Plastic Analysis of Anchorage Zones for Prestressed Concrete

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To be presented at FIP 93 Symposium
Kyoto, Japan, October 1993

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March 1993

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SUMMARY

In this paper, upper- and lower-bound methods of analysis are developed for the problem of anchorage zones in prestressed concrete. These methods are compared with and validated against experimental test results. It is shown that such methods accurately model both the failure loads and behaviour of anchorage zones for prestressed concrete.

1. INTRODUCTION

When a large prestressing force is applied to the end of a beam over a small anchorage area, tensile bursting stresses are developed behind the anchor plate. The problem of analysing the bursting stresses in concrete blocks under concentrated loading has been investigated in the past by several researchers [1,2,3,4,5,6]. Such work has led to commonly accepted empirical design methods for the detailing of steel reinforcement in end blocks.

The present experimental series was carried out in order to study the effect of steel quantity and positioning on the load-carrying capacity of strip-loaded end blocks. The results from this test series would then be used to verify upper- and lower-bound theories for the analysis of end blocks.

2. EXPERIMENTAL PROCEDURE

Sixty specimens of overall rectangular dimensions $750mm \times 250mm \times 125mm$ were strip-tested to failure through rigid steel plates of varying length. The plates were placed centrally and the ratio of loaded length, $2a_1$, to total length, $2a$ ($250mm$), was varied between 0.1 and 0.7.

The concrete used was of average cube strength $f_{cu} = 60N/mm^2$ and of average split-tensile strength $f_t = 4.0N/mm^2$. Steel stirrups were included, of varying number and bar size, spread over $2a$, $3a$ and $4a$ (250, 375 and $500mm$) lengths from the strip-loaded end of the specimens. The yield strength of the steel bars varied between 417 and $438N/mm^2$.

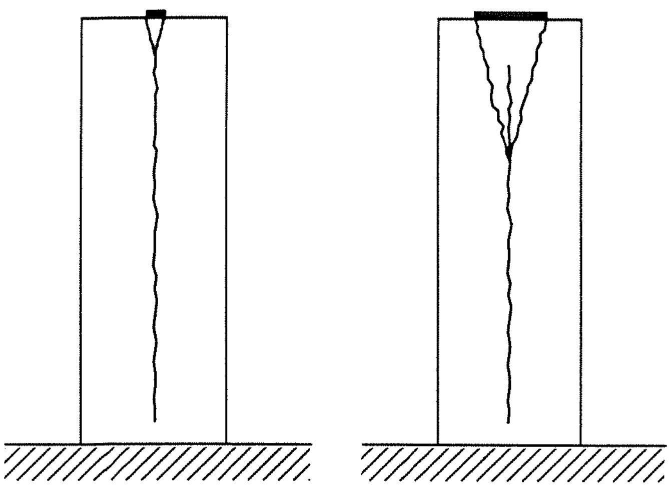
3. EXPERIMENTAL RESULTS

It was found from the tests that the provision of steel reinforcement increased the cracking load of the specimens substantially. Visible cracking was initiated along the central axis when the measured average steel strain was about $600\mu\varepsilon$ in most tests.

After this central cracking had started, downward propagation of the crack occurred. The accompanying loss in stiffness of the block created a redistributed stress condition under the loading plate, which led to ultimate wedging punch-through of the plate, as shown in figure 1. In most cases (except where $a_1/a = 0.1$) the ultimate wedging load was higher than the initial cracking load. When a_1/a was equal to 0.1, failure of the blocks was purely by splitting. The blocks were able to withstand no additional load after initial central cracking had occurred.

Figure 1.

Planar wedge failure of the test specimens.



As the quantity of steel increased in the specimens, the rate of increase in strength of the specimens dropped. This was due to third direction stresses causing out-of-plane wedging failure. This behaviour was partially prevented by providing steel cross-links between the stirrup legs to confine the concrete and maintain plane strain conditions.

