Analysis of Continuous Prestressed Concrete Beams

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1 Introduction

This conference is devoted to the development of structural analysis rather than the strength of materials, but the effective use of prestressed concrete relies on an appropriate combination of structural analysis techniques with knowledge of the material behaviour. Design of prestressed concrete structures is usually left to specialists; the unwary will either make mistakes or spend inordinate time trying to extract a solution from the various equations.

There are a number of fundamental differences between the behaviour of prestressed concrete and that of other materials. Structures are not unstressed when unloaded; the design space of feasible solutions is totally bounded; in hyperstatic structures, various states of self-stress can be induced by altering the cable profile, and all of these factors get influenced by creep and thermal effects. How were these problems recognised and how have they been tackled?

Ever since the development of reinforced concrete by Hennebique at the end of the 19th century (Cusack 1984), it was recognised that steel and concrete could be more effectively combined if the steel was pretensioned, putting the concrete into compression. Cracking could be reduced, if not prevented altogether, which would increase stiffness and improve durability. Early attempts all failed because the initial prestress soon vanished, leaving the structure to behave as though it was reinforced; good descriptions of these attempts are given by Leonhardt (1964) and Abeles (1964).

It was Freyssinet's observations of the sagging of the shallow arches on three bridges that he had just completed in 1927 over the River Allier near Vichy which led directly to prestressed concrete (Freyssinet 1956). Only the bridge at Boutiron survived WWII (Fig 1). Hitherto, it had been assumed that concrete had a Young's modulus which remained fixed, but he recognised that the deferred strains due to creep explained why the prestress had been lost in the early trials. Freyssinet (Fig. 2) also correctly reasoned that high tensile steel had to be used, so that some prestress would remain after the creep had occurred, and also that high quality concrete should be used, since this minimised the total amount of creep. The history of Freyssinet's early prestressed concrete work is written elsewhere (Grote and Marrey 2000).

Figure 1: Boutiron Bridge, Vichy

Figure 2: Eugene Freyssinet

At about the same time work was underway on creep at the BRE laboratory in England ((Glanville 1930) and (1933)). It is debatable which man should be given credit for the discovery of creep but Freyssinet clearly gets the credit for successfully using the knowledge to prestress concrete.

There are still problems associated with understanding how prestressed concrete works, partly because there is more than one way of thinking about it. These different philosophies are to some extent contradictory, and certainly confusing to the young engineer. It is also reflected, to a certain extent, in the various codes of practice.

Permissible stress design philosophy sees prestressed concrete as a way of avoiding cracking by eliminating tensile stresses; the objective is for sufficient compression to remain after creep losses. Untensioned reinforcement, which attracts prestress due to creep, is anathema. This philosophy derives directly from Freyssinet's logic and is primarily a working stress concept.

Ultimate strength philosophy sees prestressing as a way of utilising high tensile steel as reinforcement. High strength steels have high elastic strain capacity, which could not be utilised when used as reinforcement; if the steel is pretensioned, much of that strain capacity is taken out before bonding the steel to the concrete. Structures designed this way are normally designed to be in compression everywhere under permanent loads, but allowed to crack under high live load. The idea derives directly from the work of Dischinger (1936) and his work on the bridge at Aue in 1939 (Schönberg and Fichter 1939), as well as that of Finsterwalder (1939). It is primarily an ultimate load concept. The idea of partial prestressing derives from these ideas since the addition of quite significant amounts of untensioned reinforcement does not alter the logic (Emperger 1939).

The *Load-Balancing philosophy*, introduced by T.Y. Lin, uses prestressing to counter the effect of the permanent loads (Lin 1963). The sag of the cables causes an upward force on the beam, which counteracts the load on the beam. Clearly, only one load can be balanced, but if this is taken as the total dead weight, then under that load the beam will perceive only the net axial prestress and will have no tendency to creep up or down.

These three philosophies all have their champions, and heated debates take place between them as to which is the most fundamental.

2 Section design

From the outset it was recognised that prestressed concrete has to be checked at both the working load and the ultimate load. For steel structures, and those made from reinforced concrete, there is a fairly direct relationship between the load capacity under an *allowable stress* design, and that at the ultimate load under an *ultimate strength* design. Older codes were based on permissible stresses at the working load; new codes use moment capacities at the ultimate load. Different load factors are used in the two codes, but a structure which passes one code is likely to be acceptable under the other.

