ADVANCED COMPOSITE MATERIALS IN BRIDGES AND STRUCTURES
M.M. El-Badry, Editor; Canadian Society for Civil Engineering, Montreal, Quebec, 1996

MATÉRIAUX COMPOSITES D'AVANT-GARDE POUR PONTS ET CHARPENTES
M.M. El-Badry, éditeur; Société canadienne de génie civil, Montréal, Québec, 1996

EFFECT OF VARIABILITY OF HIGH PERFORMANCE YARNS ON BUNDLE STRENGTH

C.J. Burgoyne
University of Cambridge
Cambridge, United Kingdom

P.D. Mills
Alan Baxter and Associates
United Kingdom

ABSTRACT

High performance yarns, such as aramids and polyesters, are brittle; unlike steel, they have no ability to yield while still carrying load. This has an important effect on the properties of a bundle of fibres, since the weakest fibre (or more strictly, the fibre with the lowest break strain), will snap first and shed its load to its neighbours. When these in turn are unable to carry the extra load, the bundle fails.

This paper describes numerical studies, based on yarn test data, which show that bundle failure occurs when only a few of the constituent yarns have failed. It is thus very important that the distribution of failure strains of the yarns is correctly measured. Tests are described on 50 yarns of each of 5 different types of fibre. A polynomial function is fitted to each yarn load-extension curve, which is then used with a correlated random number generator to produce typical yarns taken from the full population. These are used in a Monte-Carlo simulation of bundle strength to predict both the strength of the bundles, and the variability in that strength. The variation in the slack strain is shown to be particularly important.

A number of particular topics are also discussed; variation between fibres and along fibres, transfer length, the difference between bundles with and without resins, and the effect of multiple levels of variation (such as when bundles of bundles are used, as in multiple strand tendons).

INTRODUCTION

The use of new materials for high strength tensile elements, such as prestressing tendons, has focused attention on the efficiency with which the strength of the constituent fibres is converted into the strength of the complete element. The materials used, typically aramid, glass, carbon or polyester fibres, are all brittle; furthermore, they are all made in the form of thin filaments with diameters of a few microns and strengths of a few grams. At the manufacturing plant the filaments are assembled into multi-filament yarns for ease of handling. These yarns have strengths measured in kilograms, but are weaker than the sum of the strengths of the constituent fibres. The yarns are then assembled into finished products which gives a further decrease in the strength per original filament.

The reason for the decrease in strength of the assembly is the brittle nature of the filaments and their variability. When tested, the filaments must all undergo the same strain since their ends are anchored together, either by anchorage systems or by the test grips. But because some of the filaments have a lower strain capacity, due to their variability; they snap and shed their load to their neighbours. This increase in load can cause immediate failure of other elements, which leads to catastrophic breakdown of the assembly. The result is failure of the complete assembly at a stress on the overall assembly which is only sufficient to cause failure of a few elements. [In contradistinction to this phenomenon, ductile fibres, such as steel, do not exhibit the loss of strength, provided the fibres have sufficient plastic strain capacity that the fibres remain carrying load until all have yielded.]

This problem was originally studied by Daniels (1945), who used the phrase "bundle theory" to describe the phenomenon. Amaniampong (1992) studied applications to parallel-lay ropes, and included visco-elastic effects. Assumptions had to be made about the form of the variation in the stress-strain curve for the fibres, and in virtually all cases the resulting equations have to be solved by numerical simulation using Monte-Carlo techniques.

One of the problems is the absence of sufficiently large data sets obtained in comparable circumstances. Because failure occurs when only a few of the original fibres have failed, it is the lower tail of the distribution of failure strains that is important. A small number of especially weak fibres will have little effect on the mean fibre strength, but may have a large effect on the strength of the bundle.

For the present study, 50 yarns have been tested from each of 5 different materials. Four are aramid fibres (Kevlar 29, Kevlar 49, Technora and Vectran), and the fifth is a polyester. All tests were carried out by the same person on the same test machine, and so are comparable. The yarns were obtained by cutting a length from a commercial parallel-lay rope (or, in the case of the Vectran, from a low lay-angle rope), so each yarn will have come from a different bobbin at the rope manufacturer's plant; the results obtained thus show between-bobbin variation rather than within-bobbin effects.