Spreading steel over depths of $3a$ and $4a$ was found to be beneficial, both in terms of cracking and ultimate load capacities of such blocks. Such dilution of steel content (compared with the case where steel is spread over $2a$ as suggested by European methods) could reduce congestion in end blocks, a notorious problem in the detailing of prestressed concrete structures [7].

4. UPPER-BOUND SOLUTIONS

For modelling purposes, the present problem may be considered as a case of shearing along yield lines at failure, which has been studied extensively by several researchers [8,9,10,11,12], who have obtained good correlation between such plasticity methods and test results. The following assumptions are made regarding the plasticity solutions formulated in this paper.

1. Rigid perfectly-plastic collapse occurs. Elastic deformations are negligible.
2. The Modified Coulomb failure criterion with non-zero tension cut-off is assumed for the concrete in areas of shear-tension or shear-compression failure. Figure 2 shows details of this failure surface. The internal angle of friction, ϕ, is assumed to be a constant 37° [9,11] for all combinations of stress. In separation failures, a limiting tensile strength of concrete is assumed, where applicable.
3. The steel bars carry axial tension forces only. Any dowel effects are ignored.

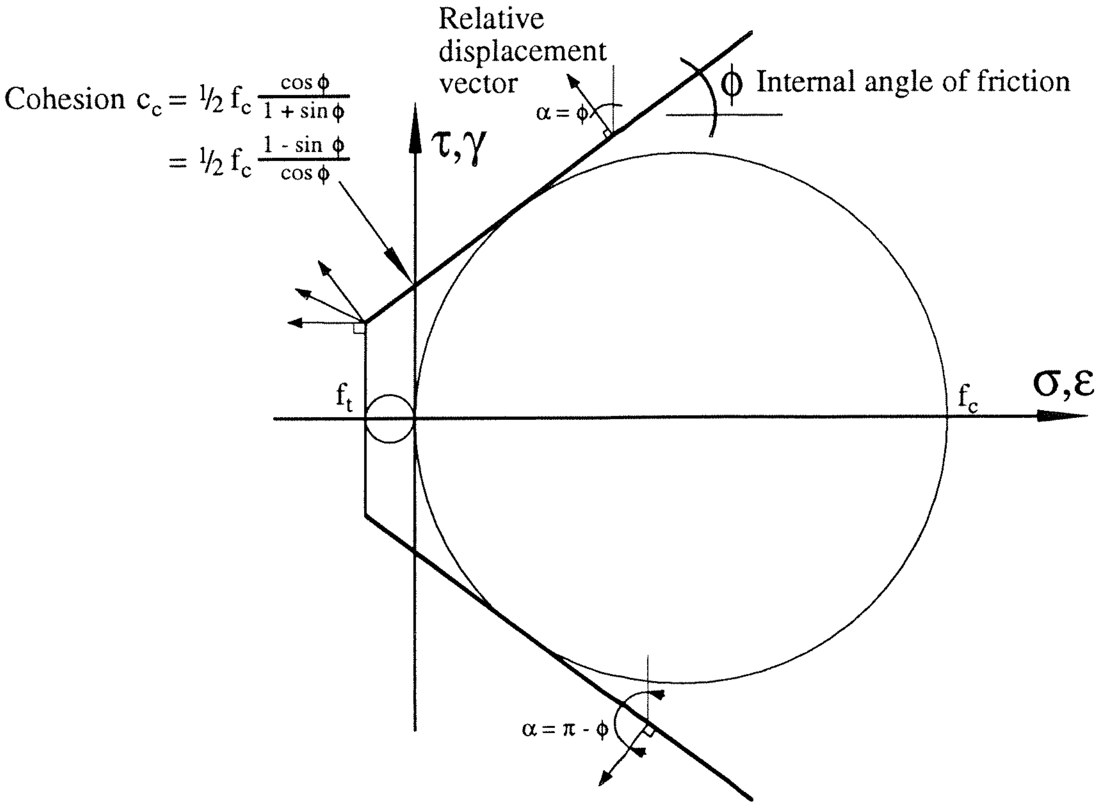


Figure 2. The Modified Coulomb failure criterion, with non-zero tension cut-off.