For prestressed concrete, those ideas do not hold, since the structure is highly

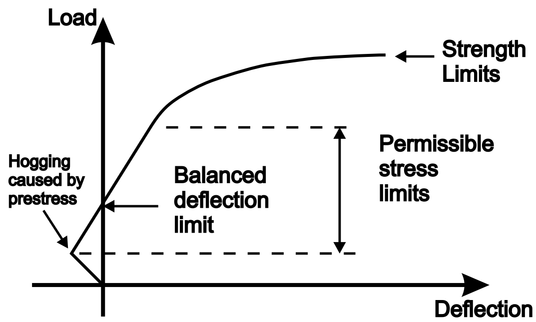

Figure 3: Load deflection curve

stressed, even when unloaded. A small increase of load can cause some stress limits to be breached, while a large increase in load might be needed to cross other limits. The designer has considerable freedom to vary both the working load and ultimate load capacities independently; both need to be checked.

A designer normally has to check the tensile and compressive stresses, in both the top and bottom fibre of the section, for every load case. The critical sections are normally, but not always, the mid-span and the sections over piers but other sections may become critical when the cable profile has to be determined.

The stresses at any position are made up of three components, one of which normally has a different sign from the other two; consistency of sign convention is essential.

If P is the prestressing force and e its eccentricity, A and Z are the area of the cross-section and its elastic section modulus (top or bottom fibres), while M is the applied moment, then

$$f_t \leq \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \leq f_c \tag{1}$$

where f_t and f_c are the permissible stresses in tension and compression.

Thus, for any combination of P and M, the designer already has four inequalities to deal with.

The prestressing force differs over time, due to creep losses, and a designer is usually faced with at least three combinations of prestressing force and moment;

- the applied moment at the time the prestress is first applied, before creep losses occur,
- the maximum applied moment after creep losses, and
- the minimum applied moment after creep losses.

Other combinations may be needed in more complex cases. There are at least twelve inequalities that have to be satisfied at any cross-section, but since an I-section can be defined by six variables, and two are needed to define the

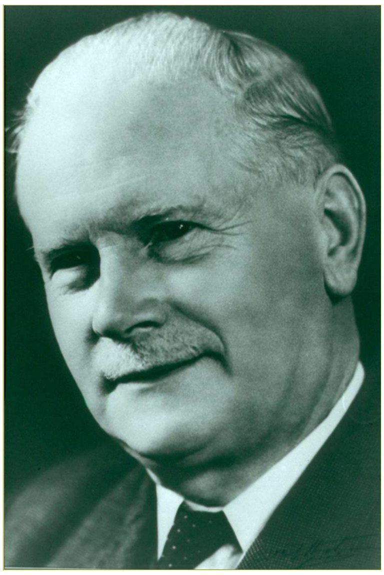

Figure 4: Gustave Magnel

prestress, the problem is over-specified and it is not immediately obvious which conditions are superfluous. In the hands of inexperienced engineers, the design process can be very long-winded. However, it is possible to separate out the design of the cross-section from the design of the prestress. By considering pairs of stress limits on the same fibre, but for different load cases, the effects of the prestress can be eliminated, leaving expressions of the form:-

$$Z \geq \frac{\text{Moment Range}}{\text{Permissible stress range}} \tag{2}$$

These inequalities, which can be evaluated exhaustively with little difficulty, allow the minimum size of the cross-section to be determined.

Once a suitable cross-section has been found, the prestress can be designed using a construction due to Magnel (Fig. 4). The stress limits can all be rearranged into the form:-

$$e \geq -\frac{Z}{A} + \frac{1}{P}(fZ + M) \tag{3}$$

By plotting these on a diagram of eccentricity versus the reciprocal of the prestressing force, a series of bound lines will be formed. Provided the inequalities (2) are satisfied, these bound lines will always leave a zone showing all feasible combinations of P and e. The most economical design, using the minimum prestress, usually lies on the right hand side of the diagram, where the design is limited by the permissible tensile stresses.

