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DUCTILITY AND DEFORMABILITY IN BEAMS PRESTRESSED WITH FRP TENDONS

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ABSTRACT

The paper addresses the definitions of ductility and deformability that are causing significant problems to advocates of the use of FRPs in prestressed and reinforced concrete, and confusion for those who are considering its application. The use of the words for steel-reinforced structures is discussed, as are the questions of plasticity theory, energy absorption and warning of failure. It is shown that structures that have a significant ability to deform, and at the same time, absorb energy, are desirable. It is shown that structures with confinement of the concrete in the compression zone can give the ideal moment-curvature relationship, with a slowly rising response (giving large displacements), and a concave unloading curve (giving large energy absorption). It is suggested that these ideas should be combined with partially bonded or external tendons to achieve optimal results.

KEYWORDS

Ductility, concrete, prestressing, confinement, aramid fibres, testing, analysis.

INTRODUCTION

Discussions with practising design engineers about the use of FRP always return to the question of the lack of ductility. At some point in the discussion someone will say "But FRPs are brittle", with the implication that they therefore cannot be used. The advocate of FRP will talk about the rotation capacity of the beams, but the initiative has been lost and it is very unlikely that FRPs will be used in the structure under discussion.

It is worth discussing what an engineer means by *ductility*. The Oxford English Dictionary definition is "capable of being drawn out into wire". This clearly applies to prestressing strands since they have been produced in this way. Equally clearly, it does not apply to FRPs themselves, since they cannot be drawn. But what does the term mean when applied to the moment-curvature (M-κ) relationship of a beam? The key is to consider, not just the response when the beam is loaded, but also that when the beam is unloaded *before failure*.

Consider a steel-reinforced concrete beam, whose behaviour is illustrated figuratively in Figure 1. The beam is initially stiff, since the concrete is uncracked. There is then a phase in which cracking takes place which is very variable – no two beams will behave the same since individual cracks are initiated by brittle flaws. However, as the cracks develop they propagate to the neutral axis of the beam, with little or no tensile stress, and similar beams would again behave in the same way. If the beam is unloaded from this stage it behaves almost linearly and with very little permanent deformation. Some energy has been dissipated, since the loading and unloading curves differ, but it has been released as part of the fracturing process of the concrete. The energy left in the beam in a loaded, cracked-elastic state is entirely elastic energy; this is illustrated by the fact that the beam can be loaded and unloaded repeatedly in this regime, which includes the normal working load of the structure.

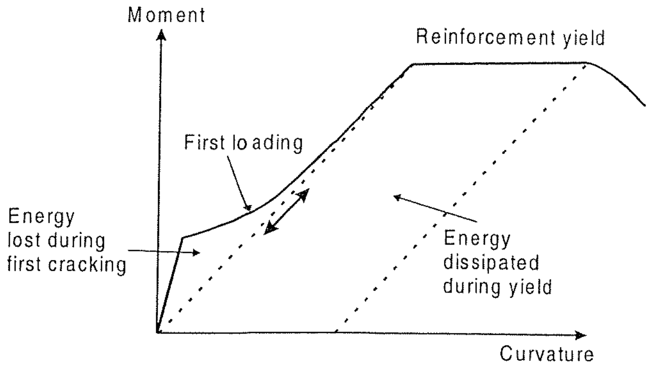

Figure 1. Moment-curvature relationship of a typical steel-reinforced concrete beam, showing unloading response and energy dissipation

Subsequent loading of the beam (which in accordance with normal design procedures may be assumed to be under-reinforced) causes the steel to yield; this leads to a plateau on the moment curvature diagram. If the beam is unloaded from this state, the behaviour is usually linear but there is a large residual curvature. The M-κ response remains flat until failure; if the beam were to be unloaded immediately prior to failure – something that is possible conceptually but very difficult to achieve in practice – it is possible to consider what has happened to the energy that has been introduced to the system.

The area under the moment-curvature response represents the energy stored in the beam at any time, per unit length of the beam; that energy has been supplied by the loads. Three areas can be considered, divided by two unloading curves. The first division occurs along the unloading curve from the point where the steel first yields; the second occurs from the point immediately before failure. The area to the right of the second line is the elastic energy remaining in the concrete and the reinforcement – this is stored in the beam and released on unloading, except at the plastic hinge. The area to the left of the first line has already been dissipated – this occurred at the moment of cracking. The area in between has been dissipated by plastic deformation of the steel – this is widely understood by engineers, and is the reason why a flat M-κ response is regarded as "a good thing".

