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# PROBABILISTIC STRENGTH ANALYSIS OF PARALLEL-LAY ROPES

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## ABSTRACT

This paper describes a model for determining the short term strength of parallel-lay ropes and bundles of parallel elements by means of probability theory. Ropes of a characteristic length are modelled from a knowledge of the statistical properties of the constituent elements. The model allows the analysis of parallel-lay ropes with a non-linear stress-strain relationship and permits the study of the variability effects due to the scatter in the elements' cross-sectional areas, failure strains, stiffnesses and random slack. The scatter in element cross-sectional area increases the bundle (rope) strength slightly, albeit by an insignificant amount. The variability in the stiffnesses however has a profound effect on the bundle strength. The results from the model are contrasted with classical bundle theory. Using experimental yarn data from parallel-lay ropes made of *Kevlar-49* aramid and high tenacity *Terylene* polyester yarns, the strength behaviour of the ropes can be predicted accurately.

## 1. INTRODUCTION

Progress in the polymer industry over the last decades has produced materials with desirable mechanical properties. These materials have found applications in diverse areas such as the aeronautical and civil engineering industries.

In the civil engineering industry, parallel-lay ropes made from high strength synthetic fibres are being preferred for use in many offshore and bridge structures. They are used to replace high tensile steel tendons and strands in many application areas due to their desirable properties, particularly where low weight and corrosion resistance are of prime concern[1]. Parallel-lay ropes have also been identified for use in cable stayed and suspension bridges, prestressed concrete structures, prestressed brickwork, cable supported roofs, deep water platforms and retaining walls[1,2,3].

Of all the various types of rope construction, parallel-lay ropes give the best conversion efficiency from the properties of the elements to those of ropes[4]. The individual elements (yarns) are arranged parallel to the rope axis throughout the entire rope length, unlike in twisted-lay (stranded) or braided rope construction where helical or serpentine yarn paths are introduced, and the full stiffness of the constituent yarns are therefore mobilized. *Parafil* ropes are one such type of rope made from aramid or polyester yarns[4].

In this parallel construction, the ability of broken elements to shed loads mainly to their near neighbours is greatly reduced, something which is achieved in structured ropes by friction between adjacent elements. There is little interaction between the individual yarns, and the ropes can be seen as an aggregate of separate elements. However, the tensile strengths of the ropes are not accurately predicted from the elements by simple averaging rules. This is especially the case when the elements are elastic, with high stiffness and a high variability in strength, as is typical in high performance ropes. A random sample of apparently identical elements displays a high variability in strength and this has a profound effect on the strength of the resulting bundle or rope[5]. The tensile strength efficiency (the ratio of the bundle strength to the mean strength of the elements) of fibre bundles is a monotonicaly decreasing function of the element coefficient of variation (ratio of the element standard deviation to the mean strength)[6]. The variability in the strength of the bundle elements is attributed to flaws which are randomly distributed within the bulk of the elements[7]. Thus, the failure event can be modelled as a stochastic process and the probabilistic approach is adopted here.

Daniels[8] used a stochastic process to study bundles of threads made by parallel construction; this is now generally referred to as the classical bundle theory. Daniels developed an asymptotic result for such a rope, but the model was simple in that it only applied to ropes with linear elastic elements and the only variable parameter was the strength of the constituent elements. An application of Daniels' classical model to *Parafil Type G* ropes by Chambers[9] proved inadequate in explaining the rope behaviour.

Phoenix and co-workers[10,11,12] dealt with more realistic models of parallel-lay ropes where the strain of the elements, rather than the stress, was used as the the main statistical parameter. They introduced random slack into the model but kept the other parameters constant and developed an asymptotic result. Their asymptotic result gives a specific value for the rope strength, but it has been observed by Guimarães[13,14] that there is a size effect associated with *Parafil Type G* ropes.

In the models described above, the area and the stiffness of the elements were considered constant, but experimental evidence shows that these do vary[9,15]. The models also assume linear elastic materials with the same stress-strain curve and cross-sectional area. However, tensile tests of polymeric fibres show that fibres exhibit a scatter in their cross-sectional areas[15]. A fibre continuum model of bundles of linear elastic elements with linearly varying stiffnesses also shows that the variation in the element stiffness has a profound effect on the strength of bundles of parallel elements[16]. Thus, the variability in the stiffness of the bundle elements cannot be ignored.