For each yarn test account is taken of the end effects in the test machine and a fifth order polynomial is fitted to the load-extension curve, the coefficients of which are stored in a data file, as is the breaking extension of the yarn. Amaniampong (1992) showed that a fifth order polynomial was needed to model polyester and Kevlar yarns, but that not all the coefficients were statistically significant - in the present study, all coefficients have been left in the data file. The full data are given elsewhere (Burgoyne and Mills (1996)).

The properties of an assembly of yarns is obtained by Monte-Carlo simulation, from which the strength of the bundle, and its variability, can be obtained. A long rope can then be considered as a chain of elements, each of a characteristic length. The individual elements are assumed to be independent, so will have different strengths; the rope will fail in

its weakest element, so it is important to be able to predict both the characteristic length and the strength variability.

Test procedure

All the yarn tests were performed in a tensile testing machine fitted with pneumatic horn grips and a gauge length of 500 mm. The break loads varied from about 80 N for the polyester to 400 N for the Kevlar 49. The samples were obtained from sheathed ropes, from which a 1 m length was cut. The sheath was removed carefully and the helical winding filament discarded. Any yarns that had been damaged when the sheath was removed were discarded at this stage. The smallest ropes had about 200 yarns, and no attempt was made to select yarns from particular zones in the cross-section. Preliminary tests were undertaken to determine the jaw effect which differed between materials. The load and extension data were recorded on a data-logger and transferred to another computer for analysis.

GENERATION OF RANDOM YARNS

The rope analysis program relies on having different yarns so that each rope analysis will be distinct, yet still representative of real ropes. A method is therefore required which will generate random yarns which have the same variability as the original population. The method used here is given in detail in Amaniampong (1992). The load-extension curves for the yarns are functions of five variables which define the fifth order polynomial, and a sixth which defines the break strain. By considering the means and standard deviations of each of these parameters, and the correlation coefficients between them, it is possible to produce yarns at random which could have come from a population with these properties. These will be referred to as *generated yarns*. Generated yarns will, in general, all be different. They can be expected to produce a smoother cumulative distribution function than the test data, but one that is a reasonable approximation to that curve.

A test of this method is to compare the properties of the generated and test yarns. The simplest test is to compare the distribution of yarn forces at failure. This is of interest in its own right, since it is easy to visualise the variability of the yarns. It is also of interest because the break load is not stored in the original data files, but is derived from all six of the yarn variables. Fig. 1 shows such a comparison for all 5 yarn types. The distribution functions for the generated yarns are taken from 5000 yarns and are smoother than the original test data. The results for the polyester and the two Kevlars are in good agreement; the Technora prediction is not so good, and the Vectran prediction is significantly worse. There is a tendency for the generated yarns to be stronger than the test yarns at the extremes of the distribution, but to be weaker in the middle. This may be of significance, since it will be shown below that rope failure occurs when only a relatively small number of yarns have failed, so the goodness of fit at the bottom end of the distribution may be most relevant.

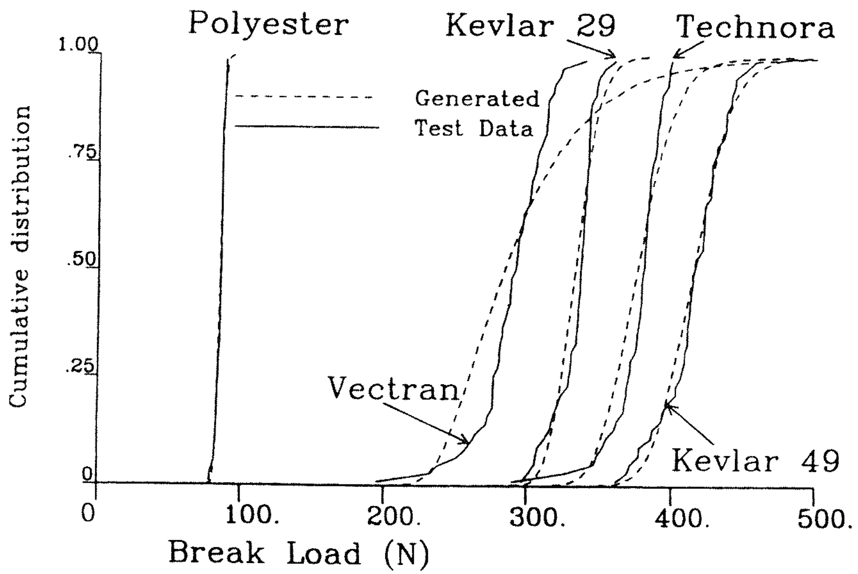

Figure 1. Cumulative Distribution of break loads for tested and randomly generated yarns.