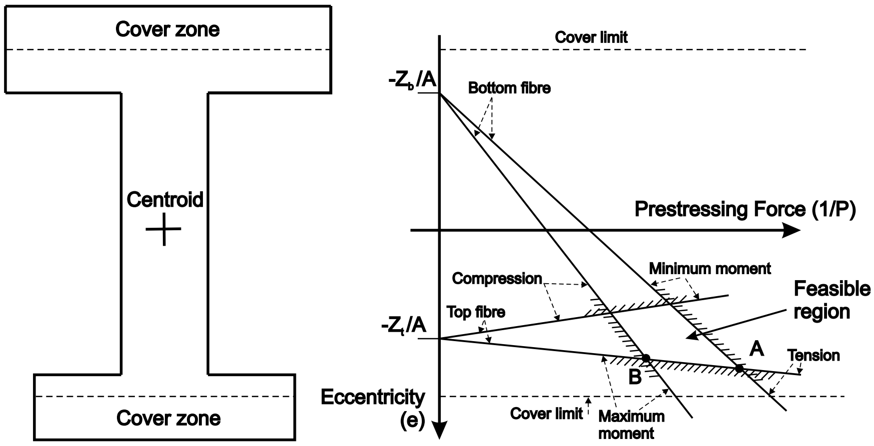

Figure 5: Magnel diagram

Plotting the eccentricity on the vertical axis allows direct comparison with the cross-section, as shown in Fig. 5. Inequalities (3) make no reference to the physical dimensions of the structure, but these practical cover limits can be shown as well.

A good designer knows how changes to the design and the loadings alter the Magnel diagram. Changing both the maximum and minimum bending moments, but keeping the range the same, raises and lowers the feasible region. If the moments become more sagging the feasible region gets lower in the beam. In general, as spans increase, the dead load moments increase in proportion to the live load. A stage will be reached where the economic point (A on Fig. 5) moves outside the physical limits of the beam; Guyon (1951a) denoted the limiting condition as the *critical span*. Shorter spans will be governed by tensile stresses in the two extreme fibres, while longer spans will be governed by the limiting eccentricity and tensile stresses in the bottom fibre (assuming sagging bending). However, it does not take a large increase in moment for point B to move outside the cover limit, at which point compressive stresses will govern in the bottom fibre under maximum moment.

Only when much longer spans are required, and the feasible region moves as far down as possible, does the structure become governed by compressive stresses in both fibres.

3 Continuous beams

The design of statically determinate beams is relatively straightforward; the engineer can work on the basis of the design of individual cross-sections, as outlined above. A number of complications arise when the structure is indeterminate which means that the designer has to consider, not only a critical section,

Figure 6: Luzancy Bridge

but also the behaviour of the beam as a whole. These are due to the interaction of a number of factors, such as Parasitic Moments, Creep, Temperature effects and Construction Sequence effects. It is the development of these ideas which forms the core of this paper. The problems of continuity were addressed at a conference in London (Andrew and Witt 1951). The basic principles, and nomenclature, were already in use, but to modern eyes concentration on hand analysis techniques was unusual, and one of the principle concerns seems to have been the difficulty of estimating losses of prestressing force.

3.1 Secondary Moments

A prestressing cable in a beam causes the structure to deflect. Unlike the statically determinate beam, where this motion is unrestrained, the movement causes a redistribution of the support reactions which in turn induces additional moments. These are often termed Secondary Moments, but they are not always small, or Parasitic Moments, but they are not always bad. They are normally denoted by M_2.

Freyssinet's bridge across the Marne at Luzancy, started in 1941 but not completed until 1946, is often thought of as a simply supported beam, but it was actually built as a two-hinged arch (Harris 1986), with support reactions adjusted by means of flat jacks and wedges which were later grouted-in (Fig. 6). The same principles were applied in the later and larger beams built over the same river.

Magnel built the first indeterminate beam bridge at Sclayn, in Belgium (Fig. 7) in 1946. The cables are virtually straight, but he adjusted the deck profile so that the cables were close to the soffit near mid-span but above the centroidal axis at the internal support (Magnel 1951). Even with straight cables the sag-

Figure 7: Sclayn Bridge

ging secondary moments are large; about 50% of the hogging moment at the central support caused by dead and live load.

The nomenclature for dealing with these reactant moments was quickly established (Guyon 1951b). The designer needed to distinguish between the actual location of the *cable profile* (e_s), and its apparent position, known as its *line of thrust* (e_p). The two profiles differ by M_2/P. The cable profile has to fit within the section and is subject to cover constraints, but it is the line of thrust which has to be used in the stress calculations.

