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Automated determination of concordant profiles

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The Paper presents a method by which a designer can automatically determine a concordant cable profile that fits within eccentricity bounds governed by the applied moment envelopes. The method works by determining loadings which cause peak bending moments at a particular point; these moment diagrams are scaled concordant profiles. By using an iterative method, these loadings can be combined to give a composite notional loading; the corresponding bending moment, when scaled by an arbitrary factor, is the final concordant cable profile. The method also allows the shape of the concordant profile to be varied over internal supports, so that a smoothly curved cable can be fitted through an allowable zone which has sharp angular changes in the limits. The engineer is thus freed from the necessity of estimating secondary moments at the beginning of the design process. Instead, a concordant line of thrust can be determined and linear transformations applied to obtain the actual cable profile.

Notation

Tensile stresses are positive.
Sagging bending moments are positive.
Positions are measured positive downwards from centroid.

A	cross-sectional area
A_{ij}	matrix relating variables to constraints
e_p	line of thrust of cable
e_r	range of permissible eccentricities
e_s	actual cable profile
EI	flexural stiffness
f_c	permissible stress in compression (− ve)
f_t	permissible stress in tension (+ ve)
L	total length of beam
l_i	lower constraint
M	applied bending moment
M_2	secondary moment
P	prestressing force
q_j	independent variables (notional loads)
r_i	notional support loads
R	radius of curvature of cable profile
s	length of each notional load
u_i	upper constraint
w_i	notional loads
Z	elastic section modulus
θ	slope of bounds on cable profile
ω	accelerating factor

Written discussion closes 18 August 1988; for further details see p. ii.
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Introduction

The design of continuous prestressed concrete beams is complex and, if not approached in a logical sequence, can lead to the designer carrying out abortive calculations which waste time and effort. As a simplification, the design process can be reduced to the following sequence of operations

(*a*) determination of moment envelopes
(*b*) choice of cross-section
(*c*) choice of prestressing force
(*d*) choice of cable profile.

2. The first stage is essential, although tedious to perform by hand. It can be automated,[1] but is purely mechanical, and will not be discussed further.

3. The second stage is much more complicated, relying on the engineer's judgement and experience, as well as calculations of strength and stiffness at critical sections. It is at this stage that ultimate strength and shear strength considerations are taken into account. Once again, this process will not be looked at in detail, although it is worthy of further study as an exercise in the rationalization of the design process.

4. Once the cross-section has been chosen, the two interrelated stages of the choice of prestressing force, and the cable position within the beam, need to be considered. In an earlier paper,[2] which was a generalization of Low's work,[3] a method was presented for the determination of the limits on the prestressing force in a continuous beam. It was shown that the designer must consider limits on the magnitude of the prestressing force governed by stress limits at every cross-section, by the need for the range of eccentricities to be less than the depth of the beam (taking account of cover), and by the need to ensure that a valid line of thrust exists. Reference 2 covers the basic theory of prestressing in continuous beams; it is assumed in the present Paper that readers are familiar with this work.

5. The final problem that remains is the choice of the cable profile, which is the subject of this Paper. However, it is firstly necessary to be precise about the distinction between 'line of thrust design', and 'cable profile design', which differ in the way they deal with secondary moments. The beam will be subjected to a set of external bending moments M, and if indeterminate, to a set of secondary moments M_2. The prestressing cable, placed at an eccentricity e_s, tries to induce in the beam a curvature of Pe_s/EI. If these curvatures are not compatible with zero displacement at the supports, a set of self-equilibrating reactions will be caused, which will induce secondary (or parasitic) moments in the beam. The total moment induced in the beam by the prestressing cable is thus $Pe_s - M_2$, so that the cable appears to act at a different position e_p, where

$$Pe_p = Pe_s - M_2$$

The designer can choose between two alternative strategies, both of which are perfectly valid.

Line of thrust design

6. The designer can treat the secondary moments as prestressing effects. There will then be a series of stress conditions of the form

$$f_c \leqslant -\frac{P}{A} - \frac{Pe_p}{Z} + \frac{M}{Z} \leqslant f_t$$

which can be rearranged to give limits on the line of thrust of the cable e_p of the form

$$-\frac{Z}{A}-\frac{f_c Z}{P}+\frac{M}{P} \geqslant e_p \geqslant -\frac{Z}{A}-\frac{f_t Z}{P}+\frac{M}{P}$$

(The direction of the inequalities and the signs of the various terms will depend on whether the top or bottom fibres are being considered, and whether hogging or sagging bending moments are considered positive. However, the generic form given here is sufficient for illustrative purposes.)

7. The designer will find a concordant profile which satisfies these conditions; an actual profile can then be chosen which varies from the line of thrust by a linear transformation. The secondary moments are taken into account automatically, but the designer is faced with the difficulty of determining a concordant profile that satisfies the limits on e_p.

Cable profile design

8. Alternatively, the designer may choose to treat the secondary moments as loads, so they appear with the applied moments in the eccentricity equations, which can now be written in terms of the actual cable profile e_s

$$-\frac{Z}{A}-\frac{f_c Z}{P}+\frac{(M+M_2)}{P} \geqslant e_s \geqslant -\frac{Z}{A}-\frac{f_t Z}{P}+\frac{(M+M_2)}{P}$$

The designer will estimate (guess?) the magnitude of the secondary moments, determine the limits on the cable position, and then find a cable profile which not only satisfies these limits, but also gives the chosen values of M_2. The successful use of this method requires a thorough understanding of the way the structure behaves, and considerable experience in dealing with similar structures.

Cable profile over internal supports

9. Before beginning the formal solution of the problem, there is one further complication that needs to be taken into account. The bending moment envelopes at internal supports will, for most structures, show peaks in hogging bending. This will be reflected in the corresponding line of thrust limits as a discontinuity in the slope of the limits, as shown in Fig. 1. The actual cable profile, however, will be a smooth curve, since minimum radii of curvature of the order of several metres are specified for prestressing cables. This restriction can be relaxed somewhat if cables are being anchored at the supports, but that is exceptional since the presence of anchorages causes congestion in what is normally already a complicated area of the structure.

10. The requirement is that the cable profile e_s should be smooth over the piers, but the line of thrust may itself have a kink at that position, since e_p differs from e_s by a linear transformation. However, the designer will probably know the magnitude of the secondary moments that are required, and since the cable force is known, this will allow an estimate to be made of the required change in angle of e_p at the support.

Definition of problem

11. In this Paper, the principles of line of thrust design have been adopted for a number of reasons.

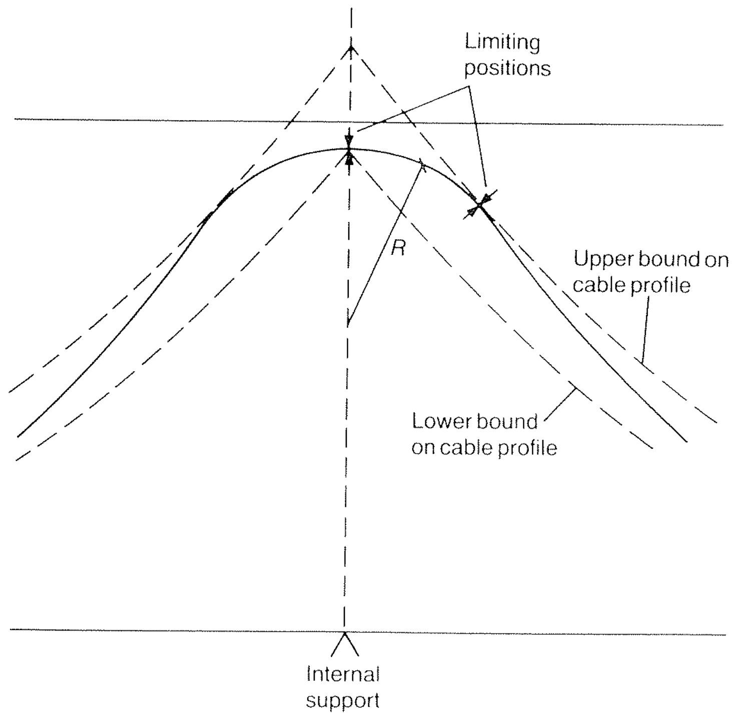


Fig. 1. Discontinuity in slope of envelopes at internal supports

(*a*) The eccentricity limits are functions only of the applied external load, and are independent of the cable profile adopted.
(*b*) Actual cable profiles can be generated by linear transformations from the line of thrust, so there is no loss of generality.
(*c*) It is possible to generate 'families' of concordant profiles, by considering the bending moments that result from the effects of notional external loads on the structure.
(*d*) Although it is simpler to seek a cable profile that has no kinks over the support, it is perfectly feasible to seek a line of thrust that has a specified kink at those positions.

