Calculation of shear and moment envelopes by Macauley's method

C. J. Burgoyne
Dept of Civil Engineering, Imperial College, Exhibition Road, London SW7 2BU, UK
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ABSTRACT

The paper describes the use of a microcomputer to determine the behaviour of continuous beams. Three procedures are described: the analysis of continuous beams subject to generalized loadings using Macauley's method, the extension to include the production of influence lines, and the analysis of these influence lines to comply with a loading code. Results are presented to show how different assumptions about partial factors applied to dead loads can produce ranges of moments for which the structure must be designed. The implications for the designer of the complexities of a modern highway design code are considered in some detail, and the methods used to produce bending moment and shear force envelopes for structures designed to these rules are discussed.

INTRODUCTION

The work described in this paper grew out of a need to be able to analyse prestressed concrete beams easily and quickly. Such beams are loaded, not only by external loads, but also by the forces imparted by the cables; if the beam is statically indeterminate, it is also necessary to determine the parasitic moments induced in the beam by these loads.

It is not always easy to determine which sections along the length of a prestressed beam are the critical ones. The cable profiles are often curved, but with fairly large radii of curvature. Bending moment envelopes often have sharp peaks in the vicinity of internal supports. The combination of these two effects means that the critical section may often not be at mid-span or over the supports (where a superficial analysis would place it), but at some other point within the beam. It thus becomes necessary to be able to calculate envelopes of bending moments and shear forces for the whole length of the beam.

Many hand methods of structural analysis were developed before the advent of computers. Almost without exception, these methods were designed either to reduce the number of simultaneous equations to be solved, so minimizing the chance of making an error, or to solve the equations by relaxation techniques, where the errors inevitable in long hand calculations would be self-correcting.

When computers became generally available, the problems associated with the direct solution of large sets of simultaneous equations vanished. The speed and reliability of calculation meant that relaxation methods (such as moment distribution) were not needed for most problems, although Successive Over Relaxation (SOR) and its derivatives remain useful tools for some applications.

Many of the old techniques are now passing into disuse in practice, and are often not taught to aspiring engineers. This is unfortunate, since the simplification of the problem was often achieved by applying structural principles which, if not absent, are at least masked by the power of the new methods. Now that we have the sledgehammer of the new methods, we are in danger of throwing out perfectly serviceable sets of nut crackers which could themselves be updated to make the most of the power of modern computers.

This paper describes the use of one of these older methods (that due to Macauley) for the analysis of continuous, prismatic beams, under generalized loadings. The extension to include the production of influence lines is then briefly covered, followed by the analysis of the influence lines to give shear force and bending moment envelopes calculated according to a complex modern loading code, such as BS5400: Part 2.

For the analysis of beams and frames, stiffness methods are now the most widely used, but there are some problems (not insuperable) regarding the assembly of the equations to be solved when the loads are spread over a number of spans and also when we wish to determine results anywhere in the structure.

Macauley's method[1] offers a practical alternative to the stiffness method if the structure can be adequately represented by a straight, prismatic beam. Strictly speaking, Macauley's method is a notation which allows the representation of discontinuous functions by a single expression. More importantly in our case, it allows these expressions to be integrated and still to be represented as a single formula. Thus, discontinuous shear forces or loads can be integrated as required to give slopes and displacements. Additional discontinuities can be incorporated into these new expressions to allow the calculation of influence lines.

This paper describes three procedures which can be built on the core outlined above.

The first routine is a continuous beam analysis procedure, which can analyse multi-span beams under a generalized combination of point loads, uniformly distributed loads and point couples, to give exact,

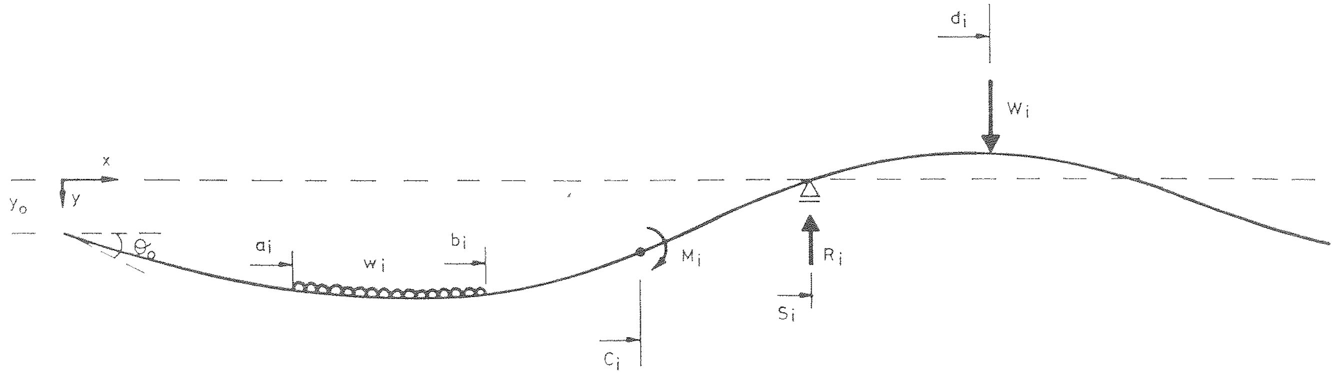


Figure 1 Beam under general loading

continuous expressions for deflection, slope, bending moment and shear force.

The second process is a modification of the first to analyse a beam loaded only by a discontinuity in either slope or displacement. By the reciprocal theorem[2], the displaced shape which results from these actions will be influence lines for bending moment or shear force at the point in question.

