BEHAVIOUR OF BEAMS WITH EXTERNAL TENDONS

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Beams prestressed with external tendons show significant differences in behaviour from beams with conventional tendons, especially after the ultimate load has been reached. The behaviour is often characterised by large deflections caused by rotations at single crack locations, and since the tendon can move relative to the concrete, there can be a reduction in the lever arm of the tendon. This leads to a sudden reduction in load carrying capacity, which is undesirable. The compression flange can also reach its limiting capacity as the beam rotates, again leading to catastrophic loss of strength.

A simple rigid-body analysis is used to highlight these phenomena, and recommendations are made as to how such beams should be detailed to overcome the problems.

INTRODUCTION

Beams prestressed with external prestressing tendons have a number of attractions for engineers. They allow a reduction in weight, since concrete is not provided merely to act as cover to the tendon or duct; they allow the tendons to be inspected for signs of corrosion (although that corrosion is rendered more likely in steel tendons because of the absence of an alkaline environment) and the tendons can be replaced or retensioned if necessary. With the moratorium on the use of grouted internal prestressing tendons imposed in the UK by the Highways Agency, and the parallel imposition of requirements for inspectability and replaceability, the use of external tendons is virtually the only way of post-tensioning highway structures that is allowed in the UK. External prestressing is also an ideal application of tendons made of new materials, such as aramids; since the tendons are brittle, it is in any event necessary to avoid the strain concentrations that occur at crack locations with bonded tendons. Since aramid fibres are non-corrodable, there is no problem about the lack of an alkaline environment.

There are, however, drawbacks. Since the tendon can move relative to the concrete, the tendon will not pick up as much strain, and hence stress, at failure, leading to lower ultimate moment capacities, which has economic consequences. This problem is made worse by the difficulty of analysing beams with external tendons, since the strain in the tendon is no longer a function of the behaviour of the beam at a particular cross-section (which allows the construction of a general moment-curvature relationship), but instead is a function of the overall beam behaviour. The consequences of this are that the beam behaviour is non-linear, its moment capacity depends on the loading geometry, and unless the designer is satisfied with simplified code rules, a complex analysis is required.

TESTS ON EXTERNALLY PRESTRESSED BEAMS

A number of tests have been carried out by the authors and their colleagues on beams prestressed with external tendons (Burgoyne et al (1), Guimaraes and Branco (2)). Although these were carried out to demonstrate the viability of prestressing with aramid fibre ropes, the beam behaviour was not dependent on the type of tendon and a number of common features were observed. These features will also be applicable to beams prestressed with tendons made from steel.

The tests were carried out on simply supported beams, with deflected tendons, subjected to four point loading; one of the test beams is shown in Figure 1.

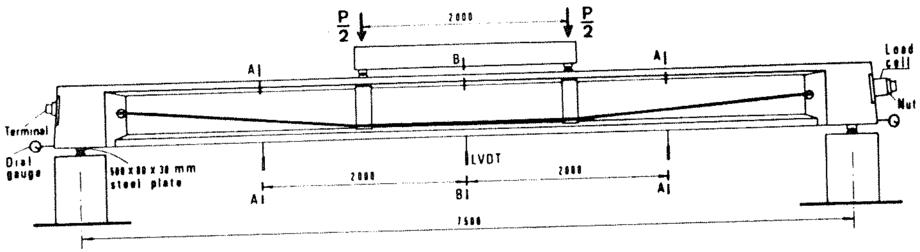

Figure 1. 8m beam with two external polyaramid tendons (from (1)).

The behaviour, prior to cracking and for moderate levels of cracking, was similar to that in beams with conventional tendons. But as the load was increased there was a tendency for one crack to open more than the others, which led to failure which can be analysed by assuming that the beam consists of a pair of rigid blocks, with all the rotation concentrated in a single hinge at the centre.

As the load was further increased, the rotation of the beam caused the tendon to move relative to the compression flange, thus reducing the lever arm of the tendon and lowering the load capacity of the beam. Final failure was caused by explosive failure of the concrete in the compression zone, leading to complete collapse of the structure.

Such behaviour is undesirable, and it must be understood in order that it can be quantified. This paper presents a simplified analysis of the rigid body mechanism of failure, which allows simple predictions to be made.