Now consider a different structure. This time the structure is a thin slab, prestressed with unbonded low-friction steel strands (Figure 2). There is an initially stiff response, followed by a zone of continuing cracking, followed by an almost flat response. The curve is not entirely horizontal, since the force in the tendon is increasing, albeit slowly, leading to final failure. In this circumstance the steel does not yield, so we cannot consider an unloading path from the point of steel yield. Unloading immediately before failure leads to virtually complete restoration of the original deflection since the prestress closes up the cracks. Virtually all the energy under the M-κ relation is elastic, apart from the small portion that was dissipated in forming the original cracks. Falsely, most engineers regard the response shown in Figure 2 as being the similar to that shown in Figure 1, in that it represents ductility. It does not.

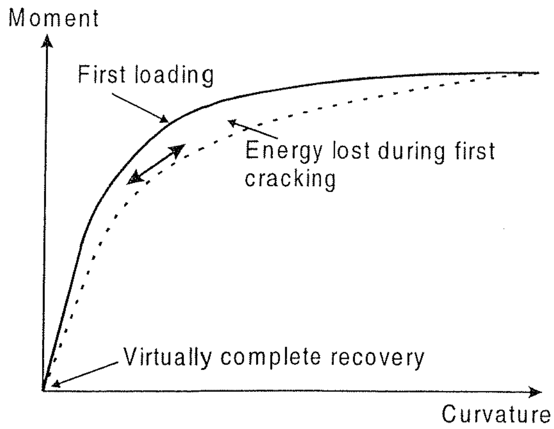

Figure 2. Moment-curvature response of a typical unbonded prestressed slab.

Consider once again the definition of ductility. If a round bar were made with a stress-strain curve that mimicked the moment-curvature response in Fig 2, could it be drawn? Clearly not, since it would deform on passing the die and then spring back to its original shape. Figure 2 thus represents deformability – the ability to be deformed – but not ductility.

LOWER BOUND THEOREM

It is important to consider why "ductility" is regarded as desirable by most engineers. Since the 1950s, plasticity theory has been widely taught and forms the basis of most codes of practice world-wide. It is based on two theorems, one of which is well-known but not used, while the other is in daily use by engineers, even if they cannot quote it. The upper-bound theorem, based on considering the energy dissipated during collapse of a structure, shows that any estimate of the collapse load obtained from a given mechanism of failure will be at or above the true collapse load. It is easy to understand, but is regarded as "unsafe" and is rarely applied, with the possible exception of yield-line analysis of slabs.

The lower-bound theorem, on the other hand, is conceptually more complex. It states that if any distribution of internal forces and moments is in equilibrium with the applied loads, and yield is nowhere exceeded, then the structure is safe. Engineers rarely use the theorem directly and few could quote it if asked. But all engineers rely on it all the time; if a structure is analysed, whether by hand calculation, frame analysis or finite elements, assumptions have to be made about stiffnesses, boundary conditions and material properties. These are often unknowable at the time of design, and frequently wrong, but the set of moments, forces and stresses that result from the analysis does satisfy equilibrium, and if the engineer makes the structure strong enough, it will satisfy the lower bound theorem. Even if engineers do not know why, the structures they design are safe.

Consider again the structures shown in Figs 1 and 2. Both have deformability, but only one has ductility. Does the lower-bound theorem apply equally to both? In practice it does, since the reduced stiffness that occurs in the more heavily loaded region allows redistribution of forces to more lightly loaded regions. So it is deformability that is important when considering the reliance that all engineers have on the lower-bound theorem.

ENERGY-DISSIPATING STRUCTURES

Structures that are designed to dissipate energy are exceptions to the discussion given above. Protective structures, such as those designed to resist blast or impact, or buildings designed for use in seismically active regions fall into this category. It might be thought that ductility was the over-riding consideration for these structures, but even here that is not clear.

An important consideration is the frequency and intensity of the event that is being protected against. If the events are rare, but potentially catastrophic, then maximising the energy dissipation is desirable. Such an event might be the explosion of a chemical storage facility, where neighbouring buildings or the surrounding plant must be protected to prevent loss of life or a chain-reaction. It would be accepted in these circumstances that the protective structure would have been severely damaged during the event, and would need to be demolished and replaced afterwards. This might also apply to those parts of the world where large devastating earthquakes occur very rarely. In these circumstances, saving a large number of lives, despite the fact that buildings will subsequently have to be demolished and rebuilt, is sensible.