12. Therefore, a procedure is sought which allows the automatic calculation of a concordant profile that satisfies the limits on the line of thrust and has a known discontinuity of slope at each internal support. It will be assumed that the cross-section, the maximum and minimum moment envelopes, and the prestressing force are all specified or have been chosen already.

Solution strategy

13. The solution technique relies on the fact that any bending moment diagram that can be produced by an external load acting on the beam will itself be a scaled concordant profile. This follows since both the external loading and the concordant profile result in identical curvatures of the beam, which are compatible with zero displacement at each support. Therefore, that notional external loading is sought which gives a bending moment diagram which, when scaled by an arbitrary factor, gives an acceptable profile. The condition that the profile has the required discontinuity of slope at internal supports can be met by specifying that the reaction to the notional load is of a specified magnitude at that support.

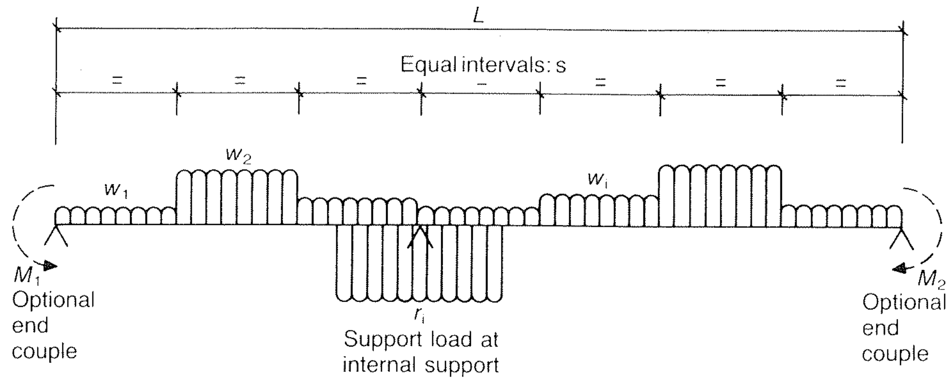


Fig. 2. Notional loading

14. The notional external load may be considered as a series of short uniformly distributed loads of varying magnitude (Fig. 2). The corresponding bending moment diagram will consist of a series of parabolae, with no discontinuities of slope, except at internal supports. To allow some control over this discontinuity, additional external loads are introduced over the area immediately adjacent to the internal supports, which act to relieve the support of its load, and hence reduce the slope discontinuity in the bending moment. Throughout this Paper, these loads will be termed 'support loads', to distinguish them from other notional loads.

15. If it is assumed that the beam has a notional prestressing force of 1000 kN (an entirely arbitrary scaling factor), then a bending moment of 1000 kNm corresponds to an eccentricity of the concordant profile of 1 m. As a consequence of the loading chosen, the bending moment at the ends will be zero, which corresponds to a line of thrust (and also a cable profile) with zero eccentricity. This is perfectly reasonable, because, if the end of the beam is pinned, there will be no bending moment resulting from the real load at that position, so it makes sense to have a prestressing cable which acts at the centroid at these positions. However, if it is considered desirable to have a cable with an eccentricity at the end, this can be achieved by adding a couple to the notional loading with an appropriate magnitude.

16. The definition of the notional loading forms the set of independent variables for the present problem. There will be L/s values for the loads w_i, $n-2$ values for the distributed support loads r_i, and 2 values for the end couples M_i. The constraints of the problem, which the independent variables are chosen to satisfy, are the required eccentricity of the line of thrust, and the discontinuity of slope of the cable at the internal supports. These are equivalent to the required bending moment and internal support reactions of our notional loading.

17. The limits on the eccentricity are continuous functions, which are thus definable by an infinity of conditions. However, the limits on the cable position can be satisfied at a number of discrete points, thus reducing the number of conditions to a manageable level. The positions at which the constraints along the beam are satisfied need not be regularly spaced; indeed, it is preferable to check the position of the cable more frequently in regions where the profile is changing rapidly, such as near a support, and less frequently in regions where the cable profile is smooth (Fig. 3).

18. For each of the independent variables (q_j, the notional loads), the effect

that a change in that variable has on the position of the cable can be determined by carrying out a structural analysis under the action of that load.

19. The end result will thus be a series of conditions of the form

$$l_i \leqslant A_{ij} q_j \leqslant u_i$$

The matrix A_{ij} is not necessarily square, and it should be noted that the conditions are inequalities.

20. Before proceeding further, it is necessary to consider the solution technique that will be adopted, as a number of alternatives can be considered. The problem concerns a set of linear inequalities, for which a feasible solution which satisfies these inequalities is being sought. Superficially, the problem is similar to the standard linear programming (LP) problem, with the important difference that it is not the intention to minimize or maximize a function.

21. The Simplex algorithm, which is the most commonly used technique for the solution of LP problems, works in two stages. Phase 1 finds a feasible solution to the problem, and is then followed by Phase 2 which optimizes that solution. Therefore, to solve the problem, a technique which went through phase 1 of the LP solution but not phase 2 could be used. However, Phase 1 is itself a complicated procedure, requiring one iteration for every constraint. In this case, that would mean a large number of iterations; if, for example, a beam were 100 m long, then to check that the cable profile was satisfactory every 5 m along the beam, would require 20 upper bound constraints and 20 lower bound constraints, giving a total of 40 iterations. Additionally, the Simplex algorithm makes heavy demands on computer memory, so the technique is not particularly attractive in the present example.

22. Alternatively, a technique such as Gauss–Seidel or Successive Over Relaxation (SOR) can be modified to cope with the present problem. If a set of linear

