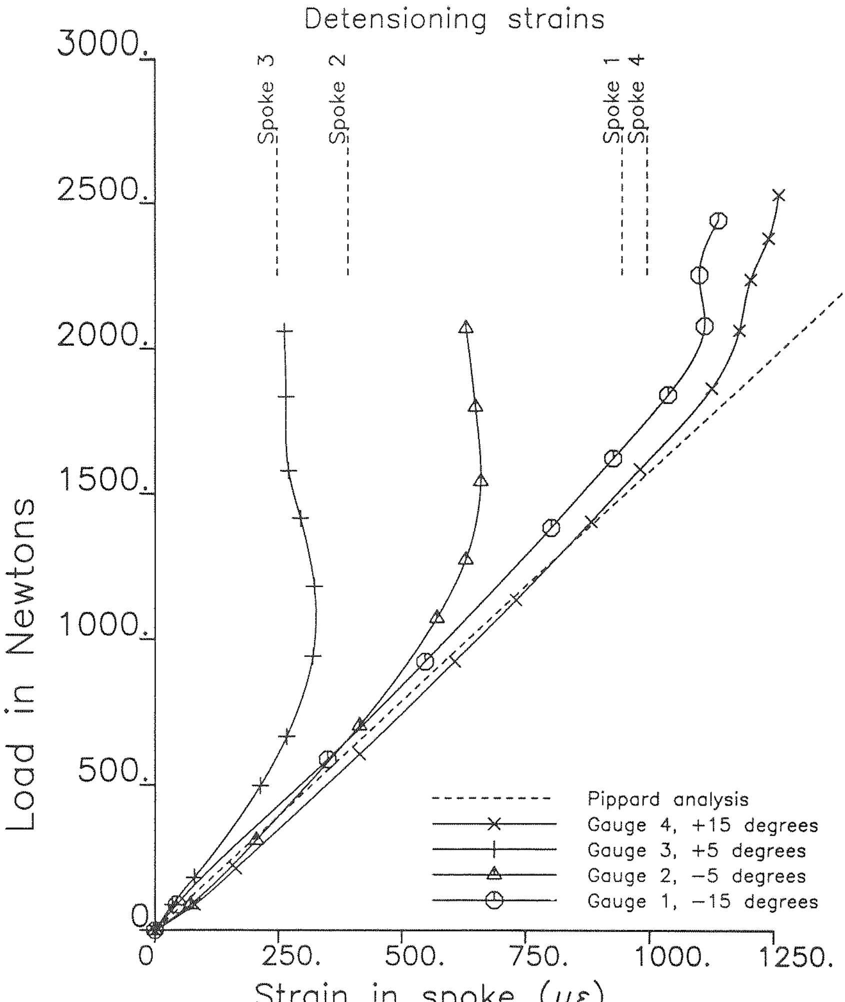

FIG. 5. Variation in Spoke Strain with Applied Load, for Bare Rim with Each Spoke at Its Lowest Point

Only the spokes in contact with the ground, or near the ground, show significant strains. For the other spokes, there is a small tensile strain, which does not vary all the way round the wheel.

Fig. 7 shows more detailed results for spoke positions close to the ground, replotted in Cartesian coordinates. The test results are clearly similar to Pippard's analysis, agreeing well at the point of ground contact, but showing a higher compressive strain change at angles between $\pm 10°$ and $\pm 30°$ away from the ground contact.

The form of the deflected shape is similar to that observed in a straight beam on elastic foundation analysis under the influence of a single point load. The results of such an analysis, with the foundation stiffness deter-