FINITE ELEMENT ANALYSIS

A finite element program has been modified by one of the authors (Campos, (3)). The program takes account of the non-linear response of the concrete (according to the stress-strain curve defined by the CEB/FIP Model Code 90 (4), and allows for compatibility between the displacement of the tendon and the concrete by considering global displacements. The program allows for slip and friction at the deflector points as the load increases. Good agreement is obtained with test results, as shown in Figure 2. The program is numerically stable for most of the beam's response, but it was not found possible, in most cases, to follow the post-failure response of the structure. This was seen as a disadvantage; even though the load capacity was expected to drop after failure, it was felt that it should be possible to model how rapidly the structure lost strength. It was in an attempt to overcome this problem that the rigid body analysis was developed.

RIGID BODY ANALYSIS

Consider a simply supported beam with a pair of external tendons, one on each side of the central web. The tendons are deflected at two points on either side of the centre line, and loaded symmetrically, as shown in Figure 3(a).

At failure, the beam deflects as shown in Figure 3(b). All deformation is assumed to take place at a central hinge, with the two blocks remaining rigid. The supports and loads are assumed to be capable of lateral movement to take up the horizontal displacements. The concrete is assumed to carry no tensile stresses, so the central hinge rotates about the neutral axis; its position is defined by the depth x which is a primary variable of this analysis. The rotations of the two blocks are both taken as Θ.

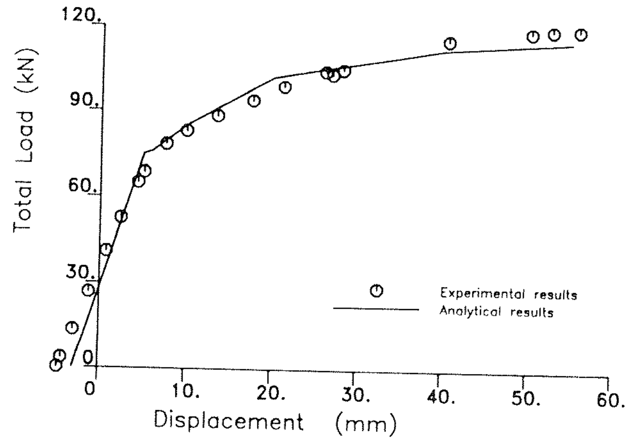

Figure 2. Typical experimental behaviour and finite element analysis of externally prestressed beam.

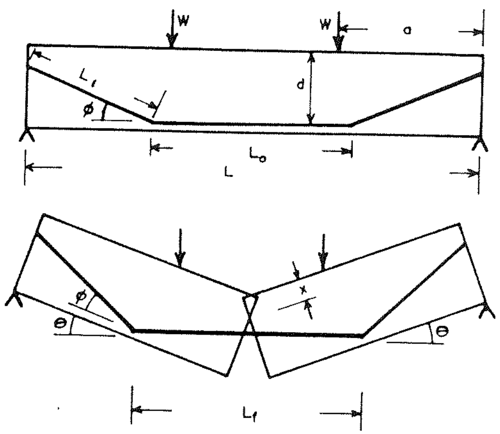

Figure 3. (a - top) Undeformed beam. (b - bottom) After rigid body movement with central hinge.

It then follows that the length of the tendon between deflector points is given by

$$L_f = 2(d-x)\sin\Theta + L_o\cos\Theta \qquad (1)$$

so the extension of the tendon is given by

$$\Delta L = 2(d-x)\sin\Theta - L_o.(1 - \cos\Theta) \tag{2}$$

The value of x is as yet undefined and depends on the force in the tendon.

Tendon force

The force in the tendon will vary as the beam deflects. The initial force in the tendon will be assumed to be F_0 everywhere. When the beam deflects, the force in the tendon will increase to F_1 in the two inclined cables at the ends, and F_2 in the central portion. Two conditions then need to be considered.

(i) If there is no slip at the deflector, all tendon extension takes place in the central portion, so

$$F_1 = F_0 \quad \text{and}$$

$$F_2 = F_0 + \Delta L\, A.E_{ps}/ L_o \tag{3}$$

where E_{ps} is the secant modulus of the tendon as measured from the initial tendon force F_0 (as in Figure 4).