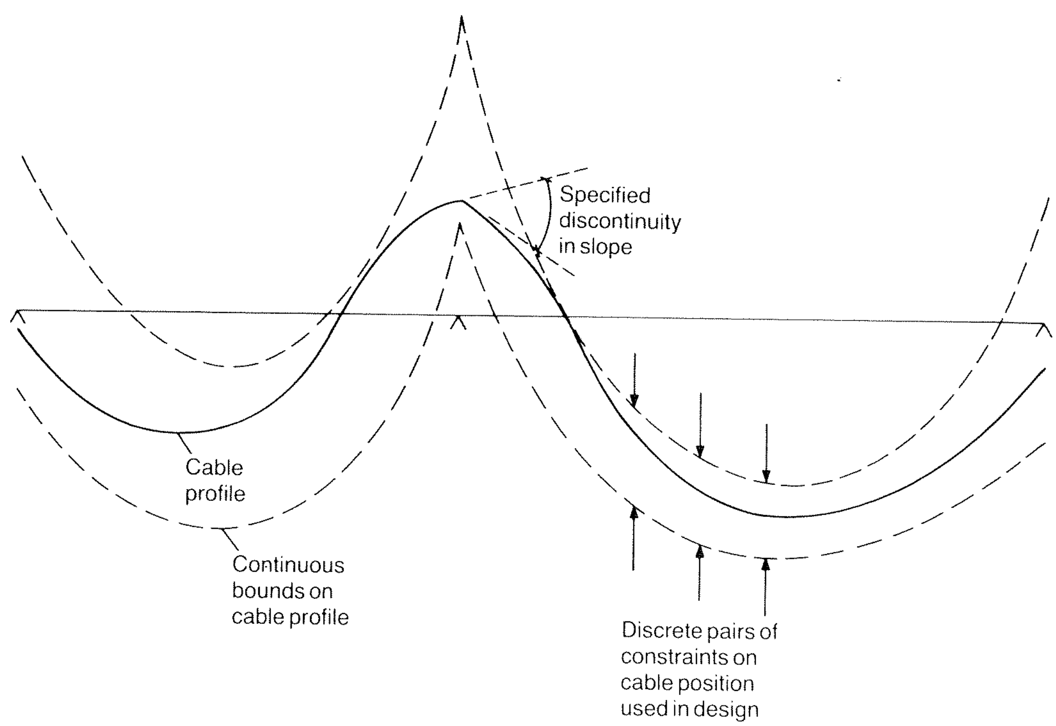


Fig. 3. Constraints on bending moment attributable to notional loads

equations had to be solved (and certain convergence criteria, which will be discussed later, were satisfied), an iterative technique to find the unique solution could be used. In essence, these methods work by determining the error in the present estimate of the solution, and calculating the change in the independent variable necessary to eliminate that error. In this case, the error can be regarded as the amount (if any) by which the inequality is not satisfied. Iteration continues until all the inequalities are satisfied. It is not possible to estimate the number of iterations required *a priori*, or indeed whether convergence will occur at all.

23. The conditions for convergence of Gauss–Seidel and SOR for the solution of equations are quite complex,[4] but a normally sufficient (although not necessary) condition is that the matrix is diagonally dominant, with the largest element of each row on the leading diagonal. Thus, if the present problem is to be solved using SOR, the problem needs to be reformulated in a diagonally dominant way. To do this, reference can be made to the mechanics of the problem rather than to the mathematics.

24. The constraints that have to be satisfied are on the bending moment produced by the notional loads. If any condition is not satisfied, a loading which will cause a bending moment at the position in question needs to be found. This can be done by determining an influence line for bending moment at that position,

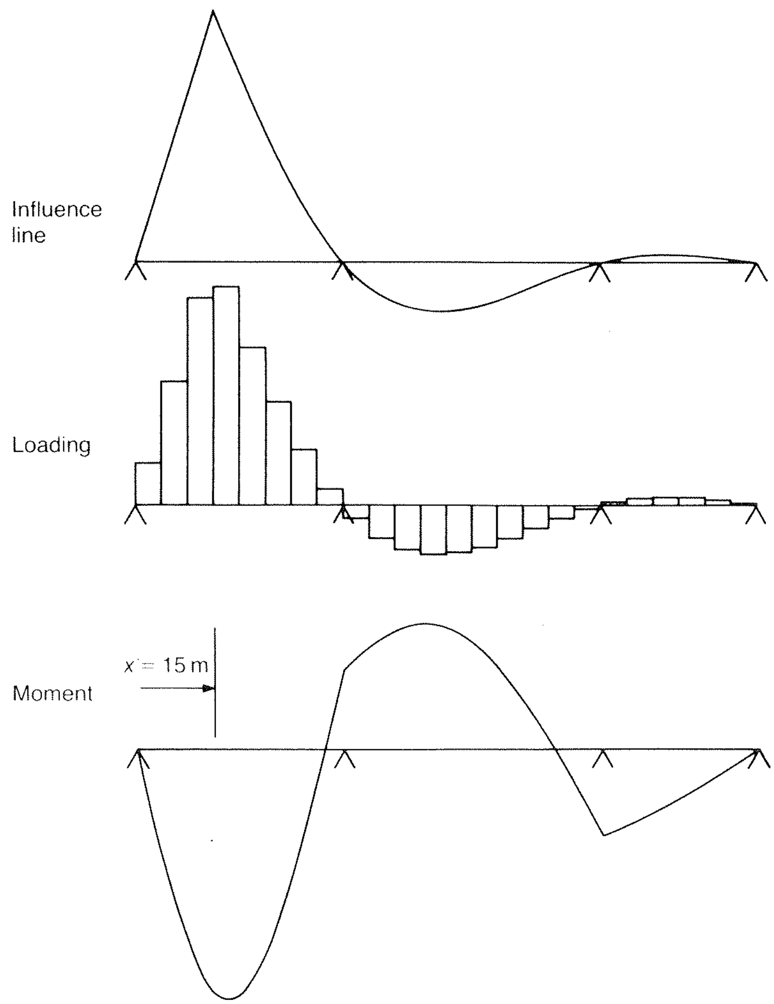


Fig. 4. Influence line at 15 m, with corresponding loading and bending moment

and then modifying the notional loads in proportion to the height of the influence line at the centre of each notional load. The process can best be illustrated by an example.

25. Consider the case of a 3-span beam, with spans of 40 m, 50 m and 30 m, for which a loading which generates a bending moment at 15 m from the left-hand end needs to be found. The arbitrarily chosen loading will consist of a series of uniformly distributed loads each 5 m long. The influence line for bending moment at that position is calculated, as shown in Fig. 4(a). A loading is then chosen in which the intensity of each distributed load is proportional to the ordinate of the influence line at the centre of the distributed load, as shown in Fig. 4(b), which results in the bending moment diagram for the whole beam shown in Fig. 4(c). (We are only concerned with distributions of loading and bending moment, since the results can be multiplied by an arbitrary scaling factor without loss of generality; thus all the plots are normalized and no scales are shown.)

26. The maximum ordinate on the moment diagram is not exactly at 15 m, but it is very close to it, indicating that the loading that has been chosen nearly satisfies the requirement for diagonal dominance. This would require that the loading chosen caused maximum moment at the point in question.

27. This technique does indeed yield loadings which are very close to the convergence requirement at most positions along the beam. Over the central portions of the spans and also directly over internal supports, the maximum moment does indeed occur very close to the point where the influence line is calculated, as shown in Figs 5 and 6. However, at positions close to internal supports the situation is not so clear.

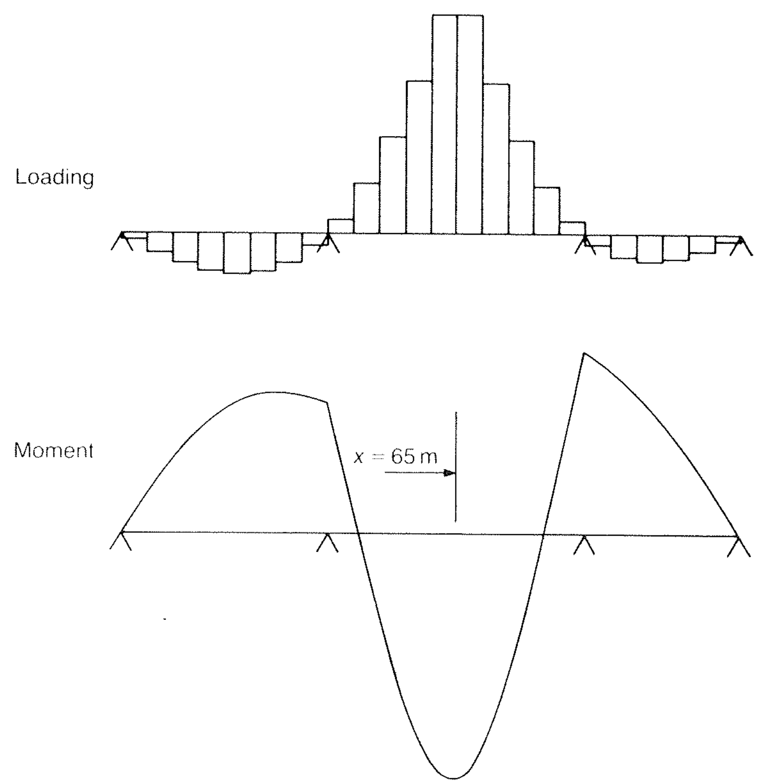


Fig. 5. Loading to give moment at mid-span

28. Figure 7 shows the results for a position 5 m from one of the internal supports. The maximum moment occurs near the middle of the adjacent span, so that this loading will not then satisfy the conditions for diagonal dominance.