The third procedure has the second routine as its core, to produce influence lines; it then determines which loading produces the worst effect at the point in question. For a point load, or a uniformly distributed load, this is not difficult, but for the load conditions now specified in modern codes of practice, the calculation is extremely complex and not amenable to hand calculation.

CONTINUOUS BEAM ANALYSIS

Since Macauley's method is well known, only a brief outline will be given here to show how it has been applied in this case.

We adopt the standard bracket notation, which specifies that anything contained within braces $\{\ldots\ldots\}$ is zero if the contents of the brace are negative, but take their normal value if positive.

Consider the beam shown in *Figure 1*. The position on the beam is specified by the dimension x measured from the left hand end. The displacement at $x=0$ is y_0, where the rotation is θ_0. It is supported on a number of supports at $x=s_i$ whose reactions are denoted by R_i. It is loaded by a number of point loads at $x=d_i$ of magnitude W_i, a number of distributed loads with intensity w_i from $x=a_i$ to $x=b_i$ and a number of couples M_i at $x=c_i$. The bending moment (M) at any point x can be found from:

$$M=\sum R_i\{x-s_i\}-\sum W_i\{x-d_i\}-\sum w_i/2(\{x-a_i\}^2+\{x-b_i\}^2)+\sum M_i\{x-c_i\}^0$$

If this equation is divided by the stiffness (assumed to be constant along the beam) to give the curvature, and integrated twice we obtain:

$$y=y_0+\theta_0 x-\sum R_i/(6EI)\{x-s_i\}^3+\sum W_i/(6EI)\{x-d_i\}^3+\sum w_i/(24EI)(\{x-a_i\}^4-\{x-b_i\}^4)-\sum M_i/(2EI)\{x-c_i\}^2$$

The unknowns in this equation are the support reactions R_i and the initial values y_0 and θ_0. These can be determined by satisfying conditions that $y=0$ at each support position, and that vertical and moment equilibrium are satisfied.

Once these values have been determined, the displacement and moment can be determined anywhere using the equations given above, and the slope and shear force can be found using the derivatives of these expressions.

A computer program has been written for a BBC microcomputer using these ideas. The simultaneous equations are solved using a simple Gaussian elimination routine with row interchange if small pivots are found. It has been found useful to normalize the equations so that the largest element in each row of the coefficient matrix is unity, but no other special techniques are used.

The program is provided with simple prompted data entry and editing facilities. Since the amount of data required is small, no facilities are provided for creating or using data files.

The advantages of this method are that loads can be applied anywhere within the beam, and that continuous expressions for the shear, moment, slope and deflection can be determined readily for the whole structure. The method is simple to program and is very flexible, as will be seen later.

The disadvantages of the method are that the set of simultaneous equations needed to solve for the unknown parameters can become ill-conditioned for beams with a large number of spans. When this occurs, however, it is easy to spot, since the results do not satisfy the boundary conditions, and in practice, no problems have been observed in the analysis of beams with up to 10 spans, unless the spans are of very unequal length. The method is difficult to generalize to frames and grillages, but for our purposes that is not a problem.

PARASITIC MOMENTS

The continuous beam program can be used as it stands to determine the line of thrust of a tendon in a continuous structure and the corresponding parasitic moments. The loads exerted by the tendon on the concrete can be calculated, by considering the changes in direction of the cable, and used as the input to the program. The resulting moments will be the product of the cable force and the eccentricity of the line of thrust, while the parasitic moments can be determined directly from the reactions.

The loads can be determined by hand, or by a

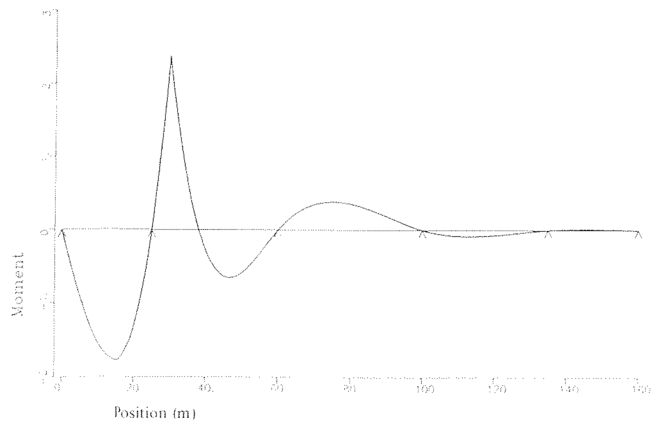


Figure 2 Typical influence line for moment

specialized 'front-end' routine which accepts the cable profile directly and itself determines the equivalent loads.

INFLUENCE LINE PROGRAM

The extension of the method to calculate influence lines is trivial. Macauley's notation can easily cope with discontinuities in slope and/or deflection; if there is a unit discontinuity in slope at $x=p$, then the deflection equation for an otherwise unloaded beam becomes:

$$y=y_0+\theta_0 x-\sum R_i/(6EI)\{x-s_i\}^3+\{x-p\}$$

which can be used to calculate an influence line for moment at $x=p$.

If there is a discontinuity in deflection at $x=p$, then the deflection equation becomes:

$$y=y_0+\theta_0 x-\sum R_i/(6EI)\{x-s_i\}^3+\{x-p\}^0$$

which can be used to calculate the influence line for shear at $x=p$.

The influence line for reaction at a support can be determined by specifying that $y=1$ at that support.