In this paper a probabilistic model which predicts the tensile strength of parallel-lay ropes and bundles of parallel elements is presented. The cross-sectional area, stiffness and breaking strain of the yarns or elements of the rope, as well as slack, are assumed as random variables. The restriction on the element stress-strain behaviour is also relaxed so that a polynomial can be used to represent the stress-strain behaviour. This allows the model to be applied to ropes with non-linear stress-strain behaviour. To extend the model to large ropes, the recently developed asymptotic result by Daniels[17] is used and the convergence is compared with Monte-Carlo simulations. The strength behaviour of large ropes can therefore be accurately predicted. The present model allows the effect of variability in the stiffness and area of the yarns on the rope strength to be studied.

## 2. STATISTICAL STRENGTH OF THE ROPES

### 2.1. Assumptions and formulation of the model

Consider a bundle of $n$ parallel elements (members) of the same type with varying cross-sectional areas and stiffnesses. Assume that the bundle is clamped in such a way that the elements have different slacks, and the load is applied to the bundle by means of extension. Let the force-strain relationship of each element be given by an $m^{th}$ order polynomial. Thus the relationship between the force, $\varphi$, and the strain, $\lambda$, of an element is given by

$$\varphi\left(a,\underline{\beta}^{\prime},\lambda\right)=\sum_{k=1}^{m} a\beta_k\lambda^k \qquad (1)$$

where $a$ is the element cross-sectional area and the vector $\underline{\beta}^{\prime}=(\beta_1,\ldots,\beta_m)$ represents the coefficients of the polynomial.

Let $Z_i(\varepsilon)$ be the force in the $i^{th}$ element at bundle strain, $\varepsilon$, and $\Theta_i$ the initial slack strain of the elements, then

$$Z_i(\varepsilon)=\begin{cases} 0 & 0\le\varepsilon<\Theta_i \\ a_i\sum_{k=1}^{m}\beta_{ik}(\varepsilon-\Theta_i)^k & \Theta_i\le\varepsilon<\Theta_i+\varsigma_i \\ 0 & \varepsilon\ge\Theta_i+\varsigma_i \end{cases} \qquad (2)$$

where $\varsigma_i$ is the failure strain of the element $i$.

Assume that each of the entities: the fibre failure strains, $\varsigma_1,\ldots,\varsigma_n$; slacks strains, $\Theta_1,\ldots,\Theta_n$; cross-sectional areas, $a_1,\ldots,a_n$; and the coefficients of the polynomials, $\underline{\beta}_1^{\prime},\ldots,\underline{\beta}_n^{\prime}$; are independent identically distributed random variables with the following density (df) and cumulative density functions (cdf):

| parameter | df | cdf |
|---|---|---|
| $\varsigma$ | $h(\varsigma)$ | $H(\varsigma)$ |
| $\Theta$ | $g(\theta)$ | $G(\theta)$ |
| $a$ | $j(a)$ | $J(a)$ |
| $\beta^{\prime}$ | $b(\underline{\beta}^{\prime})$ | $B(\underline{\beta}^{\prime})$. |

The bundle load at bundle strain, $\varepsilon$, is given by

$$L_n(\varepsilon)=\sum_{i=1}^{n} Z_i(\varepsilon) \qquad (3)$$

and the bundle strength is $S_n^*=L_n^*/n\mu_a$ where $L_n^*$ is the maximum value achieved by $L_n(\varepsilon)$, i.e., $\sup\{L_n(\varepsilon);\ \ \varepsilon\ge 0\}$ and $\mu_a$ is the mean area of fibre. If $F\left(\varsigma,\theta,a,\underline{\beta}^{\prime}\right)$ is the joint distribution function of the parameters $\varsigma_i,\theta_i,a_i,\underline{\beta}_i^{\prime}$ then the mean load at bundle strain $\varepsilon$ is given by

$$\mu(\varepsilon)=E[Z_i(\varepsilon)]=\int\varphi\left[a,\beta^{\prime},(\varepsilon-\theta)\right].dF\left(\varsigma,\theta,a,\underline{\beta}^{\prime}\right) \qquad (4)$$

and the covariance function is expressed as

$$\Lambda(\varepsilon_1,\varepsilon_2)=E\left[Z_i(\varepsilon_1)Z_i(\varepsilon_2)\right]-\mu(\varepsilon_1)\mu(\varepsilon_2) \qquad (5)$$

where $E[x]$ is the expected value of the variable $x$.