29. In practice, this does not appear to be a problem, since convergence has been observed in all cases studied by the Author. It appears that there is sufficient diagonal dominance in the conditions elsewhere in the beam to overcome any local tendency to diverge. Alternative loadings, in which the magnitudes were determined from the square of the ordinate of the influence line (as shown in Fig. 8), did not make a significant difference; the diagonal dominance was improved in regions where it already existed, but it made little difference in regions where it was absent, and no improvement was noted in the convergence rates.

30. The chosen procedure does not guarantee numerical convergence, but it is rational, in that the loading chosen is the one which is most likely to cause a maximum effect at the point in question, and it is convenient, in that it makes use of techniques which are readily available to designers.

31. One further refinement can be made. It is possible to apply a loading greater than that needed to eliminate the error at a particular position, in the manner of an over-relaxation technique. If the over-relaxation factor chosen is too high, convergence may not occur, but worthwhile benefits may accrue by an intelligent choice. This will be studied in more detail after the example has been considered.

32. The requirement to provide a line of thrust with a specified discontinuity at the support position can be met by controlling the reaction at the internal supports due to the notional loading. This can be done by providing a distributed load over a short region on either side of the internal support, which will normally be in an upward direction.

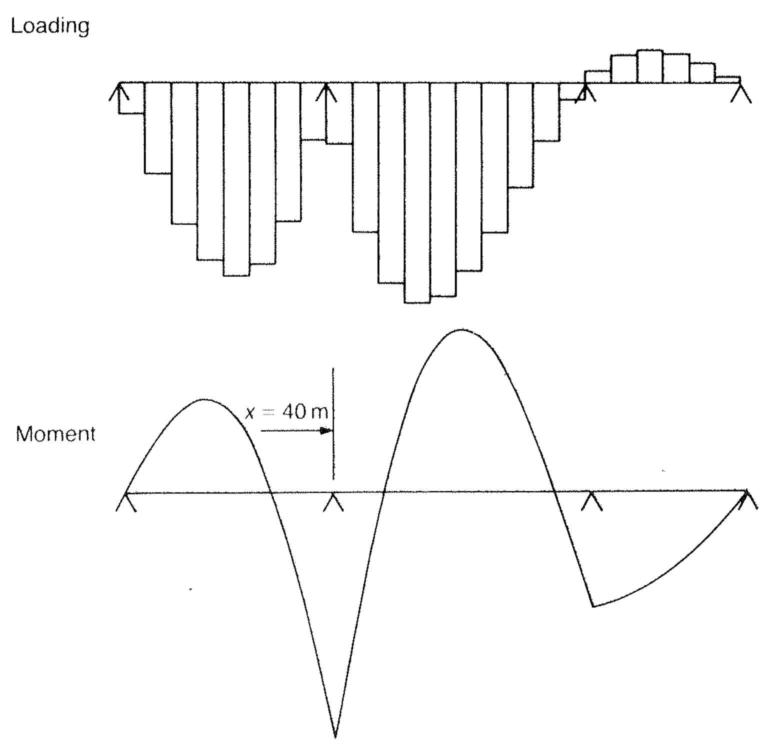


Fig. 6. Loading to give moment at support

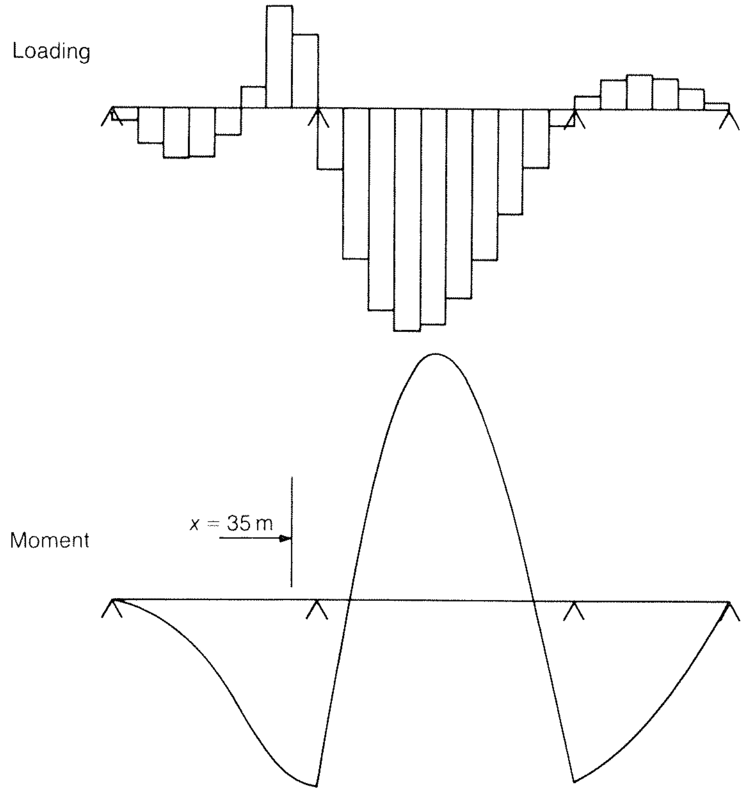


Fig. 7. Loading to give moment close to support

33. Figure 9 shows the effect of an upward load distributed over 5 m on either side of an internal support for the example considered above. The resulting bending moment has a sharp peak, and hence discontinuity, at the support in question, with relatively little effect elsewhere. Indeed, 99% of the distributed load is taken out at the adjacent support. Thus, by varying the magnitude of this distributed load, the support reaction can be altered to any desired value. The effect of the length of this distributed support load is also significant, and will be discussed in some detail later.

34. The number of the notional loads to be applied to the structure needs to be considered. In general, if equations were being solved, there would need to be as many equations as unknowns. However, in this case inequalities are being applied, with a range of possible solutions, so the conditions on the number of loads can be relaxed. It will be sufficient to ensure that approximately the same number of notional loads are provided as constraints that have to be satisfied (counting a pair of upper and lower bounds as one constraint, since we cannot fail to satisfy both simultaneously). If there are too few variables, a solution, if it exists, may not be found. There is no upper limit on the number of variables, but there is unlikely to be much benefit from taking too many.

Computer program

35. A computer program has been written to apply these principles to a particular beam. The program has the following stages.

(*a*) Read in the moment envelopes.