Consider instead the case where there are a larger number of smaller events. If fairly large earthquakes occur on a reasonably frequent basis (i.e. more than once in the expected lifetime of the structure), then having to demolish or significantly repair buildings may not be the best option. In such cases, the structure should be able to return to its initial state, without significant structural damage. Deformability is better than ductility.

There is however a price to pay for deformability. In small earthquakes, energy is stored elastically when loaded, and is thus released when unloaded. This can lead to resonant behaviour under cyclic loading. In very large earthquakes, which do lead to failure, the stored energy can be released suddenly, with potentially damaging characteristics.

Structures that Warn of Failure

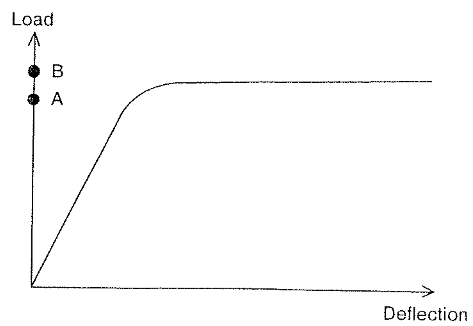

Figure 3. Does this structure warn of collapse?

It is often suggested that ductile structures are desirable since they give warning of collapse. This may be true in laboratories, where hydraulic loading is effectively under displacement control, but in real life, where most of our structures are under gravity loading, this benefit is more apparent than real. A structure with the response shown in Figure 3 would be regarded as "ductile" by most engineers, but at load "A", would anyone have any warning of collapse? By the time the load had increased to "B" it would have failed.

STRUCTURES PRESTRESSED WITH FIBRES

How does all of this apply to beams with fibrous tension elements? It is taken as axiomatic that the most sensible use of fibres in new construction is as non-corroding prestressing tendons, with partial bonding when used as internal pre-tensioning tendons, or as unbonded external tendons (Burgoyne, 1997). This allows the full strength of the expensive fibres to be used while keeping the curvatures and deflections small. The tendons may be in the form of pultruded rods, if internal, or as parallel-lay ropes if external tendons are used. Beams reinforced with FRP rods are simply too flexible for practical application.

It is also self-evident that structures should be designed, where possible, so that the tension elements do not fail, since fibres are brittle. A corollary of this argument is that the failure process of the concrete becomes much more important. In under-reinforced beams with steel tension elements, the steel yields first, thus defining the maximum moment the section can withstand, while failure of the concrete merely limits the maximum curvature that can be achieved. The failure process of the concrete if of interest, but is not of primary importance.

In over-reinforced beams, either with steel or with fibres, the maximum moment the section can withstand is governed by the failure of the concrete. Thus, perhaps paradoxically, the increased interest in FRPs is leading to more work on the failure of concrete. It also follows that enhancement of the properties of the concrete in compression is desirable.

It is also important properly to understand the question of deformability and ductility. Which do we want, and why? According to the CEB bulletin 242 (CEB-FIP, 1998), the plastic deformation capacity of reinforced members is indispensable for:

- Warning before failure of statically determinate and indeterminate structures by large deflections;
- Allowing the use of linear elastic analysis *without* moment redistribution, based on the stiffness of the uncracked section, which implicitly assumes a certain rotational capacity in plastic areas;
- Allowing the use of linear analysis *with* moment redistribution, which requires rotation capacity in the plastic areas to allow for the assumed degree of redistribution;
- Allowing the use of elasto-plastic analysis, which is based on the assumption of indefinite plasticity of the member;
- Permitting equilibrium methods which are valid only if compatibility of displacements can be achieved by plastic deformation (e.g. truss models, strut and tie models);
- Giving resistance against imposed deformations (e.g. due to temperature, support settlement, shrinkage, creep);
- Providing an ability to withstand unforeseen local impact and accidental loading without collapse (robustness);
- Permitting redistribution of internal forces in statically indeterminate structures under fire attack;
- Dissipating energy under cyclic (e.g. seismic) loading;

Of these, only the last really relates to ductility rather than deformability. In the other cases it is the ability of the beam to sustain deflection that matters, rather than its ability to dissipate energy.