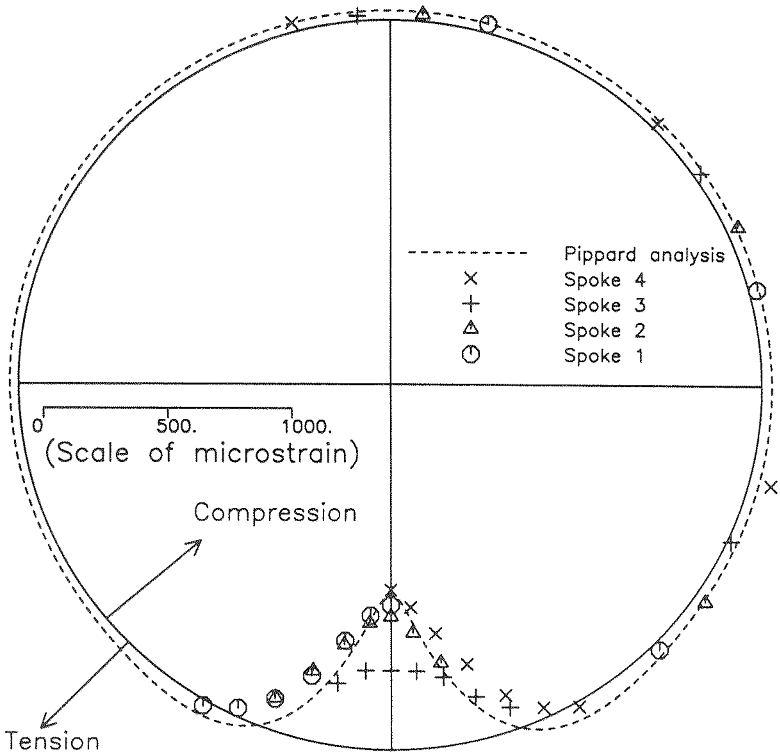

FIG. 6. Variation in Spoke Strain for Applied Load of 1,000 N on Bare Rim as Wheel Is Rotated

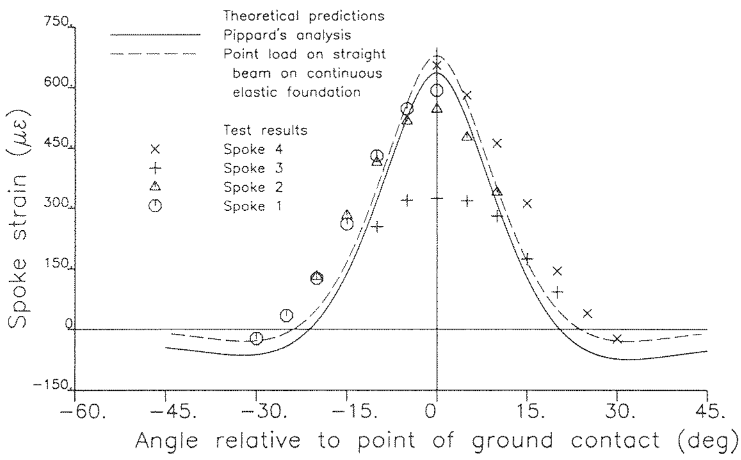

FIG. 7. Variation in Spoke Strain versus Position on Bare Rim

mined by smearing the spoke stiffness uniformly along the rim, are also shown in Fig. 7. The behavior of the circular wheel (as given by Pippard's analysis) can be seen to be very similar to the straight elastic foundation result, offset by about 40 $\mu\varepsilon$ in the tension direction. The offset corresponds to the small uniform increase in tensile strain in all the spokes away from the loading point (as seen in Fig. 6).

Tests with Inflated Tires

Tests on wheels with tires in place showed very little change in air pressure, even at very high load levels; this is slightly surprising, given that the tire was squeezed virtually flat at high load levels. The changes in the measured values were close to the sensitivity of the measuring apparatus and are negligible when compared with the inflating pressure.

Air pressure itself is not affected by the load, but it is necessary to find the effect of air pressure on the spoke tensions. Fig. 8 shows the change in spoke strain for spoke 4 (chosen since this spoke is least affected by loss of prestress), as a function of the air pressure for the spoke in its most heavily loaded position. These results are directly comparable with Fig. 5; the effect of the pneumatic tire is to reduce the peak strain, since the tire will spread the point contact over a length of the rim.