Similar appropriate changes are needed in the expressions for slope, moment and shear, but there is no difference in principle from the continuous beam analysis, so a modified version of the program was produced which could produce shear, moment and reaction influence lines.

Typically, the influence line will consist of a series of positive and negative lobes. Often, but not always, the junctions between these lobes occur at supports. *Figures 2* and *3* show typical moment and shear influence lines (for $x=30$) as calculated by the program for a beam with 5 spans of 25, 35, 40, 35 and 25 m.

DETERMINING LOADING ARRANGEMENTS FROM THE INFLUENCE LINE

Once the influence line is known it is possible to determine which portions of the beam should be loaded to produce the maximum effect. It is then a short step to put this process into a loop, so that the whole procedure is repeated at a number of regularly spaced points along the length of the beam. The loci of maximum and minimum values will, of course, be the envelopes for shear (or moment).

For uniformly distributed or point loads, the process is straightforward. If the load is of uniform intensity, but can be applied over any length(s), then all the positive lobes should be loaded to get the maximum effect, and all the negative lobes loaded to get the minimum effect. For point loads, these should be applied at locations where the magnitude of the influence line is a maximum or minimum as appropriate.

It is not difficult, once the influence line is known, to divide it into a series of positive and negative lobes whose areas are found by 3 point Gaussian quadrature. Simply by adding together the areas of the positive lobes, and multiplying by the load intensity, we can determine the maximum effect at the point in question; the minimum effect is determined in the same way.

As an example, we consider a beam acting under a uniformly distributed dead load. In a simple analysis, we might assume that the load is applied everywhere, and then multiply by a factor to allow for the structure being heavier than expected. However, in a beam with several spans, it is possible that a more severe effect would be observed if parts of the structure were lighter than intended. This is particularly important in prestressed concrete design where the structure is not unstressed at zero load.

Consider a beam with three spans of 30, 50 and 40 m; with a nominal weight of 100 kN/m. A load factor of 1.4 might reasonably be assumed for dead load to allow for overloading, but it is also reasonable to allow for the structure being lighter than expected. A factor of 0.9 might be chosen for this occurrence.

We can determine the maximum and minimum moment envelopes using the program for a uniformly distributed load of 100 kN/m; if we then add 1.4 times the maximum envelope to 0.9 times the minimum envelope, we shall obtain an envelope of the maximum moment due to dead load to which the structure could be subjected. Similarly, we can obtain an envelope of the minimum moment by reversing the factors. The results of such an analysis are shown in *Figure 4*, together with the results of a continuous beam analysis for a constant load of 140 kN/m. It is clear that at no point does the simple dead load analysis give the worst effect, and a significant range of moments is possible, which could have a major effect on the detailed section design. (It must be stressed that in reality the dead load does *not* cause a range of moments at any one section; however, at the design stage, we cannot know precisely what will be the actual weight of the

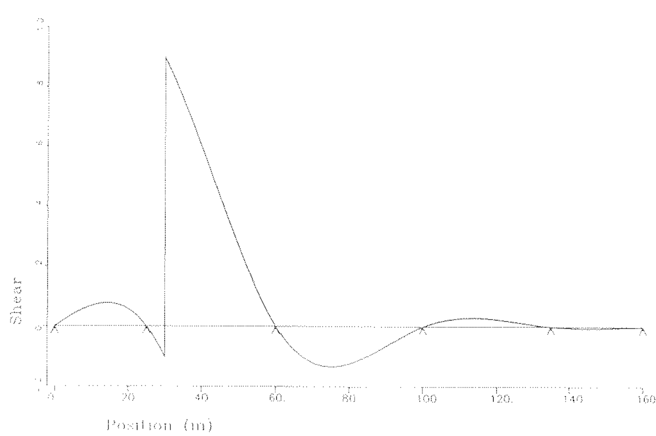


Figure 3 Typical influence line for shear

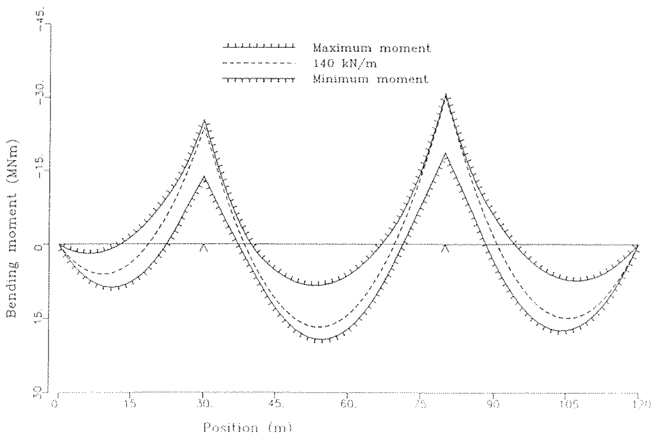


Figure 4 Moment envelope under 100 kN/m u.d. load. Minimum factor, 0.9; maximum factor, 1.4

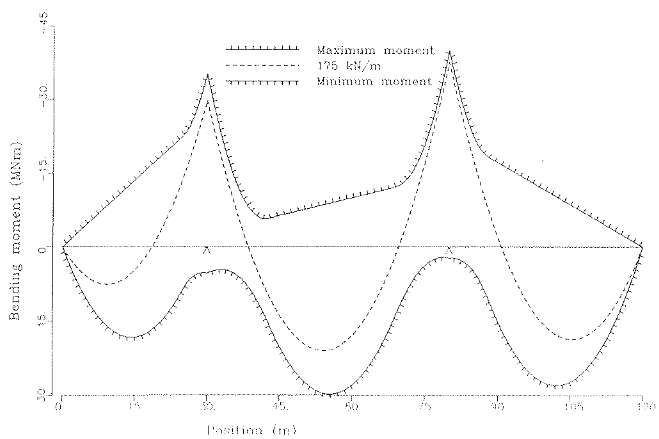


Figure 5 Moment envelope under 100 kN/m u.d. load. Minimum factor, 0; maximum factor, 1.75

structure, and it is the range of moments caused by this uncertainty which we determine here.)