UNLOADING RESPONSE

The behaviour of these beams can be investigated by considering the loading and unloading curves. It is not usually possible to unload beams during testing, and it is certainly not possible to do it after the test is complete. But it is possible to investigate this behaviour numerically and to compare different forms of construction (Morais and Burgoyne (2001)). A section is assumed, either reinforced or prestressed as required. The concrete is assumed to have a stress-strain curve, both on loading and unloading, according to the type of concrete being investigated. For any given curvature, the strain at one point in the section is assumed, which allows the strains at all positions to be fixed. The corresponding stress is then calculated, and the longitudinal equilibrium checked. The assumed strain is then iterated until equilibrium is satisfied, to give one point on the moment-curvature diagram. The curvature can then be increased (if loading), or decreased (if unloading) to build up the complete moment-curvature response.

This type of analysis allows the full moment-curvature response to be derived, from initial loading right up to final failure. Unloading responses can be studied from any point on the loading curve.

LOCALISATION

This type of analysis may not be exact (since it applies at one section rather than for the whole beam), but it is useful for comparing different responses. It also allows an understanding of why localisation occurs.

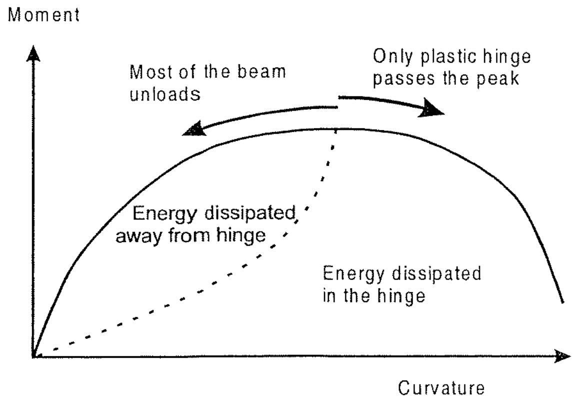

Figure 4. At failure, only one point passes the peak; elsewhere, the beam unloads.

Consider a simply-supported beam loaded by two equal point load, giving a constant moment region in the middle. Further, assume that the moment-curvature response is fairly flat at the maximum load, as shown in Figure 4. Simplistically, the same curve applies throughout the constant moment region, so the beam will fail by inelastic deformation all along that zone. In reality, however, the properties of the beam will vary along that zone, caused by slight variations in material properties or dimensions –

in the limit at a molecular level. Thus, somewhere in that constant moment region will be a section that is weaker than the others. At that section the curvature will increase, forming a hinge. At *all other sections*, the beam will unload. At one point, therefore, we are interested in the area under the complete stress-strain curve. At all other points, we are interested in the area between the loading curve and the unloading curve from the maximum moment.

Characteristic length

The area under these moment-curvature responses gives the energy stored or dissipated per unit length of the beam. If one section fails, but the remainder do not, then it is important to know the length of the failure zone. Various possibilities have been suggested, such as twice the depth to the neutral axis, implying a 45° inclination of the zone of influence above the crack tip, or the overall depth of the beam. Alternatively, the failure length might be independent of the size of the beam; since the stress-strain curves that are used in analysis are based on tests on laboratory samples typically 200 mm long, it might be sensible to take this figure, but Hillerborg (1991) has suggested that the behaviour of concrete at failure is limited by a shortening of 3 mm, which would mean that a strain at failure of about .005 would correspond to a failure length of 600 mm. At present, there is no agreed value to be taken for this length.

DESIRABLE MOMENT-CURVATURE BEHAVIOUR

What would be the optimal moment-curvature behaviour to improve both the ductility and deformability of a section made with FRP tension elements? The actual moment capacity of the section will be governed by the strength of the tension element and the lever arm to the centre of compression, but it is equally important to ensure that the full strength of the compression zone can be utilised.

Given a strength of the concrete and the tension materials, it is also desirable to

1. maximise the curvature capacity of the beam (to improve the deformability)
2. make the unloading response concave (to minimise the stored elastic energy)
3. have a steadily rising moment curvature response (to ensure that, when localisation starts, as much of the beam as possible dissipates energy).

COMPRESSION CONFINEMENT

It is well known that confining concrete enhances its performance in compression, and is frequently applied to columns. Leung (2000) has shown how the compression zone of a beam can be reinforced by means of spiral hoops. The spirals are arranged to overlap, which leads to a compression zone that has unconfined concrete outside the spirals, singly-confined concrete in most areas, and doubly-confined concrete where the spirals overlap.