A good designer can exploit this freedom, but it is also the cause of problems; the secondary moments cannot be found until the profile is known but the cable cannot be designed until the secondary moments are known. Guyon (1951b) introduced the concept of the *concordant profile*, which is a profile that causes no secondary moments; e_s and e_p thus coincide. Any line of thrust is itself a concordant profile.

The designer is then faced with a slightly simpler problem; a cable profile has to be chosen which not only satisfies the eccentricity limits (3) but is also concordant. That in itself is not a trivial operation, but is helped by the fact that the bending moment diagram that results from *any* load applied to a beam will itself be a concordant profile for a cable of constant force. Such loads are termed *notional loads* to distinguish them from the real loads on the structure. Superposition can be used to progressively build-up a set of notional loads whose bending moment diagram gives the desired concordant profile. The question of whether such a profile *can* be found will be addressed later.

Inexperienced designers often stop at this stage of the design process, but they are clearly then not making full advantage of the power of prestressing, and their structures are often uneconomic. Guyon pointed out that it was possible to move the cable profile by means of a *linear transformation* in such a way that the line of thrust remains unchanged. Having chosen a suitable concordant profile (which remains as the line of thrust), the designer can then alter the profile by linear transformations until a suitable cable profile is achieved. This freedom allows the engineer to choose lines of thrust which lie outside the cross-section.

Experienced designers often claim that they "do not bother" with concordant profiles; they simply use their judgement to choose a secondary moment that they expect to obtain in the particular structure they are designing. They can then use modified forms of the eccentricity equations (3) which allow them to produce limits on the actual cable profile directly:-

$$e_s \geq -\frac{Z}{A} + \frac{1}{P}(fZ + M + M_2) \tag{4}$$

A slightly different problem now has to be solved; how to find a cable profile that not only satisfies the local eccentricity limits but also generates the required value of M_2.

The two methods are, in fact, directly equivalent, since a profile that satisfies one set of constraints will automatically satisfy the other.

3.2 Analysis to determine M_2

The calculation of the secondary moments M_2 or the determination of the line of thrust e_p, which are equivalent, can be done in several ways. The development of these methods reflects changes elsewhere in analysis techniques, and of course the adoption of computer techniques. At the 1951 conference, for example, Guyon proposed the method of nodal points (a variation of the method of fixed points) (Guyon 1951b); as with most methods at the time the objective was to minimise the number of simultaneous equations to be solved. The most common method in use today is to determine the forces that the cable exerts on the concrete and then to analyse the beam under those loads. This can be done at several levels of detail; if the structure is analysed as a beam the resulting bending moment will be the total effect of the cable ($= Pe_p$) and the reactions will be those that contribute to the secondary moments M_2. If a more detailed method, such as a finite element analysis, is used, the distribution of cables across the width can be determined, as can local effects of the cable profile distribution, which can be particularly important with cables which have significant horizontal curvature.

Alternatively, for continuous beams, use can be made of virtual work to derive a set of equations from which the secondary moments can be determined directly in terms of the actual cable profile (see, for example, (Burgoyne 1988)). This is not usually much easier than the cable force method, but it does allow analytical formulations to be developed which can be used to derive further theories.

3.3 Existence of line of thrust?

Experienced engineers adopt their own strategies for designing complex structures. Low (1982), for example, showed that there were limits on the minimum prestressing force that is needed in a continuous beam. One minimum limit was derived directly from the Magnel diagram, while another considered the range of eccentricities that have to be allowed for between the maximum sagging region at mid-span and the maximum hogging region over the piers. If the prestressing force is not high enough, the eccentricity range is too large and it is impossible to find a secondary moment that leaves the cable profile always inside the structure.

But Low also showed that there was a third limit on the prestressing force which had to be satisfied before a solution could be found, which he called the "third equation". It was later shown (Burgoyne 1988) that this limit related to the *existence* of the line of thrust. If the prestressing force is too low, then a cable placed anywhere between the upper or lower limits on the cable profile will give secondary moments of the same sign. Under these conditions, no concordant profile can exist, so the designer will never be able to find a satisfactory solution without increasing the prestressing force. By satisfying Low's third limit, the designer is assured that a valid profile exists, even if it still has to be found.