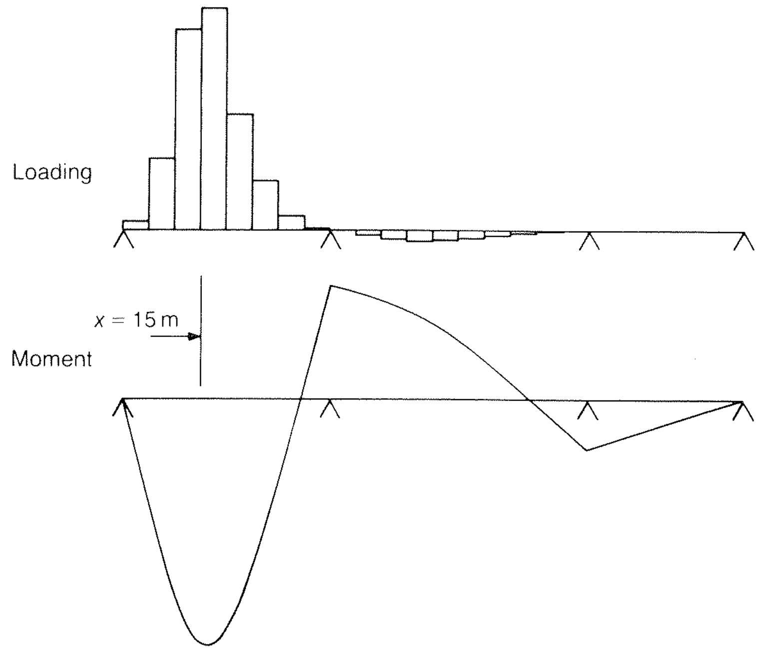


Fig. 8. Loading to give moment at 15 m, based on influence line squared

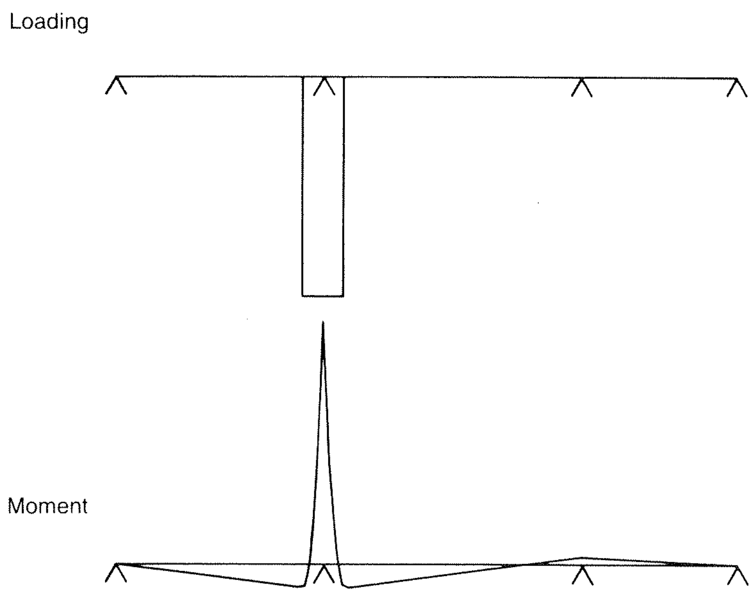


Fig. 9. Loading near support to give moment discontinuity

(*b*) Read in section property data, chosen prestressing force, etc.
(*c*) Ask user to specify the length of the notional loads, and set the load magnitudes to zero.
(*d*) Calculate upper and lower bounds on the bending moment induced by the notional loads.
(*e*) Analyse the structure under the influence of the notional loads.
(*f*) Find the position where the maximum error occurs in the bending moment owing to the notional loads. If there are no errors, go to step (*j*).

(*g*) Calculate the influence line for moment at that point.
(*h*) Determine the magnitude and distribution of the change in the notional loadings needed to correct the error in moment at the worst position, multiplied by an accelerating factor. Add to the existing loads. Modify the support loads by the error in the support reaction.
(*i*) Repeat steps (*e*) to (*h*) until convergence.
(*j*) Print and plot results.

Example

36. The procedure can be demonstrated more clearly by following a worked example. Consider the beam shown in Fig. 10, which is the same example used in reference 2. It has three spans of 40 m, 50 m and 30 m, and is subject to highway loading according to BS 5400: Part 2,[5] as modified by the Department of Transport.[6] The cross-section adopted is as shown in Fig. 11, and it was shown in the earlier paper that the prestressing force needs to be at least 51 822 kN to ensure that a line of thrust exists.

37. With a prestressing force of 52 000 kN, the upper and lower bounds on the line of thrust are as shown in Fig. 12, which are equivalent to limits on the bending moment caused by the notional load as discussed above. The notional loading will be chosen as a series of 24 uniformly distributed loads, each 5 m long, plus two uniformly distributed support loads, each 8 m long, giving a total of 26 variables. The constraints to be satisfied consist of the bending moment at 5 m intervals along the beam, with additional constraints 2·5 m on either side of the internal support positions, plus the values of the internal support reactions, giving a total of 29 constraints. In theory, there are more constraints than variables, so even if a feasible solution exists it may not be possible to find it. In practice, this does not appear to cause any problems.

38. In this example, the aim is to achieve zero reaction at the two internal supports, and an accelerating factor of 1·4 will be adopted. The initial notional loading is zero, so the equivalent cable profile is along the centroidal axis; the maximum discrepancy between this line and the required zone is at the left-hand internal support (chainage 40 m), with a value of −0·691 m. Therefore, for the first iteration, notional loads based on the influence line for bending moment at 40 m are added, as discussed earlier.

39. The results of this, and of the subsequent three iterations, are shown in Fig. 13, together with the limits on the line of thrust and the final concordant profile. After the fourth iteration, the changes being made at each stage are small, and the intermediate iterations have been omitted for clarity. Table 1 gives the position where the maximum discrepancy occurs after each iteration, and the maximum reaction. It can be seen that the maximum error in the cable profile reduces after most iterations, as does the maximum support reaction. The final iteration only involves elimination of the residual support reaction, since the cable profile is already satisfactory.

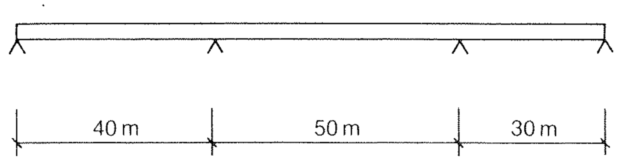


Fig. 10. Beam layout used in example

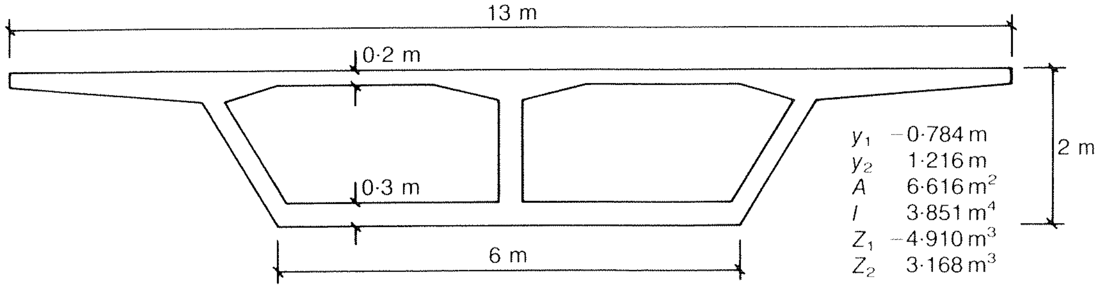


Fig. 11. Cross-section used in example

40. Figure 14 shows the notional loading chosen by the program. There are positive (i.e. downwards) loads applied over the mid-span regions in each span, together with large negative loads applied in the vicinity of the internal supports to eliminate any reactions.

41. The variation in stresses along the beam is shown in Figure 15; some of the stresses are close to zero (the specified tensile limit) at most points along the beam, indicating that a fairly efficient design has been obtained.

42. These results are typical; similar rapid convergence has been observed in a number of other test cases. It should be noted that the example given here is a fairly severe test case; the prestressing force is only just large enough to ensure that a feasible e_p exists.

Effect of variation of accelerating factor

43. The results in the above example have been obtained by the use of an accelerating factor ω of 1·4. In other words, at each stage, a correction to the notional loads is made that is 40% larger than that required just to bring the line of thrust within the allowable zone at the point in question.