For simplification purposes the following assumptions are made:

(i) the distributions of the parameters $\varsigma$, $\theta$, a, and $\underline{\beta}'$ are independent;

(ii) the coefficients of the polynomial, $\underline{\beta}'$, of each element are dependent and are related by a multinormal distribution.

The first assumption is made on the grounds that the occurrences of the element characteristics are independent. Although this assumption is made for simplification purposes, it is expected to be realistic. It is also unlikely that a joint distribution function for all the parameters involved could be obtained. The second assumption is made because it is unsatisfactory to consider the coefficients of the polynomial of each element as independent and because the multinormal distribution is the most widely used multivariate distribution.

With the above assumptions, Equations (4) and (5) can be written as[19]

$$\mu(\varepsilon) = \mu_a \int_0^{\min\{\varepsilon,\theta_{\max}\}} \left\{ \int \cdots \int \sum_{i=1}^{m} \beta_i (\varepsilon - \theta)^i \, dB(\underline{\beta}') \right\} \times [1 - H(\varepsilon - \theta)] dG(\theta) \quad (6)$$

and

$$\Lambda(\varepsilon_1, \varepsilon_2) = \mu_{2a} \int_0^{\min\{\varepsilon_1,\theta_{\max}\}} \left\{ \int \cdots \int \left[ \sum_{i=1}^{m} \beta_i (\varepsilon_1 - \theta)^i \right] \times \left[ \sum_{i=1}^{m} \beta_i (\varepsilon_2 - \theta)^i \right] dB(\underline{\beta}') \right\} [1 - H(\varepsilon_2 - \theta)] dG(\theta) - \mu(\varepsilon_1)\mu(\varepsilon_2) \quad (7)$$

where $\mu_{2a}$ is $\int a^2 dJ(a)$ and $\varepsilon_2 \geq \varepsilon_1$.

Since the bundle strength is dictated by the maximum value achieved by $L_n(\varepsilon)$, one would expect it to be given as $S_n^* = \sup\{\mu(\varepsilon)/\mu_a, \varepsilon \geq 0\}$, but this is not the case. The process is complex and the mathematical complexities and the subsequent numerical analysis required to evaluate the strength of the bundles with small and moderate number of elements, even for the "simple" cases of the classical bundle and the fibre slack model, are enormous and therefore impractical[8,10]. Monte-Carlo simulations are therefore used to study the behaviour of small and moderately sized bundles, but there is the drawback of having to specify the probabilistic distributions and the numerical values for the associated parameters of the element characteristics. For bundles with a large number of elements, the failure load is deduced from a general Gaussian process superimposed on a parabolic curve near the maximum of the process. The results of Daniels[17] on a Gaussian process whose mean path has a maximum is therefore employed.

## 2.2. Monte-Carlo study of the bundle strength

Complicated stochastic processes can be simulated by a numerical method generally known as the Monte-Carlo method, just as complex structural problems are amenable to finite difference and finite element methods. The outcome of a process of interest can be observed by randomly assigning a value to an underlying variable or vector. Such a practice is referred to as a Monte-Carlo experiment. A Monte-Carlo procedure is then composed of, say, $n$ such independent experiments and by virtue of the law of large numbers, observations made from a sufficiently large number of the experiments will be a good assessment of the statistical characteristics of the process.

For the computer model adopted (Figure 1), there is the need to specify the correlation coefficients associated with the coefficients of the force-strain polynomials of the elements. The mean and the variances of these coefficients are also required. By specifying the numerical values for the required parameters of the chosen statistical distributions representing the failure strains, cross-sectional areas and slack of the bundle elements, the outcomes, $\varsigma_1,\ldots,\varsigma_n$, $a_1,\ldots,a_n$, $\Theta_1,\ldots,\Theta_n$, and $\underline{\beta}'_1,\ldots,\underline{\beta}'_n$ are generated. Various methods for generating random numbers have been discussed by Rubinstein[18]. Equations (2) and (3) are then applied to evaluate $L_n(\varepsilon)$, the bundle load at strain, $\varepsilon$, and the bundle strength, $S_n^* = \sup\{L_n(\varepsilon)\}/n\mu_a$ is obtained. The process is repeated to generate a large number of $S_n^*$ and the data is used with standard statistical inferences about the distribution of the bundle strength.