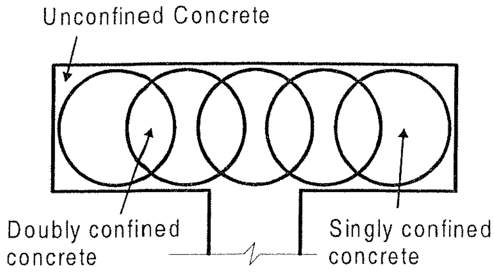

Figure 5. Compression zone with overlapping confinement spirals

The concrete can be analysed by means of Kotsovos' model of concrete under active triaxial loading (Kotsovos and Newman (1980)). The applied stresses are divided into hydrostatic and deviatoric components, and the response to these gives deviatoric and dilational strains. Making due allowance for the passive nature of the confinement, and allowing for the enhanced confining stresses that occur in the overlapping region, three stress-strain curves are produced for the compression zone. Figure 6 shows a typical response.

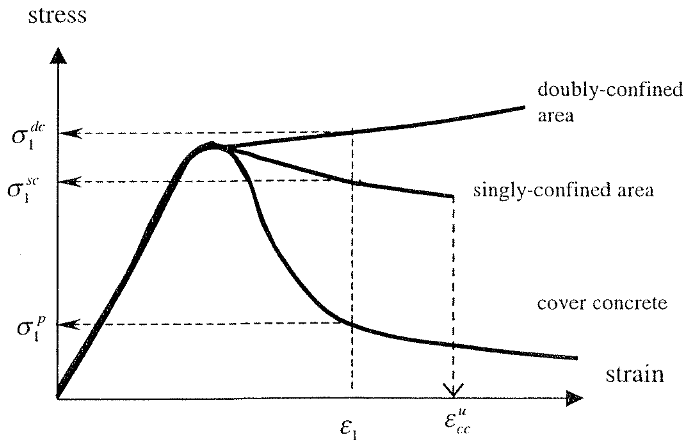

Figure 6. Stress-strain responses for different areas in the compression zone (Leung, 2000)

These stress-strain curves can then be used to produce the moment-curvature response of a beam. Figure 7 shows the response of a beam prestressed with bonded aramid tendons (Morais and Burgoyne 2001), without any enhancement from compression reinforcement, while Figure 8 shows a similar beam with enhancing spirals. The strength is enhanced by about 16% which shows that the materials are being used efficiently, but the curvature at maximum moment is increased by a factor of about 4.5. Furthermore, the moment-curvature relationship shows a rising trend to the maximum moment, and a concave unloading portion, both of which were identified above as desirable characteristics to achieve both deformability and ductility of the resulting beams.

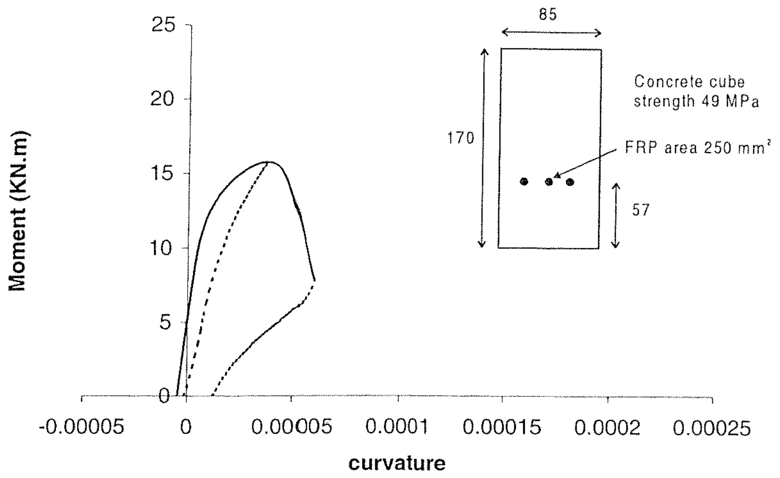

Figure 7. Loading and unloading responses of a rectangular beam prestressed with bonded aramid tendons and ordinary concrete

