PLASTICITY APPLICATIONS IN REINFORCED CONCRETE AND PRESTRESSED CONCRETE STRUCTURES

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1 INTRODUCTION

Plasticity theory has been used for many years in the design and assessment of concrete and prestressed concrete structures. While the theory gives, in many cases, optimum reinforcement based on ultimate load conditions, there are cases where higher amounts of reinforcement are required to satisfy the serviceability criterion. In this paper case studies are presented firstly to demonstrate the benefits of using simple and consistent approach to compute the reinforcement where ultimate condition governs. Secondly, case studies are presented wherein the serviceability criterion such as crack width governs. The examples presented include a range of reinforced and or prestressed concrete structures consisting of tower crane foundations, wind turbine foundations, building floors and offshore structures. The case studies have been chosen based on the non linear loading regimes that exists between ULS conditions and SLS conditions. In addition the degree of orthotropy of reinforcement ratios varies between 1.0 and 2.0 in the structures selected for case study.

2 BACKGROUND

2.1 History

In the design of structures, engineers carry out elastic analysis to find the internal forces within the structure. The reinforcement is then sized using either cracked section and uncracked section analysis or by using a non-linear theory. There has been a great deal of interest in the analysis and design of reinforced concrete structures since Nielsen and Wood [1,2,3] published a yield criterion for an isotropically membrane and later extended it to the orthotropically reinforced case by using a lower bound plasticity theory [2]. He developed procedure for finding the envelope of a given panel of known reinforcement and concrete strength, and the method of designing the reinforcement for a given set of forces. In this theory Nielsen assumed that the concrete alone would resist compression and no compression reinforcement would be provided.

Morley [4,6] included the concept of strain-rate fields which gives equal positive strain rates in each reinforcement position. He derived equations for the determination of skewed reinforcement to resist a particular in-plane force triad. For the case of the orthogonal reinforcement, these equations reduce to Nielsen's equations. According to this study, negative values of reinforcement could be obtained, implying that either no reinforcement or compression reinforcement is required along these directions. The importance of this study is the graphical approach which is used to determine the set of equations to be used depending on the loading. Nielsen [2] developed a similar set of equations by solving the force equilibrium equations. His method could deal with both moments and in-plane forces by using sandwich model. Clark [5] extended Nielsen Approach to cover the possibility that the compression reinforcement might be required. He developed three equilibrium equations for the cases when both directions had tension or compression reinforcement, The three equations are solved for nine different cases which include the possibilities of no reinforcement, tension reinforcement, and compression reinforcement. The crack direction is presumed according to the given loading. In addition, The boundary graph from the nine cases has to be constructed for each concrete strength.

Later in 1988 Zararis [7] included the effect of shear forces carried by reinforcement. Vecchio and Collins [8] demonstrated the response of cracked concrete member as much different from the uncracked member. They developed modified compression field theory using equilibrium, compatibility and constitutive relations. They developed a method to modify existing linear elastic finite element procedures to non linear analysis of reinforced concrete membrane structures. However this was successful for membrane elements and not enough tests were conducted for bending loads to

demonstrate the applicability to generalised set of all eight sets of forces (three in plane forces, three bending moments and two out of plane shear forces). Polak and Vecchio [9] tests showed that many more experimental and analytical studies have to be carried out in this field to come up with a universally acceptable solution.

A recent paper by Marti [10] presented the modified sandwich model wherein the effect of transverse shear was also considered. The paper concluded from static and kinematic methods that the limit analysis provides a simple rational approach for the ULS design of concrete and prestressed concrete structure.

2.2 Approach Adopted in Current Study

Despite the advancements made in the analysis and design of concrete structures, Clark's method and Nielsen methods give the simplest formulation and they are still acceptable for simple and detail checks. In the current study, multi layering instead of a traditional three layer sandwich model has been used to obtain the reinforcement for a given set of forces. The concrete strength reduction in each layer has been taken into account when principle strain in the perpendicular direction is tensile as per Vecchio and Collins approach. Case studies are presented here where serviceability or ultimate limit state governs the design.