## 2.3 Asymptotic strength of the bundle

The covariance function, Equation (7), can be converted to a statistical process given by[19]

$$\Lambda(\varepsilon_1, \varepsilon_2) = A(\varepsilon_1, \varepsilon_0)[1 - (\varepsilon_2 - \varepsilon_0)/d] \quad (8)$$

By superimposing the process in Equation (8) on a curve $-\eta^{1/2}(\varepsilon)$ which has its minimum at $\bar{\varepsilon}$ where $\eta(\bar{\varepsilon})=0$ and the first derivative $\eta'(\bar{\varepsilon})=0$, and then by considering the linear expansion of $\Lambda(\varepsilon_1,\varepsilon_2)$ about $\bar{\varepsilon}$ within the

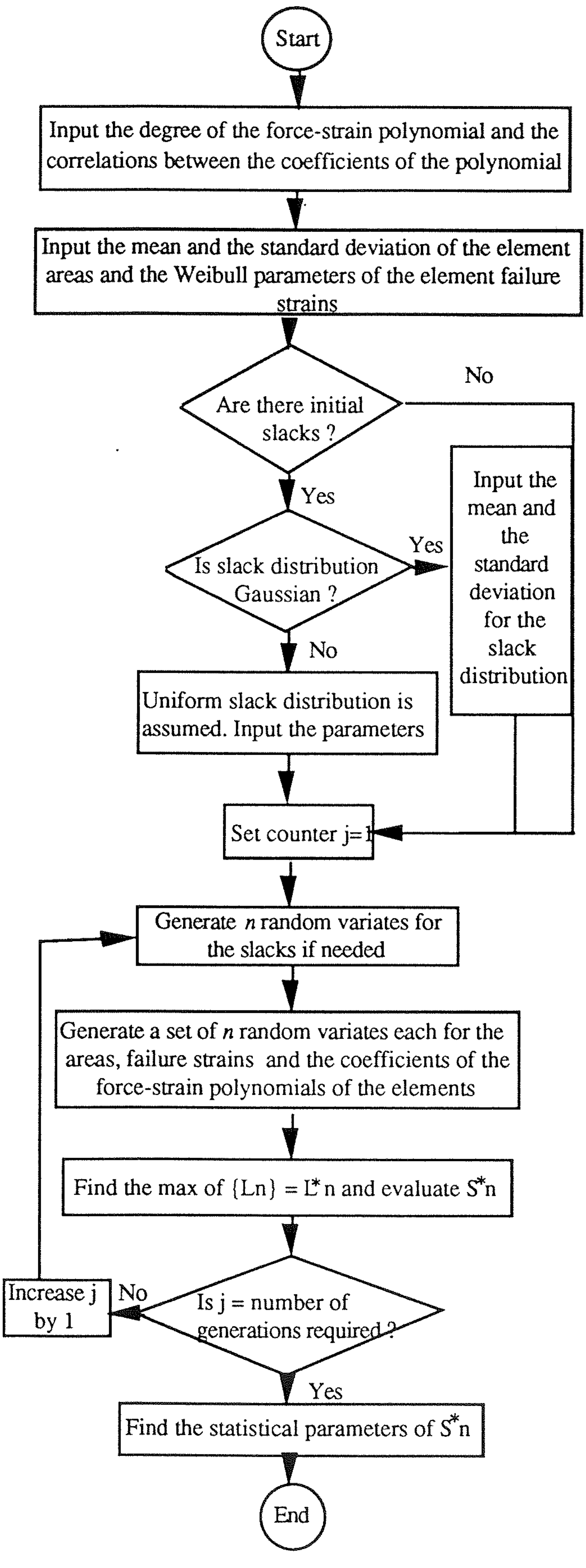



**Figure 1 A flow chart showing the algorithm for the Monte-Carlo simulations of the bundle strength**

range $\varepsilon-\bar{\varepsilon} = O(n^{-2/3})$, the bundle strength can be deduced[17].

