RIGID BODY ANALYSIS OF CONCRETE BEAMS PRE-TENSIONED WITH PARTIALLY-BONDED AFRP TENDONS

Janet M. LEES*1 and Chris J. BURGOYNE*1

ABSTRACT: An analytical model was developed to describe the flexural behaviour of a series of pre-tensioned concrete beams with partially-bonded AFRP tendons. The model was based on a rigid body formulation and procedures were incorporated which considered a number of events that could occur during testing. The flexural response of the beams was found to be particularly dependent on two key events; bond breakdown and concrete cracking. The correlation between the predicted analytical and experimental results was good.

KEYWORDS: partial bonding, pre-tensioned concrete, aramid fibre reinforced plastic, rigid body analysis

1 INTRODUCTION

Unlike steel, FRPs are elastic materials and do not yield. In a concrete beam prestressed with steel, the steel yields as the load increases beyond a certain limit and the ensuing deflections provide ample warning of failure. In contrast, the failure of FRP materials is brittle and sudden. As the material properties of FRPs are fundamentally different to those of steel, it is questionable whether conventional design methods based on the use of steel reinforcement are applicable to FRP reinforced structures.

The focus of the current work is to investigate the nature of a flexural design method which reflects an understanding of both the strengths and weaknesses of FRP materials. In particular, the influence of the bond between an AFRP tendon and concrete on the flexural response of a pre-tensioned concrete beam is considered.

---

*1 Department of Engineering, University of Cambridge, U.K., Ph.D.

1.1 EXPERIMENTAL BACKGROUND

A series of beam tests were carried out to determine the influence of the degree of bond on the rotation capacity of a concrete beam pre-tensioned with AFRP. Small scale beams ($100 \times 200 \times 2800$ mm) were cast using either FiBRA or Technora tendons. In the beam tests, the AFRP tendons were either bonded, unbonded or *partially bonded* to the concrete. *Partial bonding* was achieved either by coating the tendons with a resin of known, low, shear strength or by intermittently debonding discrete lengths of the tendon from the concrete.

Two types of intermittently-bonded beams were considered; in the first series the bonded lengths were fairly short so the bond was expected to break down during testing whereas in the second series the bonded lengths were of a sufficient length to ensure that the bond did not break down.

The beams were simply supported and tested in flexure under four point loading. It was found that by controlling the bond between the FRP tendon and the concrete, the flexural response of the beams could be optimised. The optimum flexural response was considered to be that of a beam which has both a high ultimate load capacity and a high rotation capacity. Further details of these experiments can be found elsewhere [1], [2].

A computerised analytical model was subsequently developed to clarify the behaviour of the experimental beams. The particular focus was the partially-bonded beams.

2 ANALYTICAL MODEL FOR PARTIALLY-BONDED BEAMS

Regardless of the bond condition of the tendon, the behaviour of the beams was expected to be similar until first cracking. However, after cracking occurs, the degree of bond had a significant influence on the flexural response of the beam. Although a strain compatibility approach could be used to describe the behaviour of the bonded beams, it was felt that this type of approach was inappropriate for the intermittently-bonded and unbonded beams. In these beams, sections of the tendon were not fully bonded to the concrete and hence, at a particular location, the change in strain in the concrete differed from that of the tendon. A different analysis was thus required.

The visual observation of the behaviour of the partially-bonded and unbonded beams during testing suggested that a rigid body analysis might be applicable. During testing, the flexural cracks acted as hinge locations and the deflection profile of the beam was similar to that a series of rigid blocks connected with hinges. This behaviour differed from that of the beams with bonded tendons which displayed a noticeable curvature along the length of the beam (see Figure 1).