BUNDLE ANALYSIS PROGRAM

To simulate the behaviour of bundles of yarns, a program has been written to generate random bundles, and then follow their load extension curves to failure. Two types of variability are built-in to the program. The individual yarns that make up the bundle are picked randomly, either by generating yarns as described above or by assigning each yarn the properties of one of the tested yarns. In addition, it can be assumed that the bundle is imperfectly made, by adding slack strain to each yarn. Bundles are normally made by drawing yarns from a creel under some tension; the variation in that tension will mean that some yarns will be slack after manufacture, and will not carry load until that slack has been taken up. The slack strain is generated by specifying a mean and standard deviation for a normal distribution; a random number generator then assigns a value to each yarn. Unlike the variation in the yarn test properties, no correlation is assumed between the slack strain and any other variable. The program assumes that all yarns undergo the same strain; as the strain is increased the program checks to see which yarns are slack, carrying load, or broken, and calculates the total load appropriately.

Effect of Slack

Figure 2 shows the predicted effect of different amounts of slack on ropes made from 1000 Vectran yarns; similar results are produced for the other yarns. The slack strains have been assumed to be quite high to show the effects clearly. It is the *variability* of the slack strain that affects the strength of the rope, rather than its mean value which merely produces extra

extension. Ten load-extension curves are shown for each of the three cases - no slack and slack standard deviation being 10% or 20% of the mean yarn break strain. In each case the mean slack strain has been taken (arbitrarily) as three times the standard deviation. As the slack increases, clearly the break strength of the bundle decreases, and its variability increases. However, the rope capacity drops off less quickly after the peak load has been passed. The implications for manufacturing processes are clear - control of uniformity in the fibre tension used to make the product has a very beneficial effect on the resulting strength.

The difference between generating yarns as described above, or randomly choosing a test yarn to represent each yarn in the bundle, has been investigated, and is reported elsewhere (Burgoyne and Mills (1996)). There is a small difference between the behaviour of ropes with no slack modelled from yarns chosen from the test population, and those with properties generated to give the same variabilities. However, when there is slack as well, the effects are indistinguishable. It is thus concluded that, even for the Vectran yarns which show the biggest discrepancy between the generated and tested yarns, the differences are negligible.

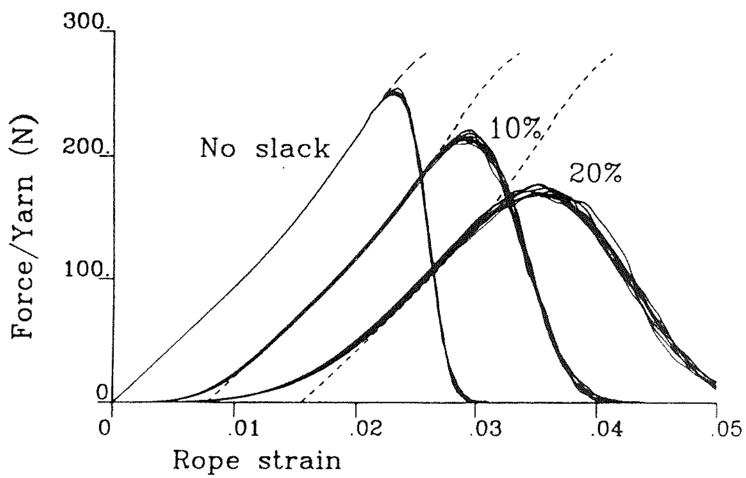

Figure 2. Load extension curves for parallel-lay ropes made from 1000 Vectran yarns, with different amounts of slack strain.