3.4 Applicability of plastic theory

Prestressed concrete beams are normally checked for ultimate moment capacity, but that is not the same thing as saying that plastic theory can be used to design such beams. Plastic theory can only be used if prestressed concrete structures have sufficient ductility to allow redistribution of bending moments as hinges form.

La Grange conducted a study on indeterminate beams and frames, where he concluded that indeterminate prestressed concrete structures attained a load at failure which was just below that predicted by full plastic theory (La Grange 1961). The small discrepancy was due to the post-peak softening that occurs in prestressed concrete structures, so that in order to allow the full set of plastic hinges to develop, the first hinges undergo some reduction in moment capacity. However he concluded that the difference was, in practical terms, negligible.

Prestressing steel is much stronger than normal reinforcing steel and does not exhibit a well-defined yield point, so the steel has to have much higher strains before significant plastic deformation occurs. Some of that strain capacity is consumed during the act of prestressing but significant elastic curvatures still have to take place before yielding can occur. This gives a lower limit on the depth of the neutral axis at failure.

There is a further complication, because higher strength steels are typically more brittle than reinforcing steels, with strain capacities of the order of 3%. Thus designers should limit the strains that develop in the steel at the ultimate load, and a limit which is frequently applied is to limit the *additional* strain, after prestressing, to 1%. This limit can be criticised as too low, but it takes account of the fact that the analysis at the ultimate load uses an average strain along the tendon, while the strain at crack locations can be higher. This condition provides an upper limit on the depth of the neutral axis.

The result of these two conditions is that prestressed concrete beams can only behave plasticly if they satisfy relatively narrow limits on the position of the neutral axis, which in turn provides narrow limits on the section geometry.

Codes of practice normally aim to force designs into this narrow band of acceptability, and they only allow redistribution if certain conditions on the neutral axis position are satisfied.

3.5 Secondary moments at the ultimate load?

A closely related question for the designer of continuous prestressed concrete is whether secondary moments should be taken into account when the ultimate moment is calculated. The question is not trivial; secondary moments can be of the same order of magnitude as the dead-load bending moments, although distributed in a different way.

The logic behind limit-state codes is to check each possible failure mechanism of the structure, such as cracking, vibration or collapse. A proper check on the ultimate limit-state would therefore require determination of the final collapse mechanism of the structure. When the final plastic hinge forms the structure becomes a mechanism, so when the penultimate hinge formed, the structure must have been statically determinate; secondary moments do not exist in statically determinate beams. The moment distribution in the beam can be found purely by equilibrium considerations which will differ from the elastic moments by a certain amount of redistribution.

However, in most cases, ultimate moment capacities are checked on a section-by-section basis by applying factored values of the elastic load distribution. Some codes make no mention of secondary moments, but others allow the inclusion of M_2 in the ultimate load calculation (Mattock 1983). In effect, this condition ensures that the first plastic hinge forms with a sufficient reserve of strength; up to this load, the structure has been behaving elastically, so secondary moments would, indeed, have been present. In a beam with sagging secondary moments the effect can be to significantly reduce the ultimate moment capacity that has to be provided over the piers, and to increase the moment capacity that is required in the span regions.

There have been some laboratory studies of continuous beams where the support reactions were measured as loads were increased until a collapse mechanism developed. The magnitudes of these reactions allowed the presence (or absence) of the secondary moments to be monitored, and the results showed that the secondary moments disappeared as the final hinge formed. The plastic hinges did not form suddenly, but slowly developed through an elasto-plastic regime (Mattock, Yamazaki, and Kattula 1971).

The existence of secondary moments is an academic question if the structure is ductile, since the moment distribution, with or without M_2, satisfies the Lower Bound Theorem if the structure is provided with adequate rotation capacity, but codes do not always allow these effects to be taken into account, or limit the amount of redistribution that can take place.

3.6 Temperature effects

Temperature variations apply to all structures but the effect on prestressed concrete beams can be more pronounced than in other structures. The temperature profile through the depth of a beam (Emerson 1973) can be split into three components for the purposes of calculation (Hambly 1991). The first causes a longitudinal expansion, which is normally released by the articulation of the structure; the second causes curvature which leads to deflection in all beams and reactant moments in continuous beams, while the third causes a set of self-equilibrating set of stresses across the cross-section.