A more severe effect is apparent if we consider superimposed dead load. Superimposed loads, such as road surfacing or railway ballast, are much more variable, with a load factor of 1.75 often being used to determine the worst effect. However, there may well be occasions when the load is removed, in whole or in part, so it is necessary to consider the range of moments which can result. *Figure 5* shows the moment envelope for a notional load of 100 kN/m, on the same structure as the last example, with a range of load factor from 0 to 1.75. The spread of bending moments is more severe than for structural dead load, and should be taken into account in design, although it may well be that the full live load case need not be considered in association with the minimum superimposed dead load case.

BS5400 HIGHWAY LOADINGS

It is much more complicated to apply 'standard loadings', as defined in Standards or Ministry directives. Highway loadings vary in intensity depending on the length of the beam that is loaded and the number of highway lanes that are present. The axle spacing of the standard vehicles are often variable, and there are areas of the beam which are not loaded when heavy vehicles are present.

The program has been written to apply the British Standard highway loading as defined in BS5400[3] as modified by the Department of Transport directive BD14/82[4]. It can also be used to apply the most recent modifications specified in the directive BD23/84[5]. It is believed that this document will form the basis of the loading provisions in the revised version of BS5400 which is due to be published soon. Two types of loading are defined in these rules; HA and HB. HA represents the maximum loading which is likely to occur under normal traffic conditions, while the HB loading represents a notional abnormal vehicle, carried on 4 axles.

HA loads

HA loading consists of a uniformly distributed load which is expressed as a load (W) per metre of lane, where W is a function of the loaded length (L), together with a knife edge load.

Under the provisions of BD14/82, the load intensity is defined in three stages:

$W = 30$ kN/m $L < 30$ m

$W = 151(L)^{-0.475}$ kN/m $30 < L < 380$ m

$W = 9$ kN/m $L > 380$ m

This has been revised upwards in BD23/84 and is now defined by two functions:

$W = 260(L)^{-0.6}$ kN/m $L < 52$ m

$W = 36(L)^{-0.1}$ kN/m $L > 52$ m

These loadings are shown in *Figure 6* for loaded lengths less than 100 m and in *Figure 7* for longer loaded lengths. The significant increase in load for lengths < 30 m and > 50 m is obvious. In both codes the knife edge load is fixed as 120 kN per lane.

Not all lanes are loaded with the full HA loading. In BD14/82, two lanes carry the full loading, while the remainder carry 0.6 HA. In BD23/84 an extremely complex specification is introduced, in which the proportion of HA on each lane depends on which lane is

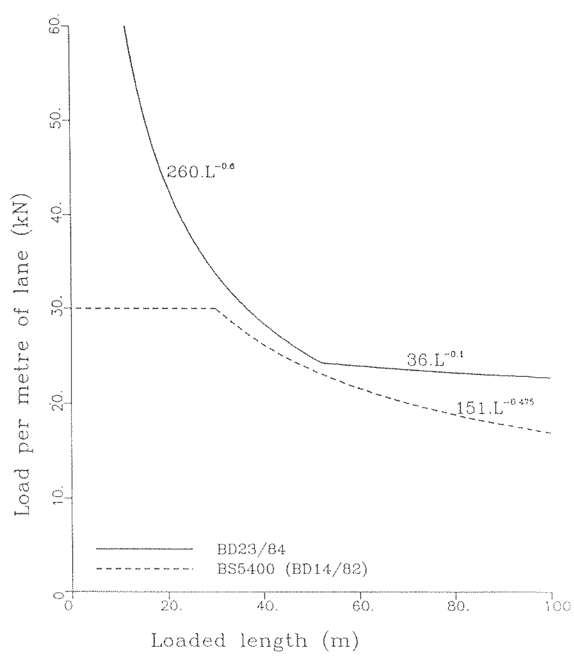


Figure 6 HA uniformly distributed load (short loaded lengths)

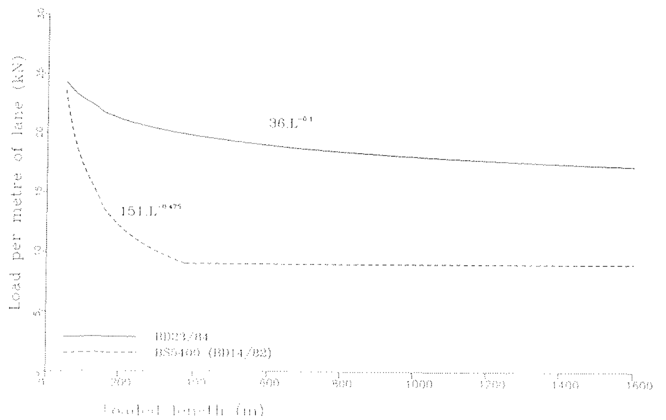


Figure 7 HA uniformly distributed load (longer loaded lengths)