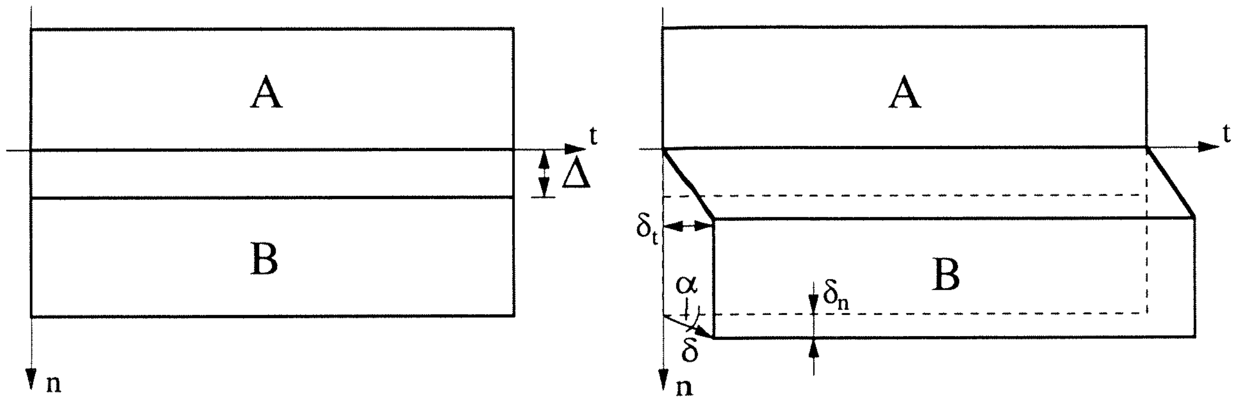


Figure 3. Discontinuity zone between two rigid blocks under relative displacement.

Furthermore, due to ductility limitations in concrete structures, a reduced strength of concrete must be assumed. The 'effectiveness factor', ν, is introduced to create an effective concrete strength, f_c, which is equal to νf_{cu}, f_{cu} being the measured cube compressive strength of the concrete. Several values of ν are assumed for a variety of problems, but for the case of shearing in concrete, $\nu = 0.67$ has been suggested [11] and is assumed throughout this paper.

Consider two rigid blocks, A and B, separated by a distance Δ, as shown in figure 3. The relative normal and tangential displacements are δ_n and δ_t respectively. The resultant relative displacement is δ, inclined at angle α to the yield line; δ and α are permitted to vary along the full length of the yield line, subject to compatibility requirements. It may easily be shown that for a plane stress problem, ignoring the tensile strength of concrete, the energy dissipated is

$$\dot{D} = \frac{1}{2} f_c \delta (1 - \sin\alpha) \text{ per unit length} \tag{1}$$

in the full range $0 \leq \alpha \leq 2\pi$.

For a plane strain problem, once again ignoring the tensile strength of concrete, equation (6) still holds, but a restriction is placed on α such that $\phi \leq \alpha \leq \pi - \phi$, where ϕ is the internal angle of friction for concrete. (See figure 2).

4.1 Uniform translation of outer blocks at failure

The two rigid outer blocks (II) in figure 4 are assumed to translate laterally on a frictionless base during planar wedging failure. Using the velocity relations from figure 4 and applying equation (1), together with a term for the inclusion of steel reinforcement across the central crack, it may be shown (11) that, if T is the total steel force in the yielded bars, then

$$P = \frac{2a_1 w}{\sin\beta\cos(\beta+\phi)}\left(\frac{1-\sin\phi}{2}\right) f_c + 2T\tan(\beta+\phi) \tag{2}$$