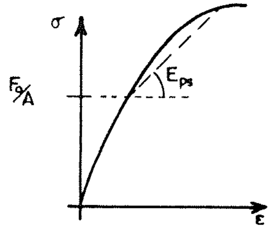

Figure 4. Stress-strain curve for tendon showing definition of secant modulus

(ii) If slip does take place at the deflector, the force in the different parts of the tendon are linked by the capstan equation and the total tendon extension must satisfy compatibility. Thus,

$$F_2/F_1 = e^{\mu(\Theta+\varphi)} \quad \text{and} \quad \Delta L.A\, E_{ps} = (F_2 - F_0).L_o + 2.(F_1 - F_0).L_1 \tag{4}$$

Taken together, these two equations allow F_1 and F_2 to be calculated in terms of the other variables.

To decide which case applies, the procedure is initially to assume that slip does not occur, and then determine if the resulting force difference is sufficient to allow slip; if so, recalculate.

Force in the compression zone for rectangular stress block

One step remains before the solution can be completed. It is necessary to calculate the stress in the concrete in the compression zone. The simplest assumption is to say that the concrete is carrying a uniform stress in compression, $k_1 f_{cu}$, where f_{cu} is the concrete cube strength and k_1 is a scalar factor. If the compression zone is rectangular with breadth b, then the compressive force, C, which must equal the tensile force in the tendon, F_2, is given simply by

$$C = k_1 f_{cu} bx = F_2 \tag{5}$$

Taken together, equations 1. to 5. form a set of non-linear equations. For a particular value of θ, a value can be found for x (if necessary by incremental search) which, when substituted into equations 1. to 4., satisfies equation 5. Once this has been found, the lever arm of the tendon can be calculated from

$$z = d - (L_o\sin\theta - x)/2 \tag{6}$$

and the moment at the centre found from

$$M = W.a = F_2.z \tag{7}$$

Figure 5 shows the results of this type of analysis applied to a typical beam. Since the rigid body analysis says nothing about the initial response, the results of a finite element analysis have been included for completeness. Good agreement is shown between the two analyses in the region where they overlap. The results show that, as soon as significant deformations occur, the lever arm reduces and the load capacity drops. Although not shown here, as L_o increases, the reduction in lever arm becomes more significant, and the loss of strength occurs at a lower deflection.

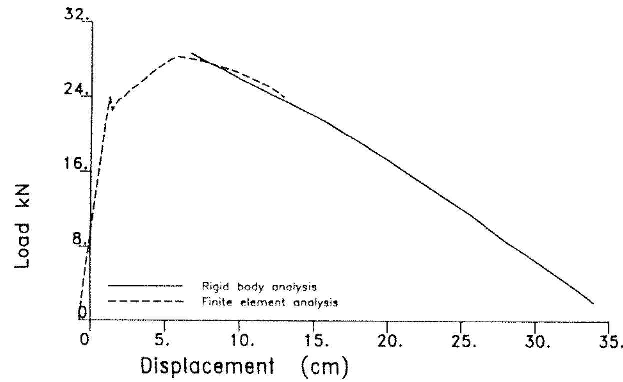

Figure 5. Results of Rigid Body Analysis, showing rapid reduction in load capacity caused by reduction in tendon lever arm.

This type of behaviour is very significant, since the response is highly unstable. Even under laboratory conditions, where loads are applied through hydraulic jacks which lose force when the structure deflects, the final failure of such structures is sudden.

One solution to the problem is to prevent movement of the tendon relative to the concrete during the collapse mechanism. This could be achieved by means of systems which would "catch" the tendon as the beam deflected past the tendon. These could take the form of corbels or ties which would ensure that the tendon eccentricity remained constant. They would not be in contact with the tendon and would carry no load, in normal circumstances. They would not interfere with the inspectability or replaceability of the tendons.

Strain capacity in the concrete

The analysis so far has assumed that the concrete in the compression zone can take whatever strains are applied to it, which is clearly not the case.