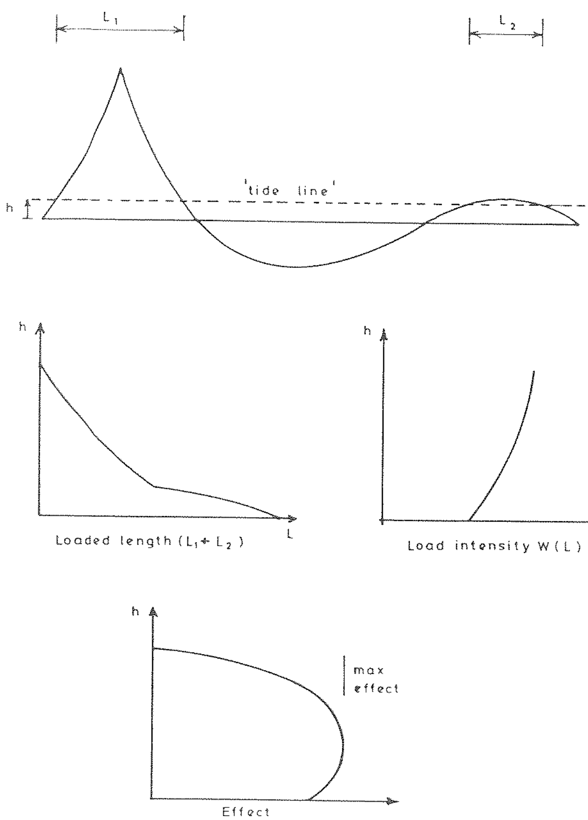


Figure 8 Determination of maximum effect using 'tide line'

being considered, the total number of lanes, the width of each lane and the loaded length. The program takes account of these factors when calculating envelopes, but it is not concerned with the distribution of the HA load across the width of the structure.

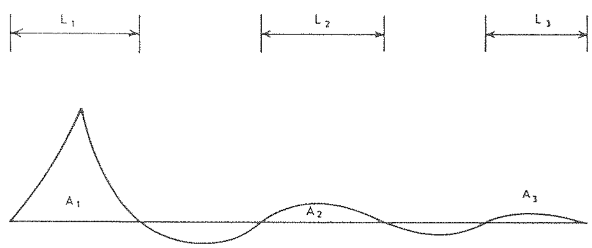
The variation of load intensity with loaded length is the major source of complexity. In areas remote from the point in question, it may be desirable not to load the structure even where the influence line indicates that the structure should be loaded, in order to reduce the loaded length and thus increase the intensity of load in regions where the load is more effective.

Strictly speaking, if the worst effect is required, one should draw a 'tide-line' on the influence line (*Figure 8*), and load those regions where the influence line is above the tide-line. As the tide-line is raised, the loaded length will decrease, so the intensity will rise. This may cause the effect of the load to get worse, and the design stress resultant is taken as the maximum value for any position of the tide-line. This calculation is tedious, since the intersections of the tide-line with the influence line have to be determined for every value of h, and the areas of the lobes recomputed.

However, in a rare act of compassion for the engineer, the Dept. of Transport have indicated in BD23/84 that the intention is that complete lobes should be included or excluded, depending on whether they make things worse or not. The problem reduces to one of finding which combination of lobes gives the worst effect. In a hand calculation, or an expert system, it should be possible to use some intelligence to decide which lobes should be included, but in a simple BASIC program, it is easiest to do an exhaustive search of all possibilities using the ability of BASIC to support recursive procedure calls. Each permutation of loads is considered; the loaded length is the sum of the base lengths of the lobes being considered, the load intensity is a function of this loaded length, and the effect is the load intensity multiplied by the areas of the included lobes (*Figure 9*).

This approach has been adopted for calculations to BD14/82, since the wording of that document was not precise about which method the engineer was intended to use.

In the present program, any lobe which has a maximum within it is considered to be two lobes, with the division between them occurring at the maximum point (*Figure 10*). This considerably simplifies the coding, and has no effect on the results except that the worst effect can be achieved by including one 'half-lobe', and leaving out the other. However, this will always be worse than that required by the code, since all the code permutations are correctly included, so the error is conservative. It will also slow the program slightly, since more permutations have to be considered.



Permutation				
1	2	3	Loaded length (L)	Effect. N.B. W(L)
1	1	1	$L_1 + L_2 + L_3$	$W.(A_1 + A_2 + A_3)$
1	1	0	$L_1 + L_2$	$W.(A_1 + A_2)$
1	0	1	$L_1 + L_3$	$W.(A_1 + A_3)$
1	0	0	L_1	$W. A_1$
0	1	1	$L_2 + L_3$	$W.(A_2 + A_3)$
0	1	0	L_2	$W. A_2$
0	0	1	L_3	$W. A_3$