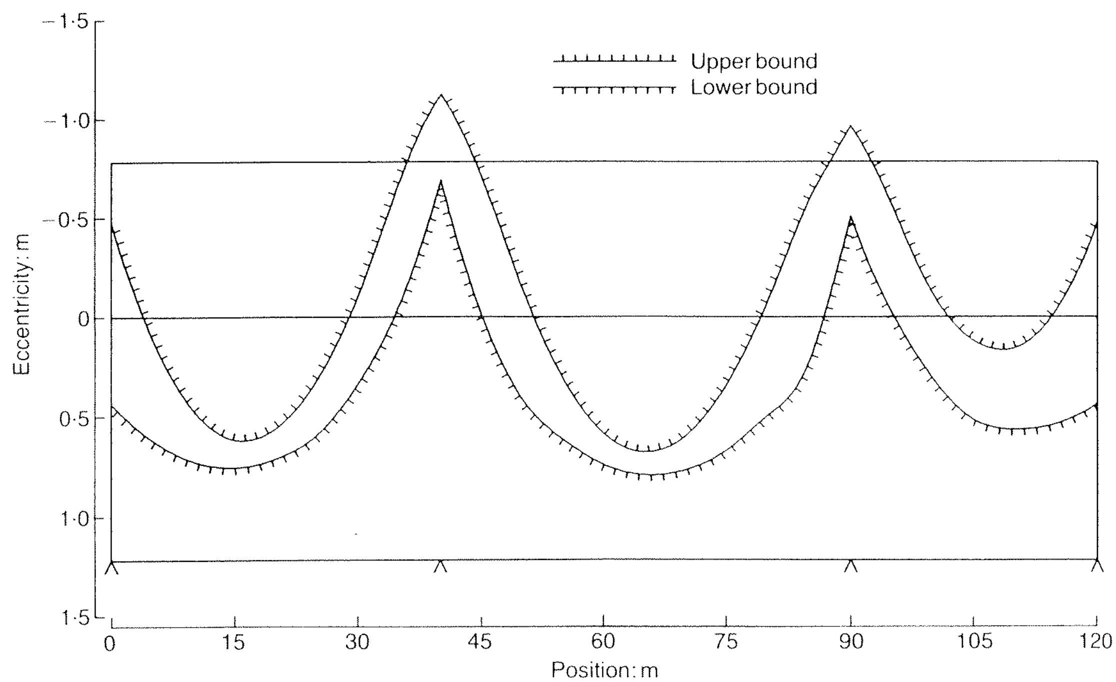
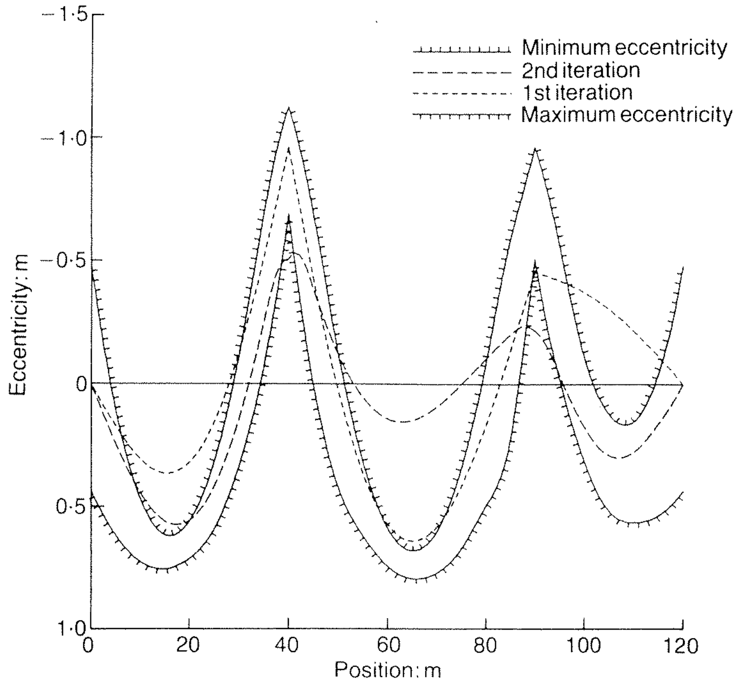
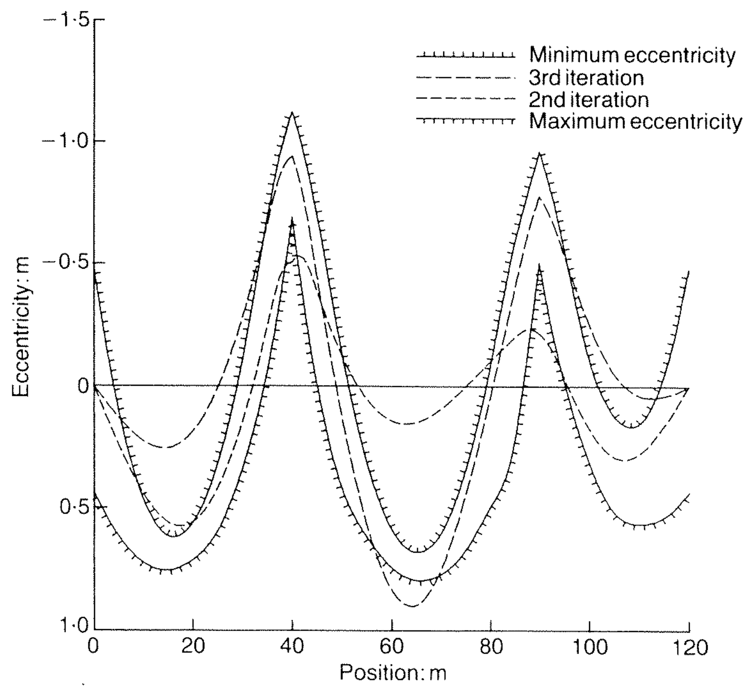


Fig. 12. Limits on e_p for $P = 52\,000$ kN



(a)



(b)

Fig. 13 (above and facing). Results of successive iterations for e_p: (a) 1st and 2nd iterations; (b) 2nd and 3rd iterations; (c) 3rd, 4th and final iterations

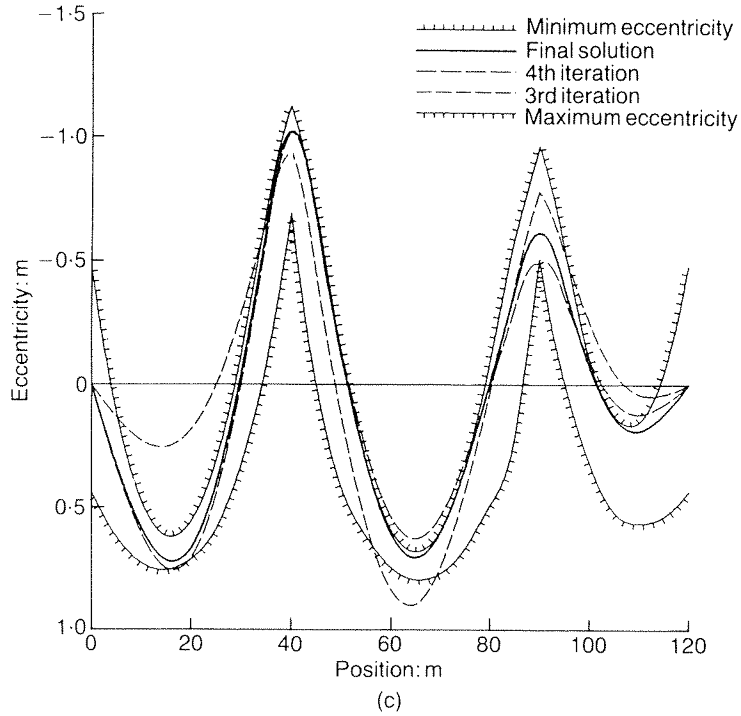


Fig. 13—continued

Table 1. Maximum errors after each iteration

Iteration	Maximum error: m	Position	Maximum reaction: kN	Support
0	−0·69100	40·0	0·000	–
1	0·42694	105·0	202·117	2
2	0·53190	65·0	−38·326	2
3	0·37371	20·0	84·592	3
4	0·06141	70·0	32·101	2
5	0·09966	105·0	10·806	3
6	0·08743	70·0	−9·116	2
7	0·02775	105·0	15·820	3
8	0·00632	42·5	−2·541	2
9	0·00656	55·0	−2·105	2
10	0·0	–	0·990	3
11	0·0	–	0·0	–