3 CASE STUDUIES

3.1 Tower Crane Base Foundations

Generally the design of the tower crane base foundations is governed by the ultimate limit state condition as they are temporary foundations thast are requiredduring the construction of 'new build', Therefore the design of such foundations can be done by using plasticity theory and optimum reinforcement can be provided. The tower crane foundations have been designed for out of service and in-service conditions. The foundations are supported on piles as shown. The predominant condition for the loading are (refer Fig. 1):

1) jib working at right-angle
2) jib working at 45 degree

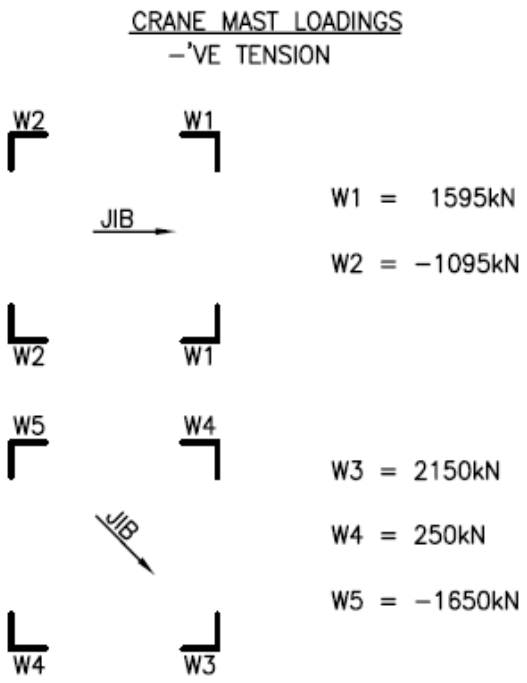


Fig. 1 Tower Crane Loading on the Foundations

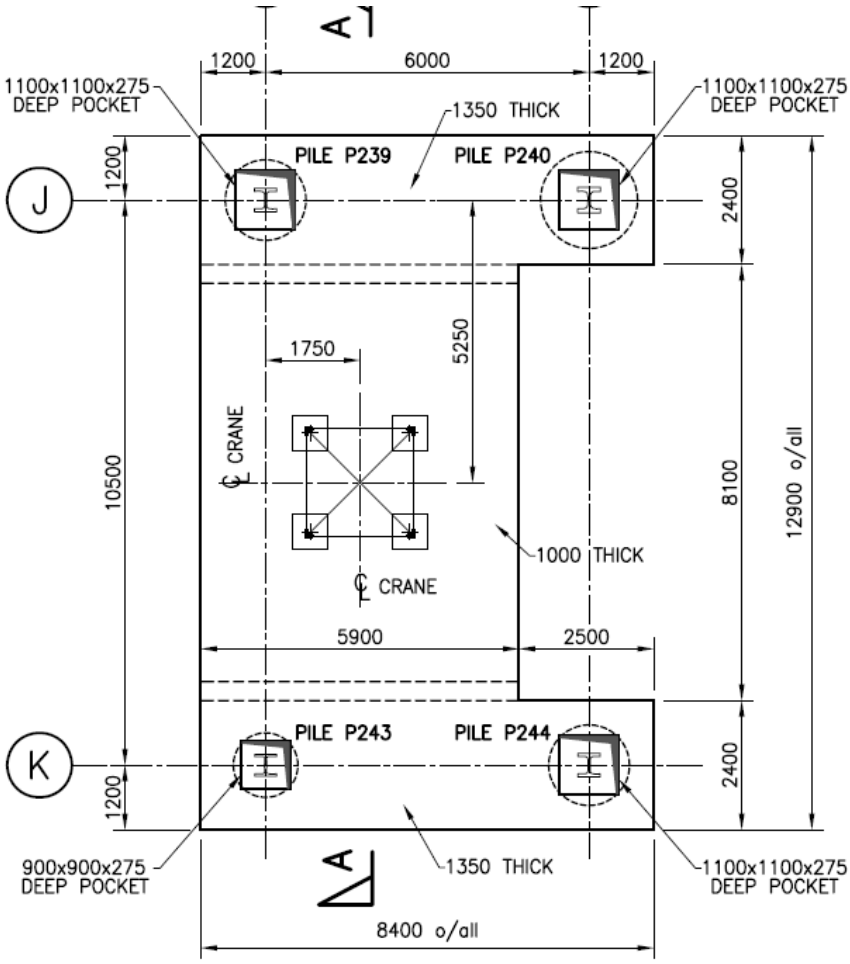


Fig. 2 Tower Crane Foundation

The size of the foundations considered in Figure 2 is 12 m x 8.4m. The foundation is supported on piles as shown. Elastic finite element analysis was used for analyzing the foundations. Eight noded shell elements were used to compute the stresses in the elements. The stress resultants were used to compute the reinforcement using the plastic theory formulation. Figure 3 shows the displacement patterns from load case 1 mentioned earlier.