The rigid body analysis imposes relative displacements on the two halves of the structure. But if the concrete has a limiting strain capacity, these displacements must be turned into strains. The literature is not consistent here. Hillerborg (5) states that in a compression test the strain between peak load and failure can be related to the displacement by considering a gauge length of 200 mm, irrespective of the actual length of the specimen. The implication is that there is localisation of failure over this length. The CEB/FIP Model Code 90 (4) seems to accept this view since it presents stress-strain curves from specimens 200 mm long but which can be used for other lengths in the absence of better data.

On the other hand, Pannell (6,7) relates the length of the failure zone to some multiple of the neutral axis depth, which is a function of section depth, so that the gauge length used to get strains from displacements would increase with beam size.

Data available at the moment do not allow this problem to be resolved. What test data are available relate mainly to small beams with compression flanges of thickness much less than 200 mm. For the moment, Hillerborg's value of 200 mm will be used to demonstrate the phenomena; similar behaviour, but not necessarily the same ultimate load, would be obtained by using Pannell's method.

The strain in the compression zone, at a distance y above the neutral axis, is given by

$$\varepsilon_c = (y.\sin 2\theta)/L_g \quad (8)$$

where L_g is the gauge length used to relate strains to displacements and is here fixed at 200 mm.

A more rigorous calculation of the compressive force in the concrete can now be undertaken, since the actual stress-strain curve for the concrete can be used. In particular, if using a rectangular stress-block it allows the upper strain cut-off at 0.0035 to be included, or it allows a more complex stress-strain curve such as the CEB/FIP model to be used.

For the purposes of the present demonstration, it is sufficient to assume that the concrete fails at a strain of 0.0035, at a position y_{max}.

$$y_{max} = L_g.0.0035/\sin 2\theta \quad (9)$$

If y_{max} is less than x, the compressive force in the concrete is limited to

$$C = k_1 f_{cu} b y_{max} = 0.0035.k_1 f_{cu} b L_g/\sin 2\theta \quad (10)$$

Thus, C *decreases* as θ increases (Figure 6), while F_2 *increases* (from equations 1. to 4.). Although x varies, which changes things slightly, this does not help much, since it only has a marginal effect on F_2. The corollary is that the compression flange can no longer resist the compressive forces to which it is subjected, and will fail. Furthermore, that failure will be catastrophic, since the majority of the force F_2, to which C must equilibrate, comes from the initial prestress F_0. Since the tendon is unbonded, all the energy stored in the tendon is free to go into the concrete causing extensive fracture.

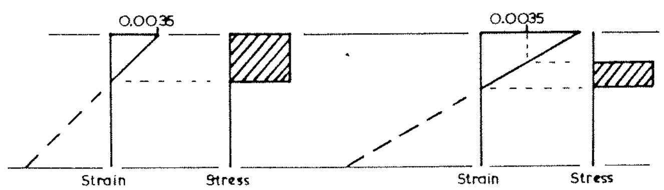

Figure 6. Reduction in compressive force at high curvature (right) in comparison to low curvature (left)

The corollary of this analysis is that the strain capacity of the compression flange should be enhanced, either by confining the concrete within links (which should go as close to the top of the beam as possible), or by the use of fibre reinforced concrete, which can have much higher strain capacity (8).

CONCLUSIONS

The failure mechanisms of beams with unbonded tendons have been considered. A simple rigid body analysis has shown that two highly undesirable effects can occur during failure, both of which need to be taken into account during design to ensure robust structures.

1. The tendon can move relative to the concrete, reducing the lever arm and the moment capacity. This leads to much more rapid unloading than would occur in beams with internal tendons. It is recommended that devices be included to force the tendon to deflect with the beam.

2. Since the rotation takes place over a very small region, the concrete at the hinge position can fail due to excessive strains in the most compressed fibres. This leads to sudden failure of the concrete, since the tensile forces are still increasing as the beam deflects. To improve the situation, care should be taken to ensure the ductility of concrete in the compression zone.

The analysis has required a method of relating displacements of two rigid blocks to strains in the concrete at the interface. It is clear that opinions still differ on how this should be achieved, and there is a case for investigating this problem so that better predictions can be made in future.

The analysis presented here has been restricted to simple, symmetrical beams under 4 point bending. However, the principles of the analysis can easily be extended to more complex structures.

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