If $\eta(\varepsilon)=\mu(\varepsilon)-\mu(\varepsilon^*)$ is chosen, the conditions for the curve $-\eta^{1/2}(\varepsilon)$ are fulfilled. Here $\varepsilon^*$ is the value at which $\mu(\varepsilon)$ achieves its maximum value. The strength of the bundle is then asymptotically normally distributed with expected value $E[S_n^*]$ and variance $Var[S_n^*]$ given as

$$\left.\begin{aligned} E[S_n^*] &= \mu(\bar{\varepsilon}) + \lambda n^{-2/3} A^{2/3}\left[-\mu''(\bar{\varepsilon})\right]^{-1/3} \\ Var[S_n^*] &= \Lambda(\bar{\varepsilon},\bar{\varepsilon}) / n\mu_a^2 \end{aligned}\right\} \qquad (9)$$

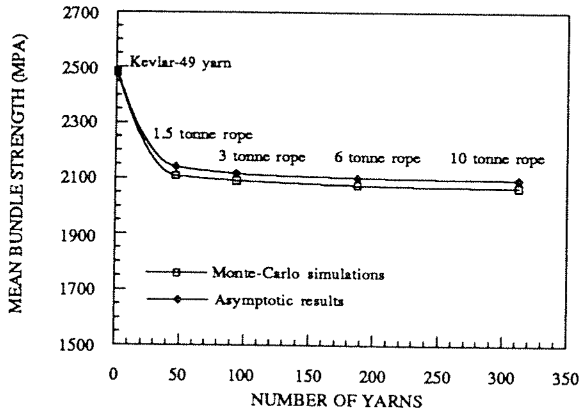
where $\bar{\varepsilon} = \varepsilon^*$ and $\lambda$=0.99615...is a constant[17].

## 3. RESULTS

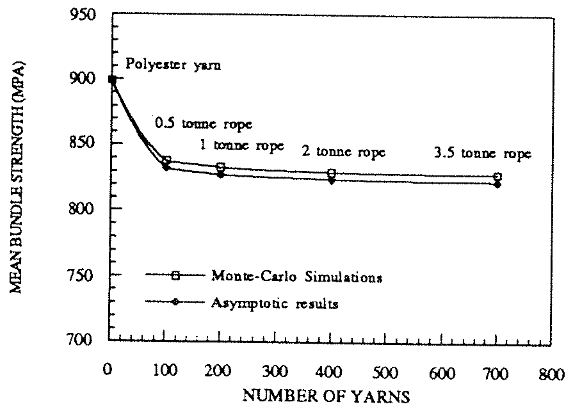
The results which follow are based on yarn data from tests carried out on *Kevlar-49* aramid yarns and high tenacity *Terylene* polyester yarns[19]. Whereas the stress-strain curve of the aramid yarns was best fitted by a third order polynomial that of the polyester yarns was modelled with a fifth order polynomial without the third and the fourth coefficients[19]. The bundles refer to parallel-lay ropes known as *Parafil Type G* and *A* ropes which have *Kevlar-49* aramid and polyester yarns as the core materials. These ropes are manufactured by Linear Composites Ltd. in the United Kingdom. A yarn is considered as the basic element of the ropes.

Figure 2 shows the comparison of the bundle strength estimates from the Monte-Carlo method and the asymptotic results for *Parafil Type G and A* ropes. The failure strains of the yarns obey the Weibull distribution[19] and Table 1 shows the summary of the results and the parameters used for the analysis. The sizes of the ropes are extrapolated from the number of yarns counted from 6 tonne *Type G* and 5 tonne *Type A* ropes. There is an initial sharp decrease in the strength of the bundle but this slows down as the bundle size increases. The asymptotic strengths obtained are higher than those of the Monte-Carlo method for all the sizes of the *Type G* ropes considered, but the reverse is seen for the *Type A* ropes. The differences in behaviour could be attributed to the different stress-strain polynomials used. Nevertheless the results are remarkably accurate with maximum relative errors of 2% and 1% for *Parafil Type G* and *A* ropes respectively. The asymptotic results can be used even for a bundle with as few as 50 elements as seen from the 1.5 tonne *Type G* rope.

Tensile tests on 6 tonne *Parafil Type G* and 5 tonne *Type A* ropes[9,19] give mean strengths of 2103 MPa and 821.6 MPa respectively. The corresponding values from the model (asymptotic results) are 2023 MPa and 820.5 Mpa. This result confirms the ability of the model to predict the bundle strength from the constituent elements.