Figure 9 Determination of maximum effect by permutation

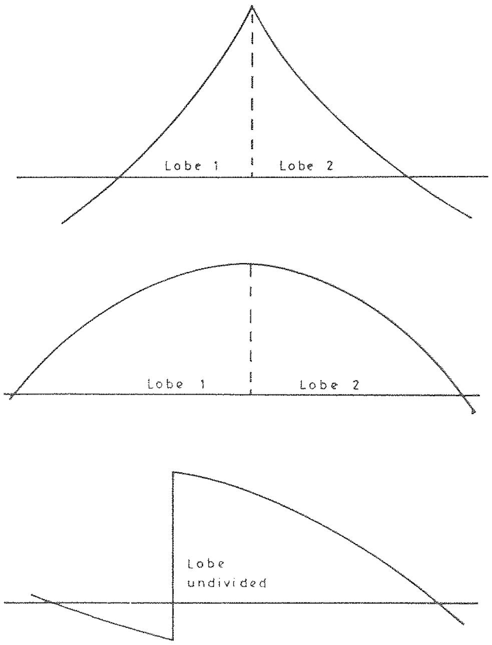


Figure 10 Lobes divided into 2 for computation purposes

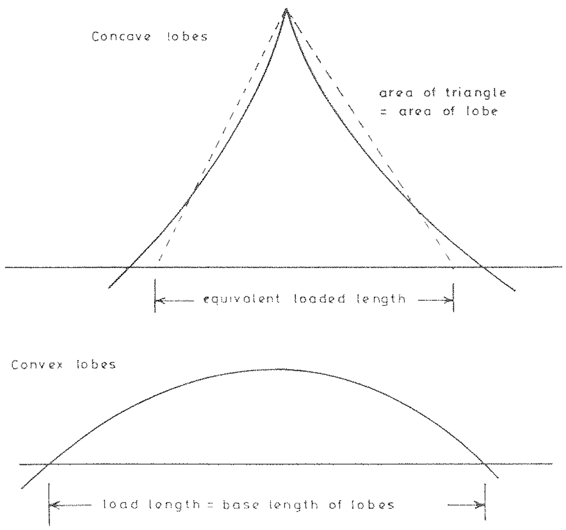


Figure 11 Convex and concave lobes

There is one further complication. If the lobes rise to a sharp peak, the permutation approach is likely to significantly underestimate the true worst effect. For these types of lobe, a modified loaded length is used. This is found by dividing the area of the lobe by one-half of the maximum height of the lobe. This is equivalent to finding the triangle with same area and height as the true lobe, and using its base as the loaded length. It will only be significant for concave lobes (*Figure 11*). The knife edge load is of course applied at the maximum ordinate of the influence line.

HB loads

The HB vehicle is regarded as being carried on 4 axles, and the magnitude of the loading is expressed as a number of units. Each unit causes a load of 10 kN on each axle; the number of units depends on the type of road. Normal roads are designed for 25 units (i.e. approx 100 tonnes total load), while motorways are designed for 45 units. The program asks for the number of units to be designed for. The axle layout is defined (*Figure 12*). The axles are arranged as two bogies, each with an axle spacing of 1.8 m. The bogies can be 6, 11, 16, 21, or 26 m apart, depending on which gives the worst effect. Normally, the shortest vehicle will give the worst effect, with the whole vehicle concentrated on the largest lobe, but for some shorter structures the worst effect over piers can be achieved by placing the two bogies at the peaks of adjacent lobes.

The program searches for the worst position of the HB vehicle in two stages. In the first, the HB vehicle is placed in a number of positions to find the worst effect according to a fixed set of rules. Subsequently, the program moves the vehicle from this position in steps of ever decreasing size to find the true position where the effect is worst.

In the first phase, the largest ordinate on the influence line is determined. The vehicle is positioned so that each axle is placed at that ordinate in turn (4 positions). This is repeated for each bogie spacing (5 cases). Normally, one of these cases will be close to the worst position and can be used for the second, refinement, phase. However, there is one case where this approach will not give the worst effect. When calculating moments close to an internal support, the influence line can look like that shown in *Figure 13*. There is a small lobe with a sharp peak at the point of interest. The program tries to put an axle at this point, but the other axles will be placed in opposing lobes, thus reducing the effect. A worse effect can be achieved by putting the HB vehicle on a remote span, where the whole vehicle acts in a positive direction. To make the program detect such cases is difficult, but it can be overcome by placing the centre of the HB vehicle, with a bogie spacing