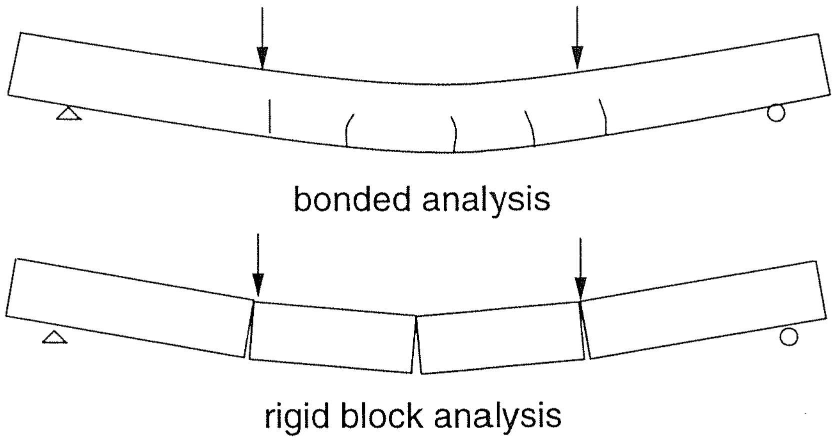

Figure 1: Schematic comparison of bonded analysis and rigid block analysis

A computerised rigid body analysis was formulated and the analysis investigated the relationship between the applied load and the rotation of the beam during testing. The starting point for the analysis was that the first central crack had occurred. The behaviour of the beams from first cracking to ultimate failure was then considered. At each stage of loading, four possible events could occur; bond breakdown, further cracking, tendon rupture or concrete crushing. The occurrence of either tendon rupture or concrete crushing resulted in the failure of the beam. However, bond breakdown or further cracking events could occur at any stage prior to the ultimate failure.

2.1 BOND BREAKDOWN

The tendon length was broken up into a series of segments. The intermittently-bonded tendons were made up of segments which were alternately bonded or unbonded whereas the adhesive-coated tendon segments were considered to be initially bonded, albeit with a reduced bond strength. A simplified force-slip bond model was used to determine whether a bond breakdown event occurred in a particular tendon segment (see Figure 2). If a bond breakdown event did occur, then the bond condition of the tendon was revised accordingly.

In the ascending branch of the assumed shear stress-displacement curve, no slip occurs up to a maximum shear stress of τ_{max}. As the bond breaks down, no further slip takes place until the shear stress drops to τ_{frict}. Once this value of bond stress is reached, the bond stress remains constant with increasing slip.

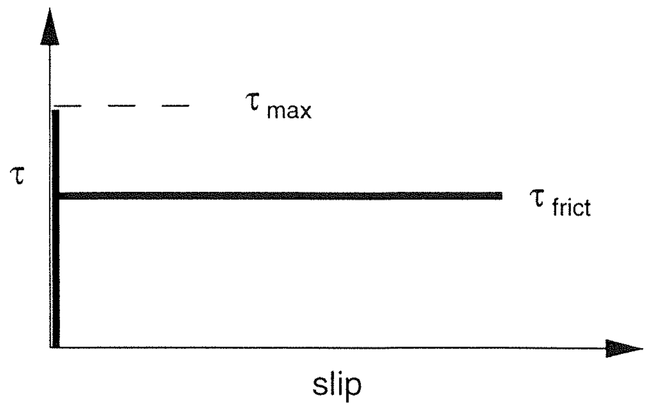

Figure 2: Schematic shear stress vs displacement curve

A further assumption was that the bond stress does not vary along the length of the bonded region and hence the bond stress was taken to be constant:

$$F = \tau \cdot 2 \cdot \pi \cdot r \cdot L \tag{1}$$

where F is the force at the loaded end of the segment, r is the tendon radius and L is the bonded segment length.

2.2 FURTHER CRACKING

To determine whether further cracking occurred a detailed analysis of the stresses on the bottom surface of the concrete is required; an analysis based solely on stress resultants is insufficient, since the local effects of the loads need to be considered. The prediction of further cracking events using a closed form solution proved to be analytically complex; thus, a finite element (FE) model was used.

The experimental beams were modelled as a cantilever with a length equal to that of half the actual beam. Four unit load cases which were analogous to the concrete compressive force, the applied load, the vertical forces at the crack location and the tendon forces through the bonded segments were applied to the cantilever (see Figure 3). For each unit load case, coefficients relating to the build-up of stress along the bottom face of the beam were determined and these coefficients were used in the main program to ascertain whether further cracking occurred.

It is of note that the fixed end of the cantilever does not reflect the actual support conditions of the experimental beams which were simply supported. However, this is not of concern since the applied loadings and internal forces will be in equilibrium. The superposition of the unit load case results, multiplied by the actual loads and internal forces, will therefore result in a solution with no moment at the support. As there is no net moment at the left hand end, the effect of the clamped end in each of the individual analyses is, by St. Venant's principle, not significant near the far end of the cantilever when the four load cases are combined.