(a) Parafil Type G ropes




(b) Parafil Type A ropes

**Figure 2 Comparison of Monte-Carlo simulations and asymptotic results**

Figure 3 shows the relationship between the bundle strength and the rope size as well as the results from the classical bundle. The failure stresses of the ropes decrease with increasing rope size and become asymptotic to the failure stress of the classical bundle.

| * Type G rope | | | | | ** Type A rope | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Bundle theory | | Monte-Carlo | | | Bundle theory | | Monte-Carlo | |
| NBL + | Mean UTS (MPa) | SD. (MPa) | Mean UTS (MPa) | SD. (MPa) | NBL + | Mean UTS (MPa) | SD (MPa) | Mean UTS (MPa) | SD (MPa) |
| 1.5 | 2140.8 | 67.19 | 2109.1 | 63.43 | 0.5 | 832.5 | 29.58 | 838.0 | 16.06 |
| 3.0 | 2117.6 | 47.51 | 2091.7 | 45.90 | 1.0 | 827.4 | 20.92 | 833.0 | 11.41 |
| 6.0 | 2103.0 | 33.59 | 2076.1 | 32.93 | 2.0 | 824.1 | 14.79 | 829.5 | 7.88 |
| 10.0 | 2095.8 | 26.03 | 2068.1 | 23.94 | 3.5 | 822.4 | 10.79 | 828.3 | 6.27 |

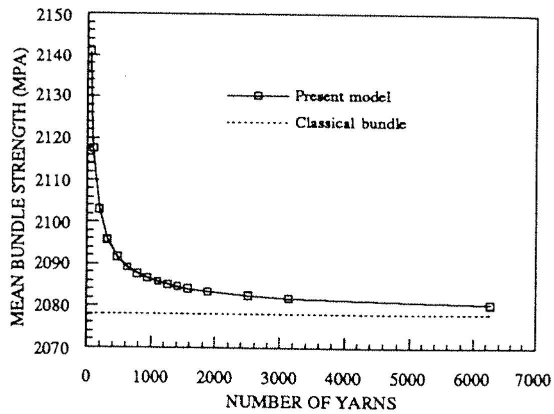
The following Weibull parameters for yarn failure strains were used:

* shape parameter=18.72
scale parameter= 1.78%

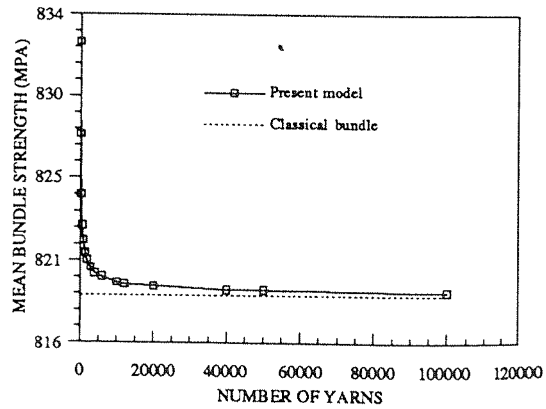
** shape parameter=15.30
scale parameter=11.29%
values correspond to rope length of 259 mm; SD=standard deviation