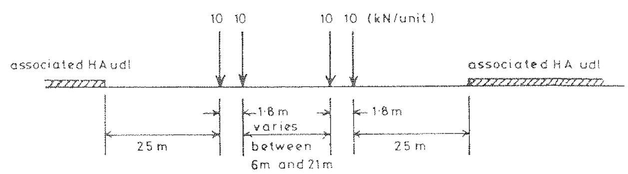


Figure 12 HB vehicle axle arrangement

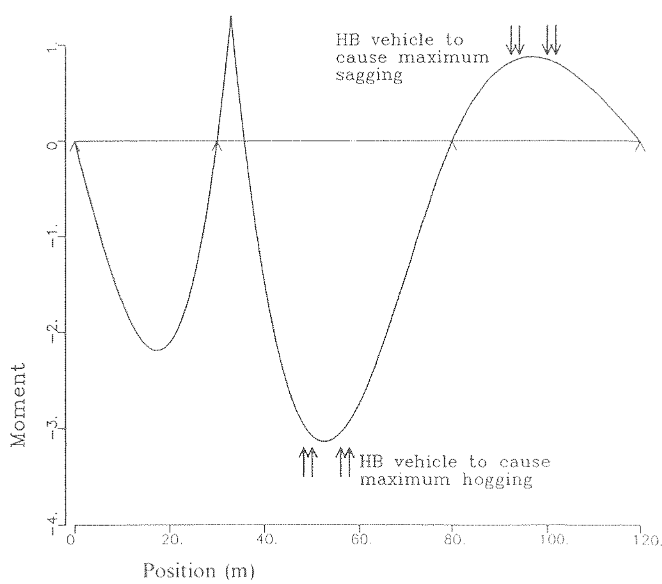


Figure 13 Influence line for moment close to support

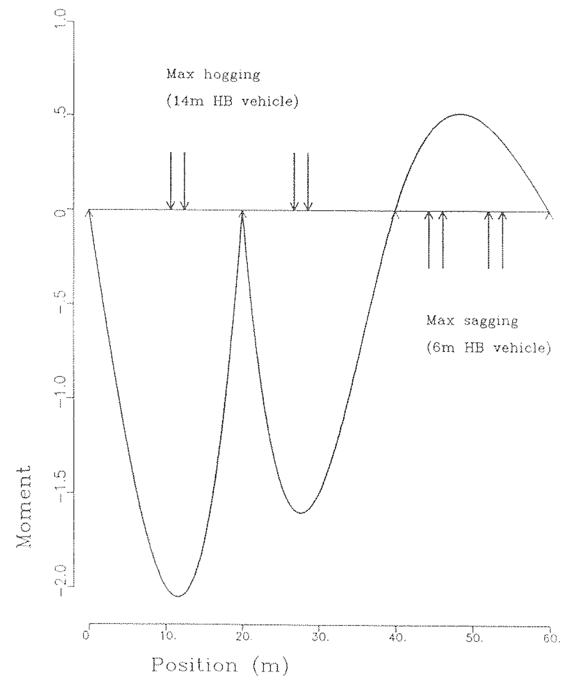


Figure 14 Variation in length of HB vehicle (beam with three spans of 20 m)

of 6 m, at the start of every lobe. Since we are dividing actual lobes into two at their maximum points anyway, this will be equivalent to placing the vehicle at the centre of each real lobe, and if one of these cases is worse than those found earlier, this will be adopted as the starting point of the refinement process.

Once the approximate worst position has been found, the program searches for the true worst position by changing the vehicle position or the bogie spacing by a small amount. Initially, movements of 0.5 m are used, but this figure is halved at each stage until the vehicle position and spacing are correct to 1 mm.

The code does not specify that the bogie spacing should be changed from one of the fixed values which it quotes, presumably to reduce the computation effort required in a hand calculation. However, the underlying idea is that the bogie spacing is not fixed, and since it is relatively easy to build into the program the extra degree of freedom, this has been done. In certain cases, this can result in a vehicle with a non-standard bogie spacing being adopted.

Figure 13 shows a case where the HB vehicle is placed away from the maximum ordinate on the influence line, while *Figure 14* shows a case where a non-standard bogie spacing is used to place the two bogies at the centres of two adjacent lobes.

HA load associated with HB load

When the HB vehicle is present, the rest of the structure may still carry HA loading. There is no HA for 25 m on either side of the HB vehicle in the same lane as the vehicle itself and the knife edge load is not applied in that lane; all other lanes are loaded with both HA UDL and the KEL. The program assumes that the HB vehicle displaces the lane carrying the smallest proportion of HA load. The remainder of the lanes carry the maximum HA load, according to the code rules.

When considering the lane containing the HB vehicle, certain parts of the lane do not carry HA load. Only the areas of lobes outside the area displaced by the HB vehicle are used to calculate the effect of that loading. However, the code specifies that the loaded length used to calculate the intensity should include those elements of the lobes that would have been loaded had the HB vehicle not been present. The program searches for the worst combination of loadings given this rule; the effect is that the worst distribution of loading on the HB lane may not be the same as that on the other HA lanes.

An important simplification that has been introduced must be made clear. The program calculates the maximum HA loading associated with the maximum HB loading. It does not attempt to find the maximum combination of HB and HA. The difference is likely to be small and the computation effort necessary to find the precise result is enormous, since the HA load associated with every HB vehicle position that was considered would have to be calculated.

EXAMPLE OF HIGHWAY LOADING

A bridge with spans of 30, 50 and 40 m with 4 lanes each 3.65 m wide has been studied. It may be loaded with 25 units of HB load.

Figure 15 shows the influence line for bending moment at a point 15 m from the left hand end, and also shows the loaded regions for both maximum and minimum effects for HA alone, HB and HA on the lane carrying the HB vehicle, as determined by the program.

Figure 16 shows the moment envelope for the beam, calculated according to BD/23/84. *Figure 17* shows the corresponding shear envelopes. These calculations required about 70 trial positions of the HB vehicle and about 35 other determinations of influence line height for each position at which the influence lines were produced. It may, with some justification, be argued that the program refines the position of the HB vehicle too accurately and allows more variability in the bogie spacing than the code requires; nevertheless, the determination of such envelopes by hand methods is practically impossible.

EXTENSIONS TO OTHER LOADINGS

The principles outlined in this paper could easily be extended to other load cases. For example, the BS5400