Effect of Size

Under conventional bundle theory, as the bundle gets larger, it also gets weaker. However, from numerical studies carried out on ropes of reasonable size, the expected reduction in strength is masked by a reduction in variability, which is very low for 1000 yarn bundles. For ropes of this size, virtually every bundle contains yarns drawn from almost every part of the infinite distribution, so the differences between bundles is small. For smaller bundles, the chance of getting a uniform rope is reduced, and the variability goes up. Figure 3 shows the mean and one standard deviation range for ropes of different sizes, expressed on a logarithmic size scale. These have been obtained from 500 randomly generated ropes. The rise in strength from 10 - 100 yarns is unexpected but repeatable. It may be due to the fact that these results have been generated with no slack assumed; yarn break strain is heavily correlated with yarn break strength. If slack is added both this correlation and the rise in strength disappear.

Effect of Length

All the results presented so far have been on the implicit assumption that the separate yarns in the bundle do not interfere with one another. Thus, when a yarn breaks it becomes ineffective over the full length of the bundle. That is reasonable for short, parallel-lay ropes, but is not tenable for long ropes or within fibre reinforced plastic rods. Tests on the force needed to withdraw one yarn from a parallel-lay rope have shown that the force increases with length; at about 3 metres the force needed is equal to the break load of the yarn (Burgoyne and Flory (1990)). It thus seems reasonable to identify a characteristic length of about 6 m; in ropes shorter than this a broken yarn will not carry any significant load, longer than this, and the force in the yarn can build up to the same value as in its neighbours.

This idea leads to a series-parallel model, in which the long rope is assumed to be made up of a series of elements, each one characteristic length long. The failure strengths of the individual elements are deemed to be independent, and clearly the rope will fail when the weakest element fails. Thus, if data are available for the distribution of individual element strengths, predictions can be made about the strength of long ropes, using a weakest link model.

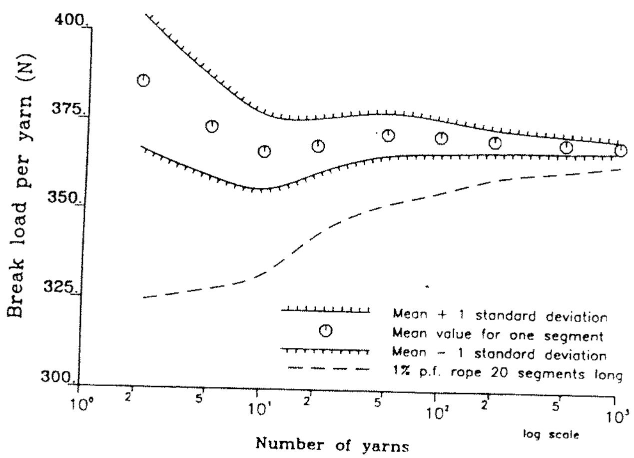

Figure 3. Effect of rope size and length on strength. (Kevlar 49 yarns, no slack)

Results for long ropes of different sizes are also shown on Fig. 3. The lower line is the assumed strength of a rope 20 characteristic lengths long, to give a 1% probability of failure. Unlike the predictions of simple bundle theory, the strength of the rope increases as the size goes up, primarily because the variability of element strengths reduces and masks the reduction in strength with size. If the rope is increased in length, the reduction in

strength continues, but at a declining rate; very long ropes are not significantly weaker than ropes of a few characteristic lengths.

This model relies on a number of assumptions; a real rope does not break itself into a neat chain of elements and there is no guarantee that the second and subsequent yarn failures will occur at the same place as the first element. However, some credence is given to the model by tests carried out by Chambers (1986), who reported tests on 2 m long ropes where yarn failures were distributed throughout the rope, and tests on 12 m long ropes where there was a tendency of the failure to localise in one location. The assumption that the element strengths are independent is also open to question; after all, the different elements are made from the same yarns by definition. To resolve the problem, data would be needed on the variation along an individual yarn, and between different yarns; this would require a large scale testing programme. Forensic analysis of test specimens which showed any yarns that had broken twice in a single rope would also be of great interest, especially if it gave data for the length between failures.