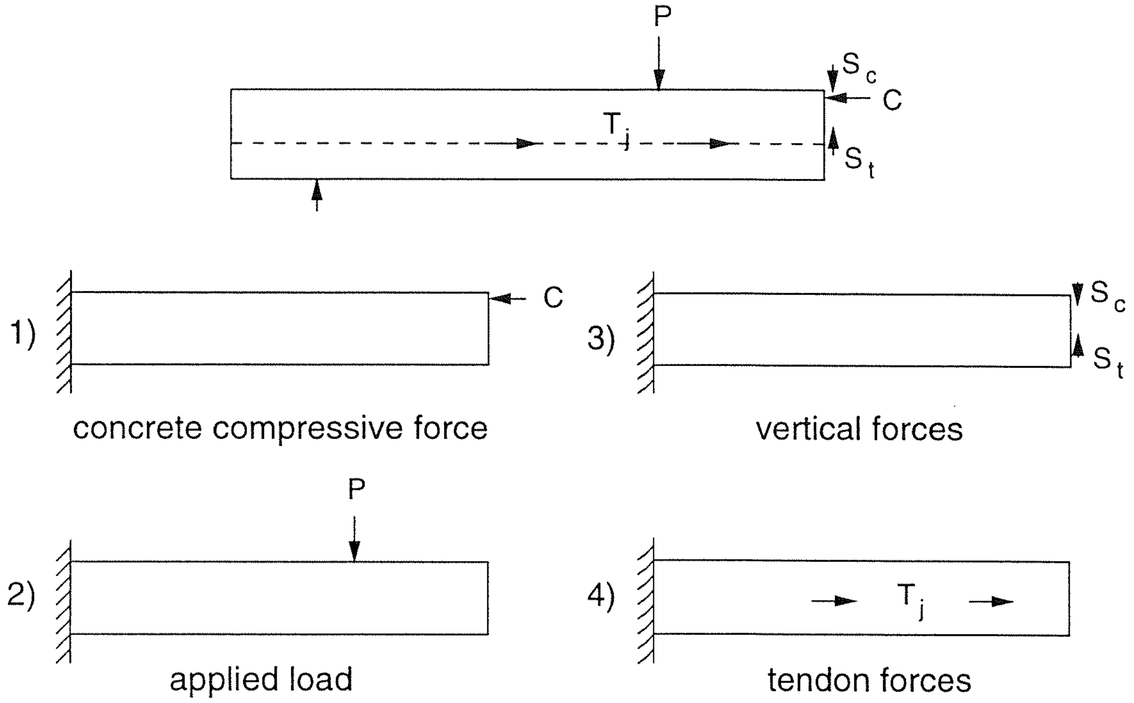

Figure 3: Forces acting on a beam

3 ANALYTICAL AND EXPERIMENTAL RESULTS

The rigid body formulation was used to analyse the partially-bonded beams and the unbonded beams. In the computer analysis of the experimental beam behaviour, specific bond breakdown and cracking events were identified. As an example of the possible sequence of events that were predicted to occur, the first series of intermittently-bonded FiBRA beams (where the bond was expected to break down during testing) is considered. In this particular beam, the tendon was modelled as a series of 11 segments where the odd numbered segments were initially bonded and the even segments were unbonded (see Figure 4).

The experimentally determined relationship between the applied load and rotation, θ, for the FiBRA beam can also be found in Figure 4 and the results of the computer analysis have been superimposed on the experimental results.

The predicted sequence of events is as follows: the bond breaks down in segments 5 and 7 at the start of the analysis; with increasing load the bond breaks down in segment 3; a crack forms at 1520 mm from the support; the subsequent increase in force in segment 8 results in the bond in segment 9 breaking down; a crack occurs at 880 mm from the support and ultimate failure is due to tendon rupture.