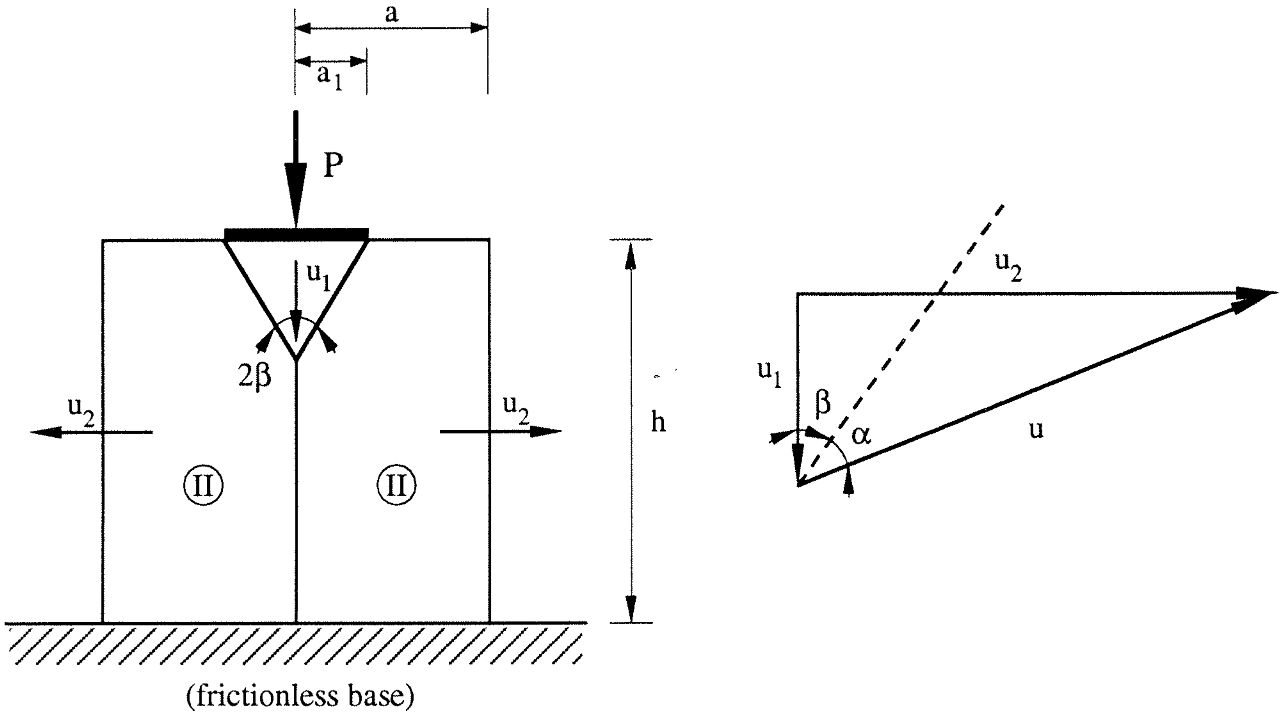


Figure 4. Failure mechanism and velocity relations under assumed uniform translation of the outer blocks (II).

Setting $\frac{\partial P}{\partial \beta} = 0$ for minimum P, it is found that

$$f_c\left(\frac{1-\sin\phi}{2}\right)\cos(2\beta+\phi) - \frac{2T\sin^2\beta}{2a_1 w} = 0 \tag{3}$$

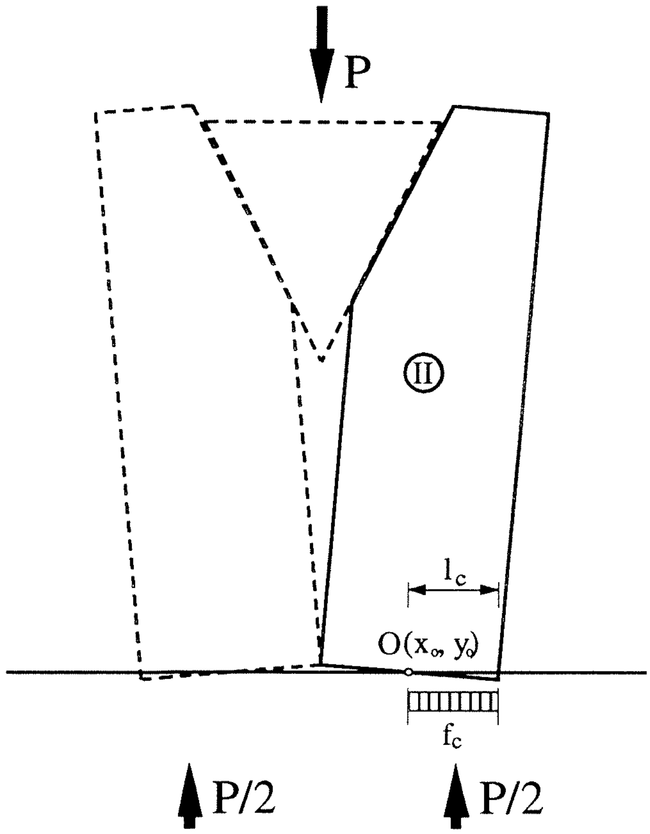
4.2 Rotation of outer blocks during failure