\+ NBL= Nominal breaking load in tonnes
UTS= Ultimate tensile stress

**Table 1 Summary of the results from Monte-Carlo simulations and the asymptotic results**



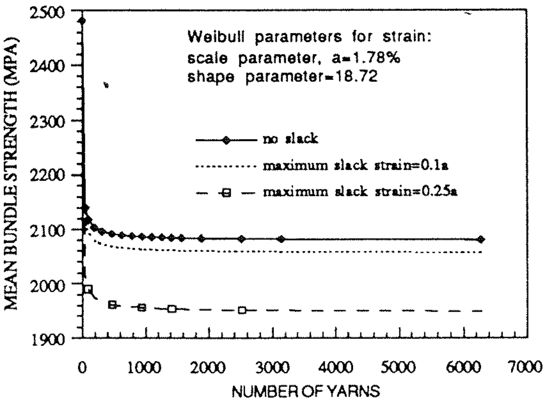

(a) Parafil Type G ropes



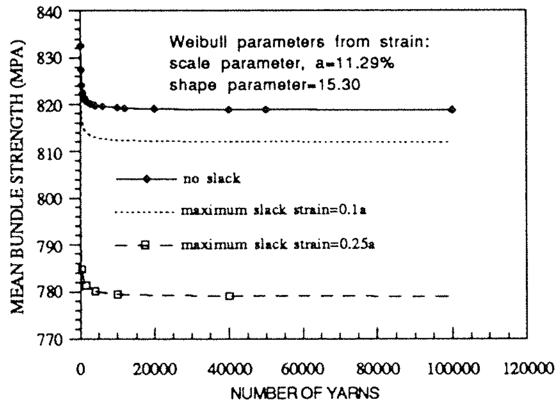

(b) Parafil Type A ropes

**Figure 3 Size effects on parallel-lay ropes**

In Figure 4 the effect of random slack on the bundle strength is shown. A uniform slack distribution was used for the analysis. Although other distributions can also be used, the trend of the results will be essentially the same. The variability in the element slack has a profound reducing effect on the strength of the bundle (rope). For instance, the introduction of a uniform slack distribution with a maximum slack of a quarter of the strain Weibull scale parameter reduces the bundle strength by about 6%. The introduction of a constant slack, however, has no effect on the bundle strength. This was also observed by Phoenix[12] and is to be expected.



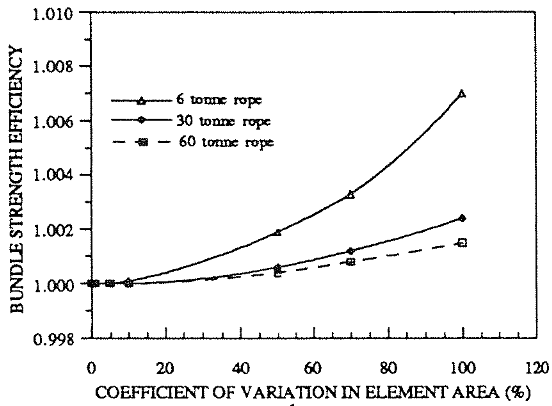

(a) Parafil Type G ropes



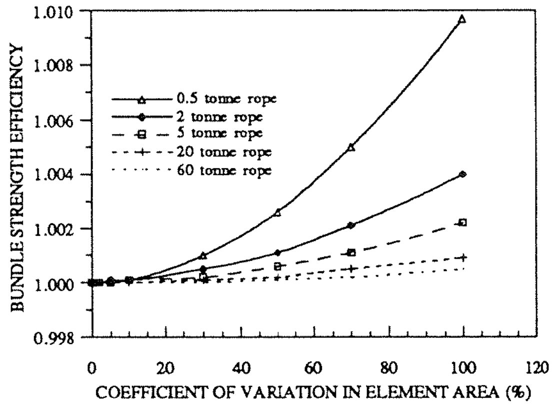

(b) Parafil Type A ropes

**Figure 4 Effect of slack on parallel-lay ropes**

The effect of the variability or scatter in the element cross-sectional area on the strength of parallel-lay ropes is shown in Figure 5. At low variability (coefficient of variation <10%) there is an insignificant effect on the bundle strength, but there is a mild increase in bundle strength as the scatter in the cross-sectional areas of the elements increases. The resulting increase in the bundle (rope) strength is less for large ropes. In fact, an appreciable effect is observed only for small bundles with very high coefficients of variation.




(a) Parafil Type G ropes




(b) Parafil Type A ropes

**Figure 5 Effect of the variation in area on the strength of parallel-lay ropes**

Different effects of the variability in elements' stiffnesses are observed for the *Type G* and *A* ropes; whereas the strength of *Type G* ropes increases with increasing scatter in the element stiffness, the reverse is observed for *Type A* ropes. This probably has something to do with the order of the stress-strain polynomial used. The effect is not negligible and therefore any calculation based on the assumption that elements have constant stiffness may grossly over or under-estimate the bundle strength.

## 4. CONCLUSIONS

A model has been presented for the analysis of the short term (tensile) strength of parallel-lay ropes and bundles of parallel elements in general. The model allows the analysis of parallel-lay ropes with non-linear stress-strain relationships and permits the study of the variability effects as a result of the scatter in the elements' cross-sectional areas, failure strains, stiffnesses and random slacks. The following conclusions can be drawn from the work described in this paper:

1. The present bundle theory or model predicts reasonably well the strength of parallel-lay ropes.

2. The variabilities in the elements' stiffnesses, cross-sectional areas and random slacks affect the bundle strength. The introduction of variable random slack reduces the strength of the rope. The scatter in the element cross-sectional area increases the bundle strength slightly, however this increase is only significant at very high variabilities and therefore the effect can be considered to be practically insignificant. The scatter in the stiffness (expressed through the coefficients of stress-strain polynomial) has different effects on the ropes; whereas *Parafil Type G* ropes gain strength, *Type A* ropes lose strength with an increasing scatter in element stiffness.

Acknowledgements

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