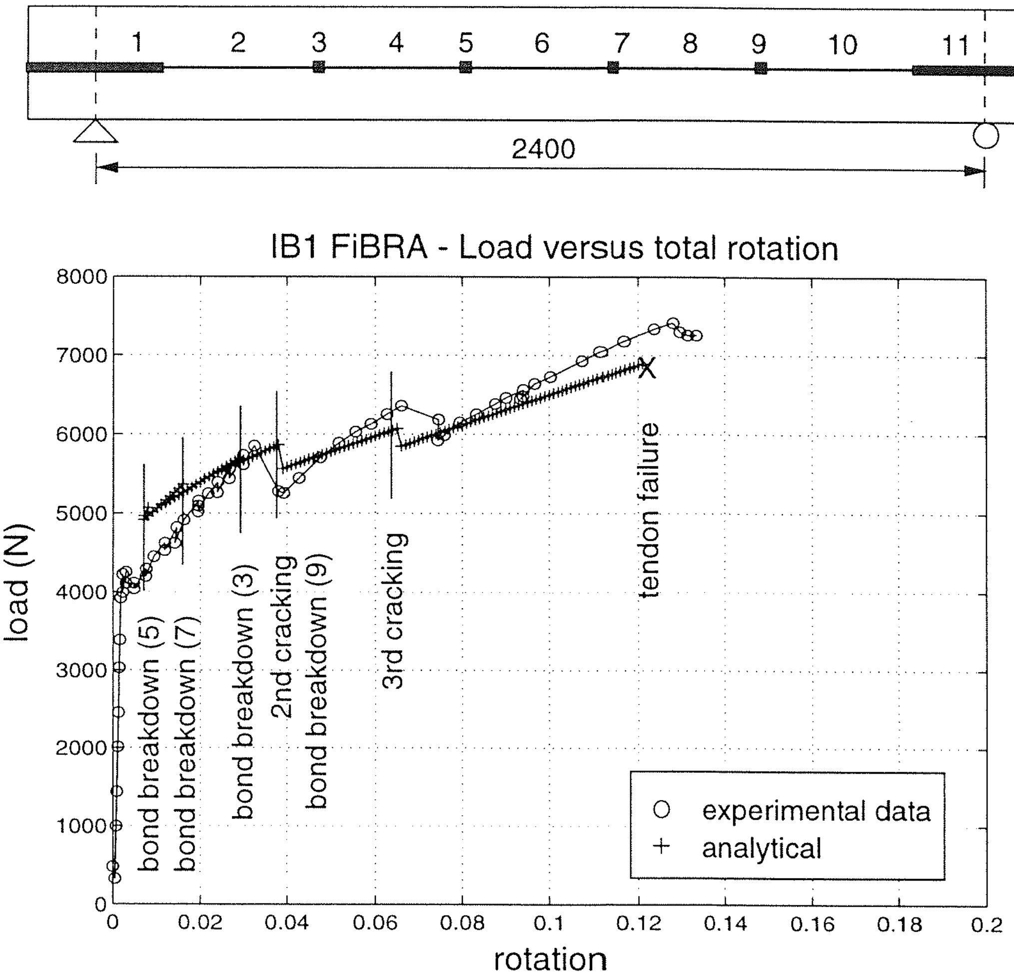

Figure 4: Load rotation curves - Intermittently-bonded series 1

The computerised analysis also provided insight into the predicted cracking behaviour of the beams. A comparison was made of the predicted stresses along the bottom face of the beam based on the FE unit load results described earlier and the stresses determined using simple beam theory (see Figure 5). The results shown are for an intermittently-bonded Technora beam just before 2nd cracking. As expected, the stresses away from the crack are similar in both cases which increases the confidence in the accuracy of the FE model. However, spurious results due to end effects in the FE model are noticeable in the first few nodes away from the support; but can safely be ignored.

The FE tensile stresses are minimal at the crack face and build-up to a peak tensile stress approximately 360 mm away from the centreline crack. In contrast, simple beam theory predicts that the tensile stress is constant between the centreline and the location of the bonded region closest to the crack. At this point there is an increase in tensile stress due to the bond force but the stress then remains constant until the tensile stress reduces due to the applied load. It can be noted in Figure 5 that the bond in segment 5 has broken down before 2nd cracking and hence the frictional bond force is transmitted