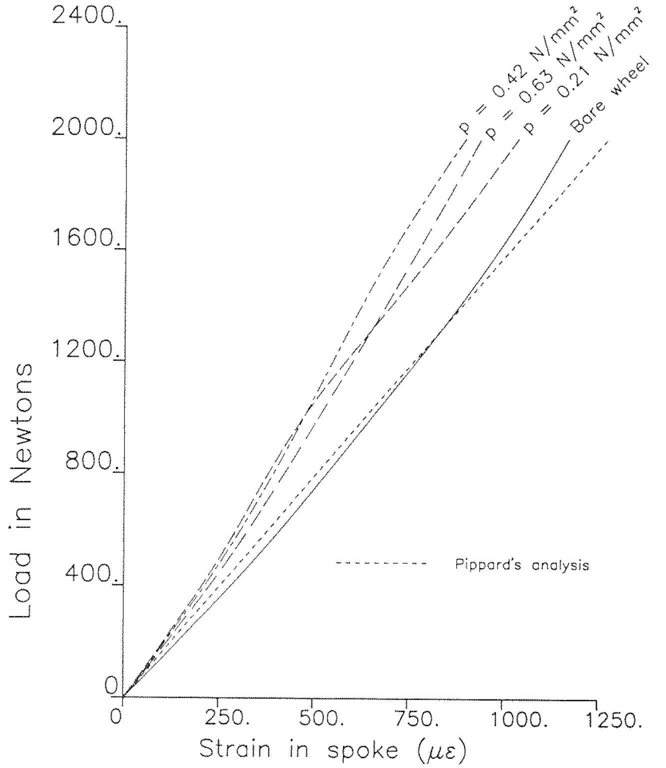

FIG. 8. Variation in Spoke Strain with Load for Spoke 4, with Different Tire Inflating Pressures

This effect can be seen more clearly in Fig. 9, where the distribution of spoke strains in the vicinity of the point of contact is shown, for a load of 1,000 N. The results are given for gauges 1 and 4 (the most heavily tensioned spokes), for the three different tire pressures tested. The bare rim results, and Pippard's analysis (which only applies to the bare rim), are also shown for comparison. Inflation of the tire broadens the peak, and reduces its intensity by about 20%, but subsequent changes in the pressure do not significantly affect the distribution of forces in the spokes.

The straight beam on elastic foundation analysis can be used to investigate the sort of loading that would cause the observed spreading of the results in the inflated tire tests. The dotted line on Fig. 9 shows the effect to be expected if the load of 1,000 N is spread over a length of 118 mm (which corresponds to 21.8° in the circular measure used here). This shows a good agreement with the test results.

Bending Moments

The bending moment in the rim can also be determined. Two strain gauges were attached to the rim midway between spokes 2 and 3; one on the centerline of the rim on the inside of the wheel, while the other was placed on the flat side of the rim as close to the outside as possible (see Fig. 1). The latter gauge was not ideally placed, since the gauge width was not negligible with respect to the specimen being measured. The results presented are based on the assumption that the strains recorded for this gauge are the strains in the specimen on the centerline of the gauge.

The measured values of the bending moment in the rim are shown in Fig. 10, with some theoretical predictions for comparison. Pippard's predictions are shown, and the bare rim test results agree well. The analysis of a straight beam on a continuous elastic foundation under a point load is omitted, since it matches almost exactly Pippard's analysis. This is to be expected, since the difference noted between the two theories when considering the spoke strain was a virtually constant offset, which would not change the rim bending moment.