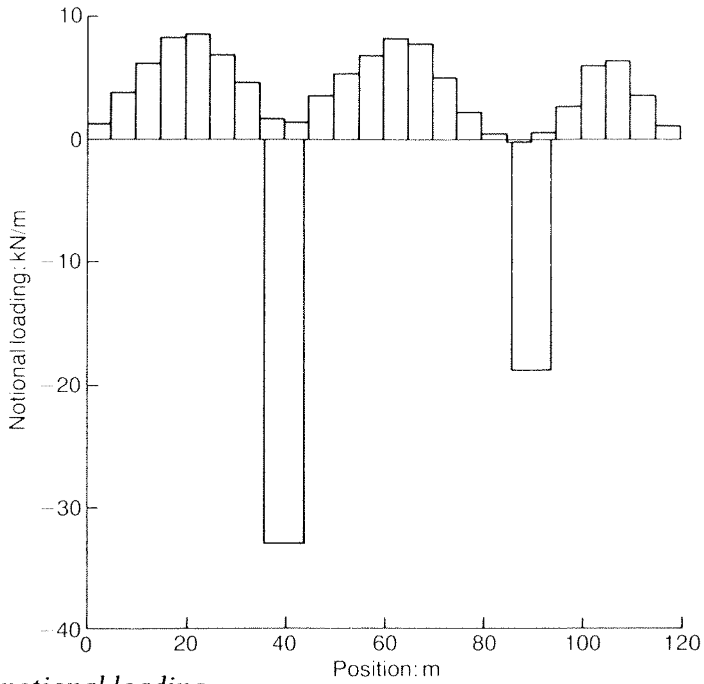


Fig. 14. Final notional loading

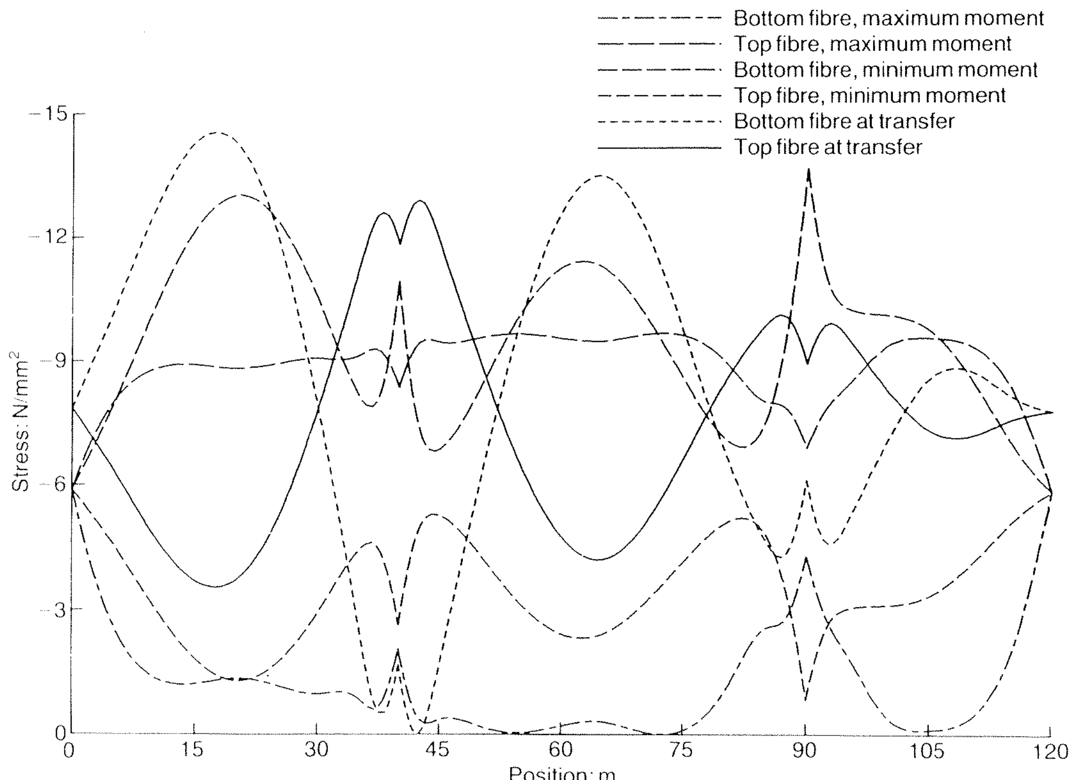


Fig. 15. Resulting stresses

44. The same problem has been studied with various accelerating factors between 1·0 and 2·2. Fig. 16 shows the number of iterations required to achieve convergence in each case. Superficially, it would appear that any factor between 1·3 and 2·0 will achieve satisfactory results, but some further observations are relevant.

45. With $\omega = 1{\cdot}0$ or 1·1, the iterations were converging towards a valid solution, but were always outside the acceptable limits. Therefore, the results shown in the figure are for results where the line of thrust was within 1 mm of the acceptable bounds. For all practical problems, this would be acceptable.

46. At the other extreme, with $\omega = 2{\cdot}2$, the results were diverging, and with $\omega = 2{\cdot}1$, they were converging only very slowly. The solution for $\omega = 2{\cdot}0$ was obtained after 12 iterations, but these iterations nearly all involved successive corrections based on errors in the line of thrust at one position. The corrections being made were so large that if, for example, the maximum error in the line of thrust lay above the upper bound for one iteration, it would lie below the lower bound at the same position at the next iteration; this results in successive corrections being made using notional loads derived from the same influence line, which is clearly wasteful. Similar waste was observed for all values of $\omega \geqslant 1{\cdot}6$.

47. Therefore, the optimum accelerating factor to use appears to lie in the range $1{\cdot}3 \leqslant \omega \leqslant 1{\cdot}5$, where the corrections made are large enough to ensure that the new solution lies within the bounds, but small enough to preclude the necessity of wasteful corrections. It should be noted that the final line of thrust produced by

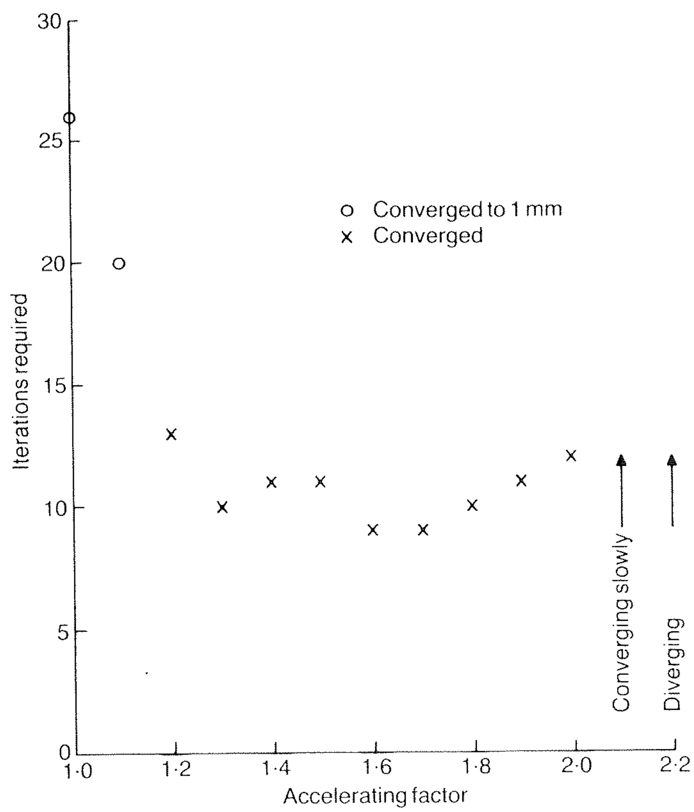


Fig. 16. Effect of altering accelerating factor

each of the successful iterations with different values of ω differed only by a few millimetres.