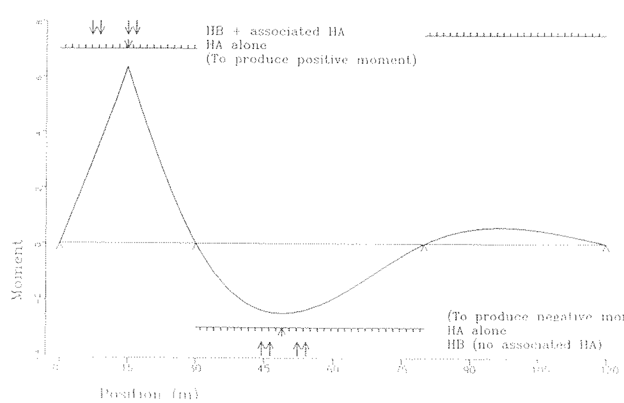


Figure 15 Determination of loading from influence line

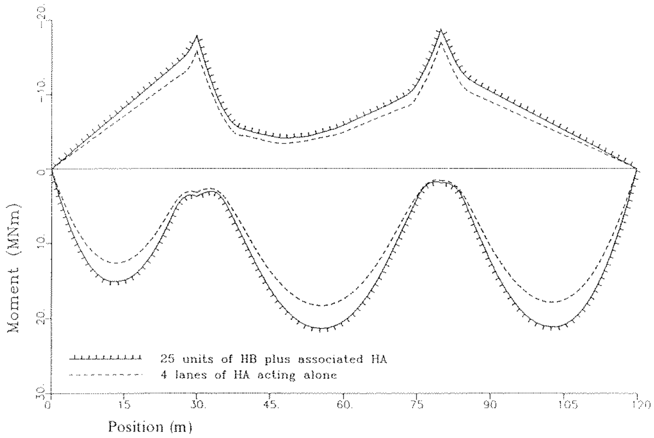


Figure 16 Bending moment envelope (beam with spans of 30, 50 and 40 m)

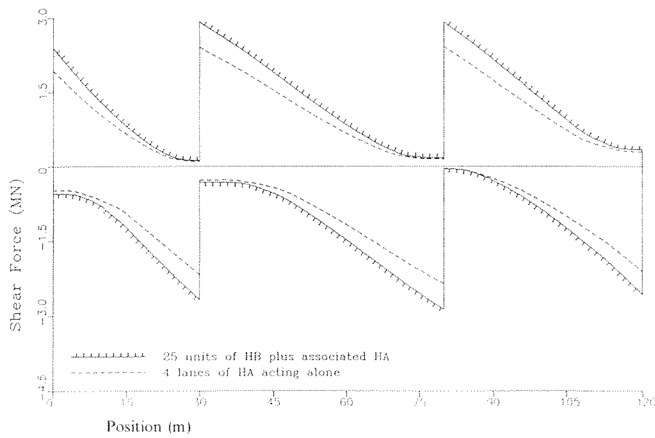


Figure 17 Shear force envelope (beam with spans of 30, 50 and 40 m)

Railway loadings (RU and RL) could also be handled in this way.

The loading used on most main line railways (RU loading), specifies 4 axles carrying 250 kN at a spacing of 1.6 m, together with a uniformly distributed load of 80 kN/m elsewhere. An approach similar to that outlined for the HB vehicle and its associated HA load could be used in this case. There is an additional complication, in that the load determined in this way must be multiplied by a dynamic factor which is a function of the length of the influence line. The effect is similar to that of the loaded length in HA loading, in that short loaded lengths are more heavily loaded than long ones, but the cause is different and the calculation not carried out in quite the same way. Nevertheless, it would be quite feasible to incorporate such an allowance in a program.

The lighter railway loading (RL), used for rapid transit railways, is specified in a different way. There is a load of 50 kN/m, applied over a maximum length of 100 m, and a distributed load of 25 kN/m elsewhere; there is also one point load of 200 kN. A 'tide-line' approach is needed to determine the worst position of the higher intensity load, while the effect of the other loads is fairly simple to calculate using other techniques outlined in this paper.

EXTENSIONS TO OTHER STRUCTURES

The biggest drawback of the system outlined here is that it only applies to prismatic sections. This restriction arises because of the difficulty of integrating terms which include $(1/EI)$ when EI varies. A number of possible ways round the problem exist.

If the structure has piecewise constant stiffness, it can be divided into regions within each of which the stiffness is fixed. The integration within each region can then be performed analytically, but care is needed to satisfy compatability conditions at the junction of regions. A technique similar to this has been applied to a beam partially supported on a Winkler foundation[6].

Other methods have been considered for applying Macauley's method to non-prismatic beams[7], but these rely on the $(1/EI)$ term being of a form that is integrable over specified regions.

If the integrals needed in the system cannot be performed analytically, they can be done using approximate numerical techniques, but care must be taken with the double integration needed to obtain an expression for the displacement from the bending moment. The techniques outlined by Brown and Trahair[8] could be suitably modified.

In the case of a beam which is haunched for a relatively short distance over the piers, an estimate of the effect of the additional stiffness can be made.

The structure can be analysed for a few specific loading cases using a more accurate program which takes account of the variation in stiffness, and the results compared with the simpler prismatic beam results. An estimate may then be made of the transfer of moment from the span region to the pier region (which must correspond to a variation of the support reactions) and similar corrections made to the envelopes produced by the present method.

The techniques for analysing influence lines once they are determined can equally well be applied to influence lines obtained by a stiffness analysis, although the calculation will almost certainly take longer.

CONCLUSIONS

A modern application of an old technique has been presented which reduces the computation of bending moment and shear force envelopes for continuous prismatic beams to manageable levels.

The implications of the complexities of modern loading codes are clear. The amount of computation required to comply with the terms of the code is already excessive even for structures which can be adequately represented by this program. For hand calculation, such effort is not possible.

More importantly, how can the worst load case be determined for a structure that must be analysed by a more complex program (such as finite element analysis)? For complex structures, the determination of influence surfaces is not possible, or at least impractical, so how can the maximum effect be determined?

Surely it is time that code writers produced loading standards with which engineers have a reasonable chance of complying?

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