Theoretical results are also given in Fig. 10 for a straight beam on discrete

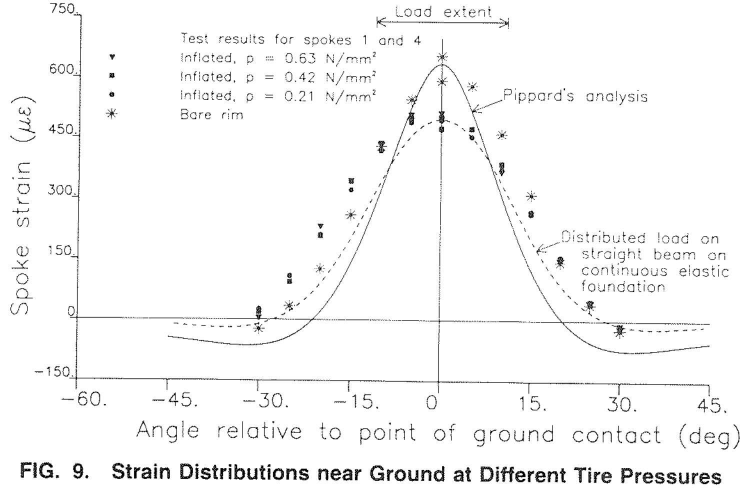

FIG. 9. Strain Distributions near Ground at Different Tire Pressures

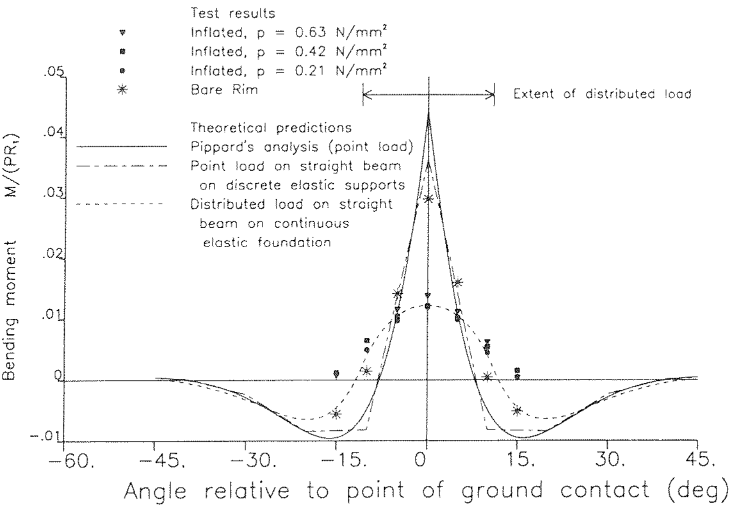

FIG. 10. Bending Moment in Rim

elastic supports, which shows a series of straight lines between support positions (assumed to be at 0°, ±10°, ±20°, . . .).

Finally, Fig. 10 shows the predictions for a straight beam subjected to a uniformly distributed load over a length of 118 mm; as with the spoke strain results, this gives good agreement with the observed behavior of the wheel with an inflated tire.

The value of 118 mm for the length of the distributed load was chosen as the value that gave the best prediction for the magnitude of the peak strain in the spokes and peak moment in the rim. Other load lengths, and different load distributions (e.g. triangular or sinusoidal) might have given better predictions for other values, such as, for example, the places with zero strain or zero moment.

The analyses of the beam on elastic foundation were carried out using standard formulas given by Hetenyi (1946). The analysis for a beam on discrete elastic supports was carried out using a standard stiffness-based frame analysis program.

The change in the axial force in the rim as measured in the experiments was quite high, being about twice the magnitude of the applied load (in tension), for a few degrees on either side of the point of ground contact in a bare rim, but dropping off rapidly to about one-tenth of the applied load in compression in parts of the rim remote from the point of contact (for both the bare wheel and the wheel with an inflated tire).

Three Prestressing Systems

Spokes as Prestressing Elements

The results of the tests, which broadly confirm Pippard's analysis, make the prestressing behavior of the spoke/rim system clear. The rim does very little to carry the load. Its main function is to provide the reaction system for the tensile prestress in the spokes, which, as a result of the pretension, are able to carry significant compressive forces before they buckle. The rim also acts in local bending to distribute load to the spokes. The spokes at or near the point of the ground contact do virtually all the work; the remainder only see a small tensile strain, which equilibrates the increase in axial compression in the rim.

In prestressing system 1, the system that carries the load is the group of lower spokes, which are prestressed by the rim and the remaining spokes.

The effect of the pneumatic tire on the stresses in the wheel must also be considered. Not only does the tire serve to make the journey more comfortable for the cyclist, but it also distributes the load around the wheel, thus eliminating concentrated loads. A bare wheel with no tire, in contact with a hard, flat surface, will be subjected to a concentrated point load (as analyzed by Pippard). However, the tire flattens in contact with the road surface, thereby spreading the load on both the road and the rim.

One common, but quite fallacious, belief is that the load is carried to the rim by air pressure. The contact area between the air itself (in the case of a tubeless tire) or the inner tube and the rim is fixed, and is the same for all points around the circumference of the wheel. Thus, the air pressure acts equally in all directions and contributes nothing to the net force in the rim.