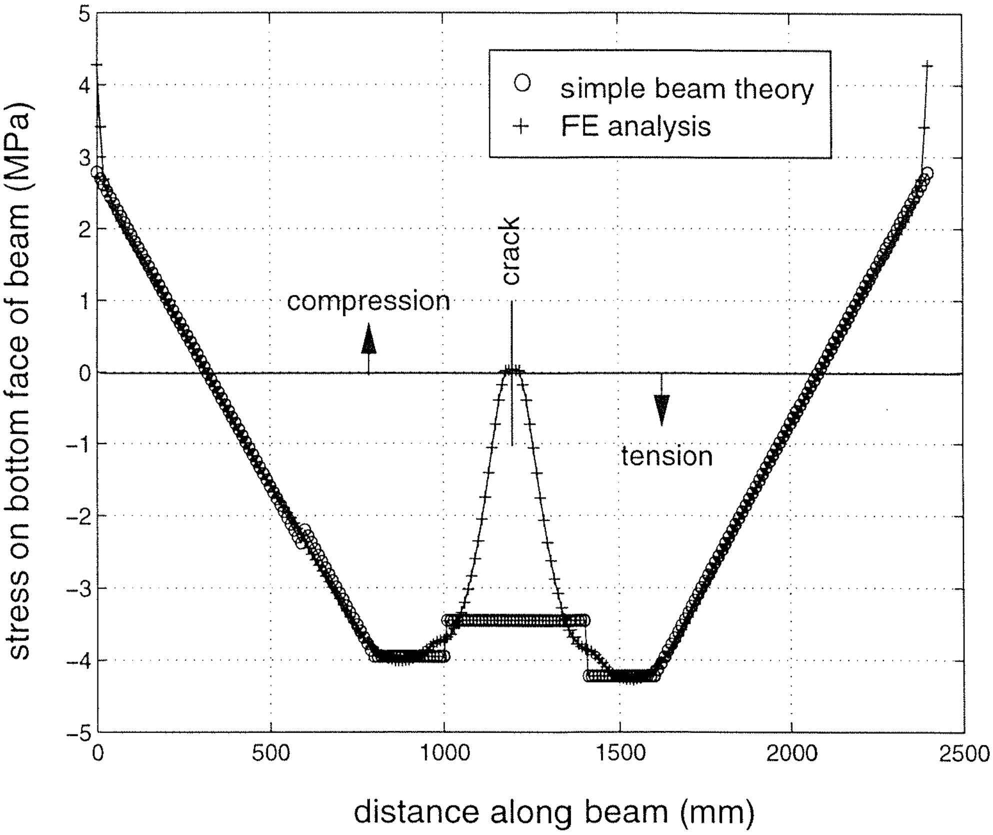

Figure 5: FE and simple beam theory results - Intermittently-bonded series 1

through this section. In contrast, the integrity of segment 7 has been maintained and as a result a higher bond force is transferred through this section.

4 DISCUSSION

The correlation between the experimental and analytical load-rotation curves was encouraging and in the intermittently-bonded FiBRA beam, bond breakdown and cracking events were identified.

The comparison between the bottom face stresses predicted by simple beam theory and those predicted using the computerised analysis indicated that the localised behaviour is an important factor in determining where the next crack will form. A knowledge of the cracking behaviour is important since multiple cracks are essential in order to ensure that large rotations can occur prior to the occurrence of premature concrete crushing at a single crack location.

In a sensitivity analysis of the analytical results, it was found that both the ultimate capacity and the rotation capacity of the partially-bonded beams were highly dependent on the interaction between the bond breakdown events and cracking events. In addition, the analysis showed that the sequence and occurrence of the bond breakdown and cracking events were very sensitive to both the bond properties of the tendon materials and the tensile strength of concrete.

Although the model was initially developed for the partially-bonded beams, it was also found to be applicable to the unbonded beams.

5 CONCLUSIONS

(1) The use of partially-bonded tendons could be incorporated in a new design method for FRP-prestressed concrete structures which would ensure large rotations before failure and significant warning of collapse.

(2) The predicted behaviour of these beams is highly dependent on the assumed material properties.

(3) The method of analysis and the comparison with the experimental results indicate that, although the details of the approach might be slightly inaccurate, the general principles of the behaviour of the partially-bonded and unbonded beams are well understood.

ACKNOWLEDGEMENTS

The authors are grateful for the support of Teijin Ltd., Mitsui Construction Co. and Tarmac Precast Concrete Ltd. One of the authors (JML) was sponsored by the Natural Sciences and Engineering Research Council of Canada (NSERC) and is appreciative of NSERC's financial assistance.

REFERENCES

[1] Lees, J. M. and Burgoyne, C. J., "Influence of Bond on Rotation Capacity of Concrete Pre-tensioned with AFRP", Procs. Advanced Composite Materials in Bridges and Structures (ACMBS-II), Ed. M. El-Badry, The Canadian Society for Civil Engineering, 1996, pp. 901-908.

[2] Lees, J. M., "Flexure of Concrete Beams Pre-tensioned with Aramid FRPs", PhD Thesis, University of Cambridge, UK, March 1997.