The reactant moments can be calculated and allowed-for, but it is the self-equilibrating stresses that cause the main problems for prestressed concrete beams. These beams normally have high thermal mass which means that daily temperature variations do not penetrate to the core of the structure. The result is a very non-uniform temperature distribution across the depth which in turn leads to significant self-equilibrating stresses. If the core of the structure is warm, while the surface is cool, such as at night, then quite large tensile stresses can be developed on the top and bottom surfaces. However, they only penetrate a very short distance into the concrete and the potential crack width is very small. It can be very expensive to overcome the tensile stress by changing the section or the prestress, and they are normally taken into account by the provision of a mesh of fine bars close to the surface.

A larger problem can arise if thermal stresses act as a trigger for more damaging cracking, such as the release of locked-in heat of hydration effects which can occur when a thick web is associated with thin slabs.

3.7 Construction sequence effects

Prestressed concrete tends to be used for the longer-span bridge structures, which often means that they are built sequentially. As a result, the bending moments at the end of construction differ from those which would be expected if the bridge had been built in one go (the *monolithic moment*). As an example, balanced cantilever construction builds out from a central pier, so the structure is inevitably in hogging bending throughout. When the tips of two cantilevers meet they are joined by an in-situ stitch, or sometimes by a short suspended span that is usually made fully continuous. The cantilever will have prestressing cables at the top to resist hogging bending, while continuity cables will be introduced across the joint to resist the sagging bending that will occur later.

The designer has to allow for the temporary condition, and also for the *trapped moments* that are induced by the construction sequence. These trapped moments can be large, and obey the same rules as the secondary moments, in that they are brought about by a redistribution of the dead-load support reactions. The designer may deliberately choose to use the continuity cables to induce a secondary moment that reduces the trapped moment.

Further trapped moments can be induced by the use of temporary prestressing cables which are introduced when the structure is in one configuration, and then removed later after the support conditions have changed. For example, in span-by-span construction, where a long viaduct is built one span at a time, it is sometimes necessary to introduce temporary cables to resist sagging bending moments that occur during construction but which will be removed later. Putting a cable into a two-span structure (for example), and then removing it once the structure is more indeterminate, does not leave a zero stress state; these effects should not be overlooked.

3.8 Creep effects

The final effect that needs to be considered is, appropriately enough, creep (Bazant and Wittmann 1982). It was Freyssinet's original observation of creep that made prestressed concrete possible since he managed to reduce the loss of force caused by creep. In simply supported beams creep causes some loss of prestress and increased deflections, which may need to be taken into account, but it does not alter the distribution of bending moments so the design remains relatively straightforward.

If the structure is indeterminate there is always the possibility that the bending moments may be altered by redistribution of the support reactions. If the structure is built in one piece, all the concrete will be of the same age, and its effective modulus will change uniformly throughout the structure. No redistribution of forces is to be expected under these circumstances.

However, if the concrete is of different ages, the amount of creep that can occur in the various parts of the structure will vary, which allows redistribution of moments. It is now well-established that the structure will creep towards the monolithic state, and the designer can take the as-built condition (including trapped moments) and the monolithic state as limiting conditions for the behaviour of the beam. This simplifies the design process.

England has studied the effect of temperature variation through the depth of the beam. Creep is temperature dependent and takes place more quickly on the warmer side of a structure than on the colder side, which can significantly alter the load distribution. This work was originally applied to nuclear reactor containment vessels, where the temperature variation across the thickness can be of the order of 100°C (England, Cheng, and Andrews 1984). The work makes use of the concept of a steady-state, when creep can continue but without redistribution of stress. More recently, it has been shown that the much smaller temperature variations that can be expected through the depth of a bridge deck, which may be of the order of 5°C, can also have a significant effect. The speed with which creep occurs is very heavily dependent on the relative ages of the concrete in different parts of the structure (Xu and Burgoyne 2005).

4 Conclusion

The successful design of continuous prestressed concrete beams cannot be divorced from the techniques used to analyse the structure, and the way these have developed in the 60 years since the first indeterminate structures were built is a fascinating reflection on the way structural analysis has developed over the same period.

It remains the case that designers cannot blindly use analysis programs without fundamental understanding of the way prestressed concrete behaves.

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