A simple analysis of the zone of contact between the tire and the road surface shows that the area of contact must be equal to the load on the wheel divided by the tire pressure, and that the pressure exerted by the tire on the ground must be uniform and equal to the tire pressure. This analysis is based on the assumption that the tire is flattened against the road, and has negligible flexural stiffness, so no bending of the tire occurs and there are no shear forces.

Such a simple analysis is probably valid for bicycle wheels, where the tires are thin and the inflation pressures fairly high. For cars and other motor vehicles, the same assumption will not be valid, and there is test data (Davies 1988) that shows that the contact pressures can significantly exceed the inflation pressure (with a corresponding reduction in the area of contact); this implies that there must be significant shear stresses set up within the tire, particularly at the edges of the contact zone.

The distinction between tubeless tires (common in motor vehicles), and those with an inner tube (common in bicycles) is of no concern here. Although the inner tube will itself act to resist the effects of the air pressure, the resistance it imposes on the air pressure will be much less than that imposed by the tire and rim. The inner tube acts solely as an airtight sealing system, rather than as a structural element, and in what follows, the inner tube itself will be ignored.

Sidewalls as Prestressed Elements

The compressive contact force between the wheel and the road has to be transmitted from the wheel, through the tire, to the surface. The sidewalls of the tire can clearly carry no compression; they are enabled to carry the force by means of the pretension in the side walls caused by the inflation pressure.

Consider the two free-body diagrams shown in Fig. 11. The first shows an inflated tire, not in contact with the ground. The tension in the sidewall (per unit length along the tire), will be given by

$$2P_s = wp \qquad (1)$$

where w = maximum width of the tire; and p = inflation pressure.

When the tire is in contact with the ground, over a width w_c (where the

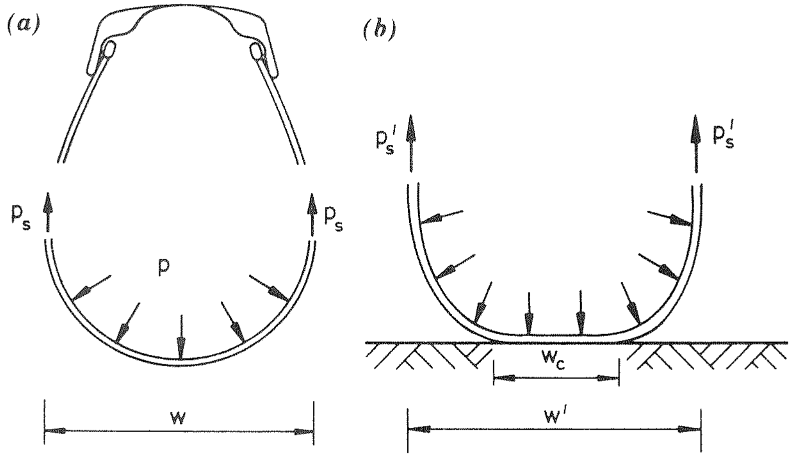

FIG. 11. Forces in Tire Sidewalls: (*a*) Tire Inflated; (*b*) Tire Inflated on Ground

inflating pressure balances the external load P), the tire expands slightly to a maximum width of w', giving a new tire wall tension of

$$2P_s' = w'p - P \quad (2)$$

This assumes that the inflation pressure remains sensibly constant, as was observed in the test.

Combining these two results gives

$$P = 2(P_s - P_s') + (w' - w)p \quad (3)$$

The load is thus being carried by a reduction in the initial pretension of the sidewalls (from P_s to P_s'), and by an increase in the width of the tire (from w to w'). The second prestressing system can thus be identified.

In prestressing system 2, the load is transmitted from the road surface to the rim by compressive forces in the tire sidewalls, which are held in tension by the prestressing action of the air pressure.

Tire Cord as Prestressing Element

The final element of prestressing in the system occurs at the junction between the tire and the rim. The forces P_s in the sidewalls of the tire are tensile stresses, which have to be developed where the tire comes into contact with the rim. This is done by the tire bead, which runs around the edge of the tire. This is shorter than the perimeter of the rim on which it fits; the tire bead is in tension when in place, and thus exerts a compressive force on the rim. When the tire is inflated, the tension induced in the sidewall acts to reduce this compressive force.