48. It is of interest to make some comparison with the accelerating factor used in the successive over-relaxation of sets of equations. The choice of the optimum factor is difficult, even when equalities are concerned, but it can be shown[4] that the accelerating factor must be less than 2·0; for large sets of equations, the optimum value is often greater than 1·9. However, in the present case, we are dealing with inequalities, and we are not correcting every condition at each iteration, only the one with the largest error.

Effect of support load length

49. The example has been calculated on the assumption that the internal support reactions are distributed over a length of 8 m by the support loads. This figure was arbitrarily chosen, and other values are feasible. Fig. 17 shows the profile of the line of thrust in the vicinity of the left-hand internal support when the support reaction is distributed over various lengths.

50. The line of thrust will be close to the top of the allowable zone away from the supports, since the requirement that e_p is concordant forces the profile to be fairly high. Therefore, the cable has to change direction by a fixed angular amount over the pier. If the reaction is taken over a short length, such as 5 m, the radius of

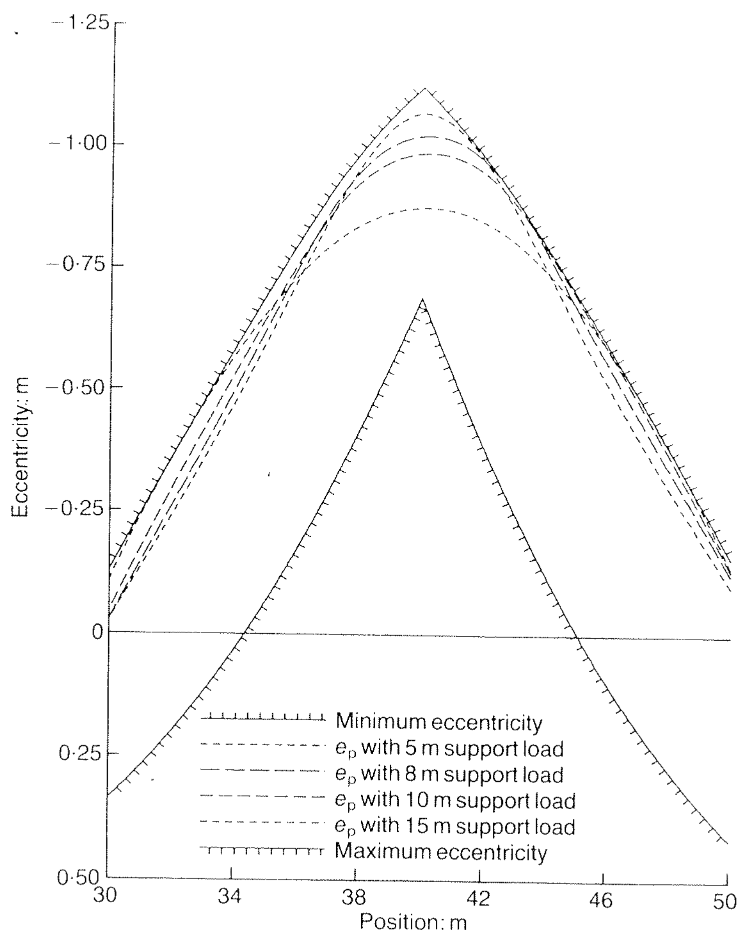


Fig. 17. Effect of varying equivalent support loading length

curvature will be relatively small, and the cable will be quite high over the pier. On the other hand, if the support reaction is spread over a longer length, such as 15 m, the radius of curvature of the cable is quite large, and the line of thrust is lower at the pier position. To compensate for the lower value of e_p at the pier, the line of thrust must be higher elsewhere, in order that the profile may remain concordant. In the example, it can be seen from Fig. 16 that the line of thrust is against the upper limit of the zone away from the support when the support load length is specified as 15 m. Indeed, no valid solution could be obtained for longer lengths of the support load. In certain circumstances, this could be a constraining factor on the design, since the cable profile must have a certain minimum radius of curvature to avoid damage to the tendon or duct during installation and stressing.

51. The minimum radius of curvature of the cable can affect the design in another way. If the width of the allowable zone is quite narrow, it is conceivable that it will be impossible to fit a suitable profile with an acceptable radius of curvature through the zone in the vicinity of the support.

52. Consider the case shown in Fig. 18. The allowable zone is narrow and, for the sake of this example, will be assumed to have constant depth. It will also be assumed to be straight on either side of the support, with angles to the horizontal of $\pm\theta$. The maximum radius of curvature R that can be fitted through the support zone occurs when the cable is at the top of the zone on the approaches to the support, but is at its lowest position over the pier. Under these conditions, and if it is assumed that the cable profile is circular and that θ is small enough to make this also a reasonable approximation to a parabola, then it is possible to show that

$$e_r \geqslant (R/2)\tan^2\theta$$

53. The above forms another constraint for the cable designer, but this time it primarily affects the bounds on the line of thrust, which is a function of the prestressing force, rather than the line of thrust itself.

Possible extensions

54. The method, as presented, assumes a constant prestressing force. It would not be too difficult to include a variable prestressing force, resulting from variations in the number of cables, by including suitable forces and moments at the positions where the cables were to be anchored. It would be necessary to choose criteria which the program would use to alter the applied moment and force at these positions, but they need not be very complex.

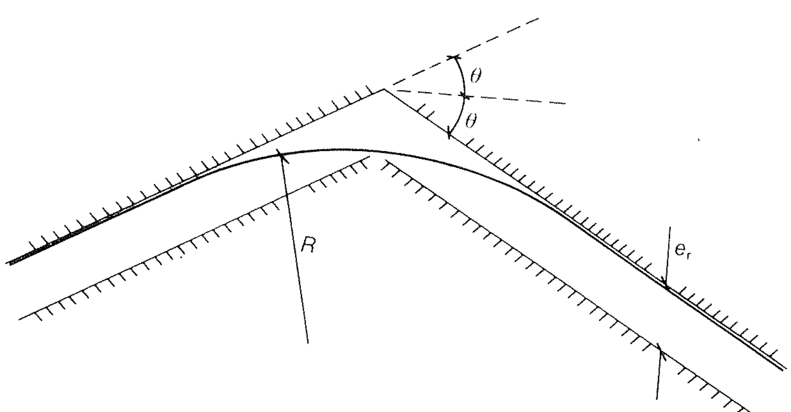


Fig. 18. Narrow cable zone at support

55. It would also be possible to include within the method, variations in prestressing force due to friction losses, which would cause the bounds on e_p to vary along the length of the beam. Care would be needed to ensure that correct allowance was being made for the fact that the program was determining the concordant line of thrust e_n of a cable profile, and that subsequent transformations to obtain the actual cable profile e_s would no longer be linear, since P was not constant.

Conclusion

56. A method has been presented which allows the automatic generation of concordant cable profiles. This allows the designer to specify limits on the line of thrust of his cables, which can be done in terms of the real loads on the structure, which are known *a priori*. Once the line of thrust is determined, it is then a simple matter to use linear transformations to generate acceptable profiles for the cables themselves. Secondary moments will thus be taken into account automatically; the designer does not have to calculate them directly, nor even choose their magnitudes, although it will still be of benefit if their existence is appreciated and understood.

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