Effect of Material

The effect of the material variation is shown in the Table 1, which shows the break load data from the tests on the yarns and the predicted strengths of 1000 yarn ropes made from those yarns. 500 analyses have been carried out for the ropes from each yarn. As expected, the rope strengths are considerably below those of the constituent yarns. The sixth column shows the number of standard deviations of the yarn strength by which the mean rope strength lies below the mean yarn strength. With the exception of the Vectran ropes, the mean rope strength lies between 2.1 and 2.5 standard deviations below the mean yarn strength; for normally distributed yarn strengths, this implies that the rope breaks when between 0.6% and 1.8% of the yarns have broken.

The importance of this result is clear. A small number of weak elements will significantly reduce the strength of bundles made from those elements, which has implications for quality control during manufacture. It also implies that large numbers of tests are needed to ensure that the lower tail of strength distributions is adequately described - even the 50 tests used here is probably not sufficient for this purpose.

Yarn	Yarn data (experimental)		Rope data (1000 generated yarns, no slack, 500 analyses)			
	Mean μ_y	St. Dev. σ_y	Mean μ_r	St. Dev. σ_r	$(\mu_y-\mu_r)/\sigma_y$	σ_r/σ_y
Polyester	85.4	2.17	80.0	.23	2.48	0.108
Vectran	288.9	24.64	251.7	1.27	1.51	0.051
Kevlar 29	332.7	13.39	303.2	1.13	2.21	0.084
Technora	375.5	18.70	336.4	1.45	2.09	0.078
Kevlar 49	414.7	21.64	368.4	1.53	2.14	0.071

Table 1. Break loads of yarns and generated ropes (in Newtons, expressed as force/yarn).

Behaviour of FRP Pultrusions

The data presented here have been obtained on multi-filament yarns, as used in parallel-lay ropes. The same yarns are used in pultrusions and braided systems, and it is worth considering whether any of the ideas presented here apply to these systems.

The idea of a characteristic length is again important. Yarns with a small degree of twist can be regarded as a bundle of filaments. Predictions show that the characteristic length of these systems is of the order of a few millimetres. The tests carried out here, on 500 mm long yarns, can thus be regarded as tests on "very long" yarns where the bundle effects between filaments and yarns have now been taken into account.

But when FRP rods are tested, the situation is different. When one filament breaks, it can pick up load very quickly due to the presence of resin. This leads to an apparently much higher strength on short specimens than is observed for yarns with no resin. But it will also lead to a very high variability between the strength of sequential short specimens, which should lead to a much higher variation along the length of the rods. In addition, bundles of FRP rods will have to be used to make tendons of reasonable size, so the strength of these tendons will be affected by the sharp drop in strength associated with a relatively small number of elements. There is also unlikely to be much significant interaction between separate rods and there will almost certainly be a large variation in slack. It is thus quite likely that there will be a reduction in bundle strength for multiple FRP rods of the same order as that observed here for yarns. However, the present authors do not have sufficient test data to be able to model this behaviour properly.

CONCLUSIONS

The variation in strength of the smaller elements (filaments and yarns) has been shown to have a very significant effect on bundles made from them. Quality control during manufacture, both of the filaments, the yarns and of ropes or rods, is very important for ensuring high conversion efficiencies from filament to bundle.

It has been shown that the difference between small ropes and large ropes is not large, and that the reduction in variability of large ropes means that the reduction in strength with length is less than for small ropes.

The variability in strength of FRP rods needs to be carefully established, as does the ability to transform the properties of laboratory scale elements to practical sizes.

References

AMANIAMPONG G. (1992). Variability and viscoelasticity of parallel-lay ropes. *PhD Thesis*, University of Cambridge.

BURGOYNE C.J. and MILLS P.D. (1996). Variability of yarns and implications for the strength of bundles. Cambridge University Engineering Dept. *Tech. Rep.* CUED/D-Struct/TR.161.

BURGOYNE C.J. and FLORY J.F. (1990). Length effects due to yarn variability in parallel-lay ropes, *Procs. MTS-90*, Washington DC.

CHAMBERS J.J. (1986). Parallel-lay aramid ropes for use in prestressing tendons in prestressed concrete. *PhD Thesis*, University of London.

DANIELS H.E. (1945). On the strength of classical fibres and fibre bundles. *J. Mech. Phys. Solids* **7**, 60-70