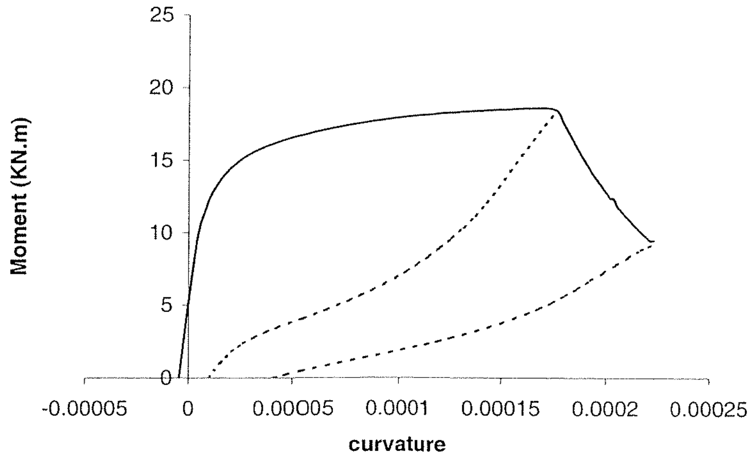

Figure 8. The same beam as in Fig. 7 but with confined concrete in the compression zone

Figure 9 shows a beam constructed using these principles, and tested by Leung and Burgoyne (2001). The top flange of the beam contained six spirals of aramid, arranged with their axes parallel to the length of the beam, which was prestressed with two external aramid ropes. The beam failed, as expected, by slow crushing of the top flange. A similar beam, without compression spirals, and tested by Burgoyne and Guimaraes (Burgoyne, 1992), failed by sudden disintegration of the compression flange.

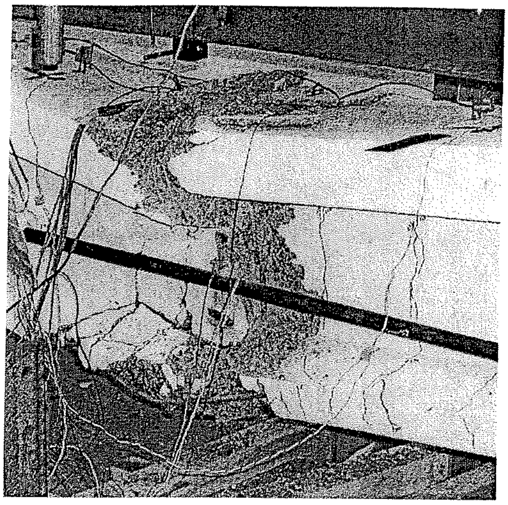

Figure 9. The failure zone of a beam prestressed with unbonded aramid ropes and with confining spirals in compression. Note the significant damage that has occurred in the top flange.

Related work by Lees and Burgoyne (1999) has shown that the moment-curvature response of beams with bonded tendons can be significantly altered by controlled debonding of the tendons. Figure 10 shows the responses of beams with fully bonded, unbonded and partially bonded tendons. The bonded beam has high load capacity, but low deflection at failure. The unbonded beam has high deflection, but low load capacity. However, the three beams with different degrees of partial bond show that both high load and high deflection can be obtained by careful design.

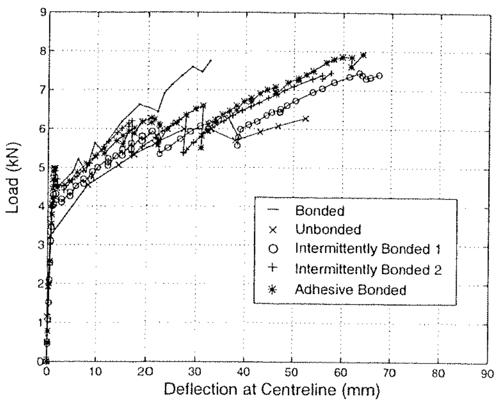

Figure 10. Load-deflection response of bonded, unbonded and partially bonded beams with aramid pretensioning tendons

CONCLUSION

The use of a brittle material for the tendons of a prestressed beam, which is itself made from concrete that is inherently brittle, does not mean that the resulting beam is also brittle. By properly understanding the way the two materials act together, and also by careful consideration of the distinction between ductility and deformability, it is possible to design structures that have the required characteristics.

The use of fibre spirals to act as containment of the concrete in the compression zone allows the moment curvature relationship for a beam to be significantly altered, and the use of external tendons or partial bonding of internal tendons allows further freedom to the designer.

These results show that beams prestressed with FRP or fibre rope tendons offer a non-corroding alternative to the use of steel tendons. However, the best results are not obtained by simply replacing the steel tendons with fibres, but by considering the fibres as materials in their own right and designing to make best use of their inherent strength and strain capacity.

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