It is worth considering how the contact force between the tire and the rim varies with load. There will be a force P_r in the tire sidewall at the rim, which, because the tire has negligible bending stiffness, must be tangential to the tire at that point. This will be superimposed on the prestressing force between the cord and the rim, which acts purely locally.

This force will be inclined at an angle θ to the radial direction (Fig. 12), so the inward component of the force will be $P_r \cos \theta$. When the tire is loaded, this force will alter to a new value P_r', and the inclination angle will

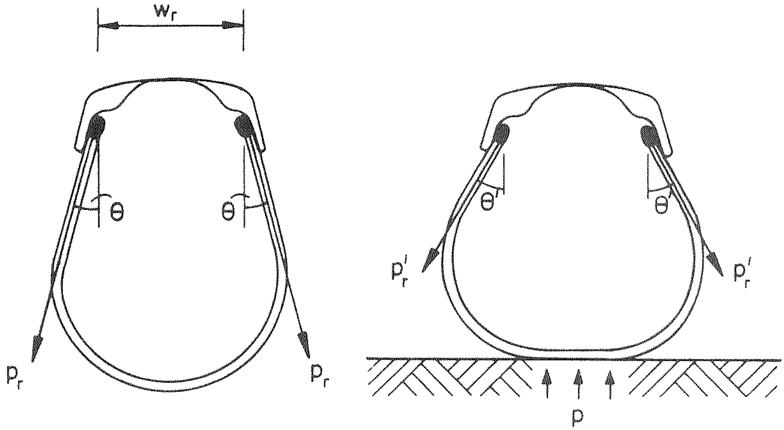

FIG. 12. Forces between Tire and Rim: (*a*) Tire Inflated; (*b*) Tire Inflated on Ground

change to θ'. The applied load P will thus be carried by the difference between the two forces

$$P = 2P_r' \cos \theta' - 2P_r \cos \theta \qquad (4)$$

The tested wheel had a lip on the outer edge of the rim, which may carry some of the outward force, but many bicycle rims and almost all car wheels have no lip. Thus, on these rims, all of the force P_r must be resisted by the inward force of the bead.

In prestressing system 3, the tire bead causes a compressive force between the tire and the rim, which resists the tensile stresses in the sidewall.

Conclusion

The paper has demonstrated that the bicycle wheel achieves its remarkable structural efficiency by the use of prestressing, which occurs in three distinct ways, each developed by a different inventor.

- Starley developed the tensioned spoked wheel (prestressing system 1) in 1874, so that compressive forces could be carried by a reduction in the spoke prestress.
- Dunlop developed the pneumatic tire (prestressing system 2) in 1888, but had to devise a complicated method for keeping the tire attached to the rim.
- Welch, in 1890, overcame the deficiencies of Dunlop's tire by additionally prestressing the tire against the rim (prestressing system 3).

Tests carried out on a wheel have shown the validity of Pippard's analysis on spoked wheels, for the case of the bare rim without any tire. It has also been shown that the good agreement is also achieved by a simpler analysis that assumes that the rim can be regarded as a straight beam resting on a uniformly distributed elastic foundation.

It has been shown that the use of a pneumatic tire reduces the peak stress changes in the spokes, and that this can be reasonably modeled by assuming that the applied load is uniformly distributed over a short length of the rim.

Once an inflated tire is in place, the effect on spoke loads and rim moments of changing the inflation pressure is not significant. It has been noted that the air pressure does not change measurably when a load is applied to the wheel.

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Appendix II. Notation

The following symbols are used in this paper:

M = bending moment in rim;
p = tire inflation pressure;
P = applied load on wheel;
P_r = force between tire and rim;
P_s = force in vertical sidewall;
R_1 = radius of rim;
R_2 = radius of hub;

w = width of tire;
w_c = width of contact area with road;
ε = angle of inclination of spokes to radius;
Θ = angle of inclination of sidewall to rim; and
$'$ = primes (′) denote that wheel is in contact with road, unprimed quantities denote no road contact.