During failure of the test specimens, it was noticed that the outer blocks rotated outwards about the base, allowing the wedge to penetrate the prism. It was decided to include this rotation of the outer blocks in upper-bound analyses of the problem. Figure 5 shows the general failure mechanism assumed. In the figure, a straight wedge yield line has been shown, but a curved yield line could also be considered (see later in this section).

Rotation of the outer blocks is assumed to occur about point O (x_o, y_o) on the base. Point O is the innermost limit of compressive stress on the base, allowing rapid calculation of the resultant reaction force on the base, assuming pure crushing of the basal concrete over this region, with $f_c = 0.67 f_{cu}$.

Figure 5.

Rotational model of the experiments, showing position of point O, and equilibrium of forces.



The wedge is assumed to move down by unit displacement. A rotation (η) about point O is chosen and the relative displacement vectors δ_i are found at discrete points p_i along the length of the yield line. The angle between the slope of the yield line and the relative displacement vector is α_i.

Work is considered to be done by concrete shearing along the sloped yield lines, by steel stretching and by crushing of concrete on the base. Two specific formulations were considered, using different assumptions.

1. The wedge yield lines are straight, and the bottom point of the yield line has $\alpha = \phi = 37°$, with α increasing in value up the wedge, in accordance with $\phi \leq \alpha \leq \pi - \phi$, and

2. The wedge yield lines are curved such that $\alpha_i = \alpha_{si} = \phi = 37°$ at every point along the wedge planes.

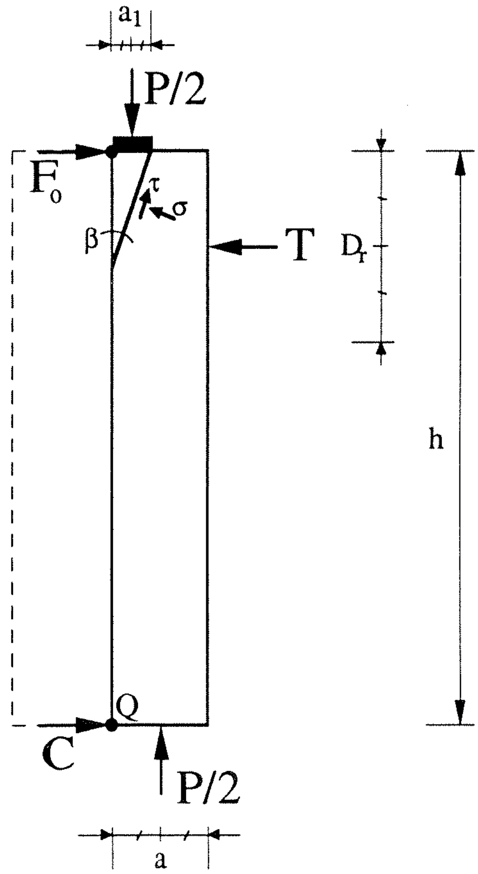
Results from all the above upper-bound analyses were found to model the test results fairly accurately [13,15]. In general, it was shown that the simple translational model produces sufficiently accurate predictions and that the added sophistication of the model which included rotations of the outer blocks produced little additional benefit.

5. LOWER-BOUND ANALYSIS

Consider the failure model of a prism shown in figure 6, where the concrete is isolated from the steel. It is assumed in this model that central cracking has occurred along the entire length of the specimen (an equivalent assumption was made for the upper-bound analysis) and that the position of the basal force $P/2$ is central on the the half-prism. It is further assumed that an equilibrium force F_o acts on the half-wedge at the level of the loading plate [16] and that horizontal equilibrium is maintained by a frictional force C on the base of the specimen and the applied steel force T. All bars are assumed to have yielded.

Figure 6.

Lower-bound equilibrium model of failure.



For overall equilibrium of the half-prism shown, the following conditions must be satisfied.

$$T = C + F_o \quad \text{(horiz. equil.)} \tag{4}$$

$$\text{and, } \frac{P}{2}\cdot\frac{a}{2} + T(h - \frac{D_r}{2}) = F_o h + \frac{P}{2}\cdot\frac{a_1}{2} \quad \text{(mom. equil. about Q)} \tag{5}$$

where all symbols are as shown in figure 6.

From consideration of the resultant stresses acting on the failure wedge, the following expression for σ may be found.

$$\sigma = \frac{a_1 w h c_c + \frac{1}{2} a a_1 w c_c \cot\beta - \frac{1}{2} a_1^2 w c_c \cot\beta + T\left(h - \frac{D_r}{2}\right)}{a_1 w h \cot\beta - a_1 w h \tan\phi - \frac{1}{2} a a_1 w \tan\phi \cot\beta + \frac{1}{2} a_1^2 w \tan\phi \cot\beta - \frac{1}{2} a a_1 w + \frac{1}{2} a_1^2 w} \quad (6)$$

Further consideration of equilibrium yields the expressions for τ, F_o and C. This solution procedure is carried out numerically under varying values of β until a minimum failure load for each specimen is reached.

The equilibrium method is clearly simple and was found to be fairly accurate when compared with the test results. It is therefore useful for general wedging analysis of prisms, and it suffers from similar limitations to those encountered in the upper-bound methods developed earlier. It is a purely planar analysis technique. However, out-of-plane effects ought to be checked in practice to ensure that unexpected failures do not occur.

6. FINITE ELEMENT BASED SOLUTIONS

In order to be able to incorporate all types of failure encountered in the tests [13,14] into an analysis, it was decided to use finite elements. In addition, it was decided to treat the failure of the prisms as occurring in two distinct phases (initial cracking followed by wedging failure), each represented by a linear elastic analysis.

In the initial analysis, load is applied to the wholly intact prism. All steel crossing the potential crack plane is assumed to be strained to an average $600\mu\varepsilon$ (see earlier) and the resulting steel force is applied to the prism. Stresses are found along this plane and compared with a simple tension cut-off criterion. In this way, an idea of the cracking resistance of the concrete prism is found.

Thereafter, this central crack is assumed to propagate the full length of the specimen. Continuity between the elements along this plane is broken. All steel is assumed to have yielded at this stage and stresses are again found in the region of wedging (directly beneath the loading plate). Because we are interested in an ultimate failure load of the prism, these stresses are averaged along many potential wedging planes and are compared with the Modified Coulomb failure criterion for concrete. In this way, the actual failure plane and load may be found for each particular specimen. This method was generalised to three dimensions and out-of-plane wedging was studied in the same way. Results from these analyses were compared with the test results and favourable correlation was obtained [13,17]. The method was shown to be able to predict not only the failure loads of the specimens, but also the behaviour at failure, which is vitally important to the design engineer.

7. CONCLUSIONS

It has been shown, from plasticity analyses conducted on concrete prisms, that there exists much potential for the use of these methods in the analysis and design of anchorage zones for prestressed concrete. They have been shown to accurately predict behaviour and strength of concrete prisms, based upon a single failure criterion for concrete.

ACKNOWLEDGEMENTS

The experimental work was carried out in the Cambridge University Engineering Department. The technical assistance of Mr C J Mason and Mr V L Piper is gratefully acknowledged.

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