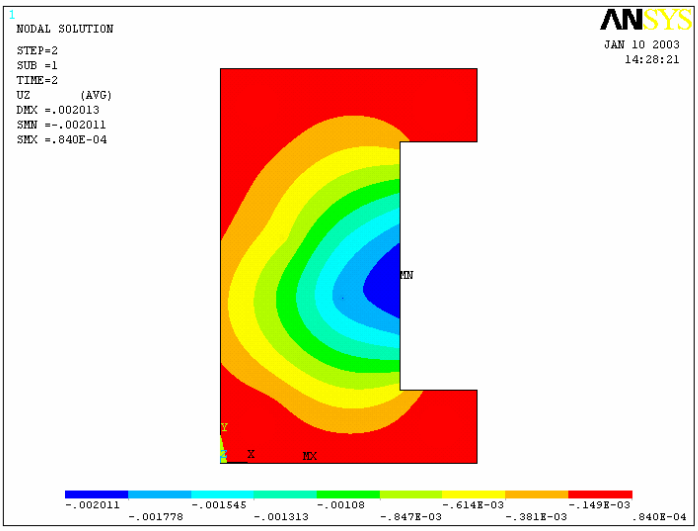


Fig. 3 Displacements in metres

Photo 1 below shows the foundation and reinforcement arrangement.

Photo 1 Tower Crane Foundation

Figure 4 shows results from another foundation but with a different geometry of the base. This foundation is also supported on piles. These two cases demonstrate the benefit of using the sandwich model approach to obtain optimum reinforcement. Orthogonal reinforcement is generally used in the foundations. Both these foundations were analysed using moment triad together with out of plane shear forces that have been obtained from linear elastic analysis. The results from FE analysis were then fed into a post processing software to predict the envelopes for the design groups. A design groups is defined as set of finite elements that have same geometry (thickness) and reinforcement.

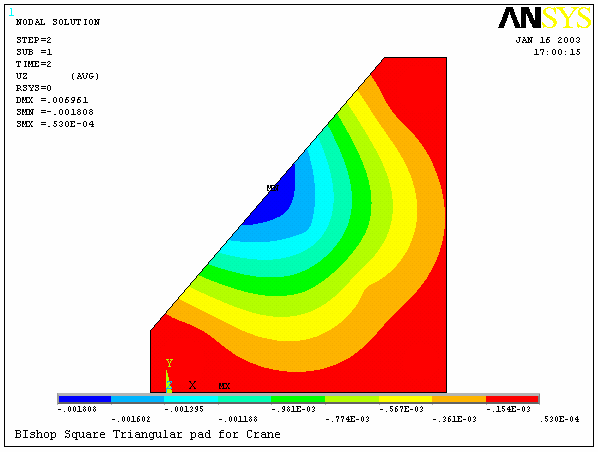


Fig. 4 Displacements for pile supported crane base foundations.

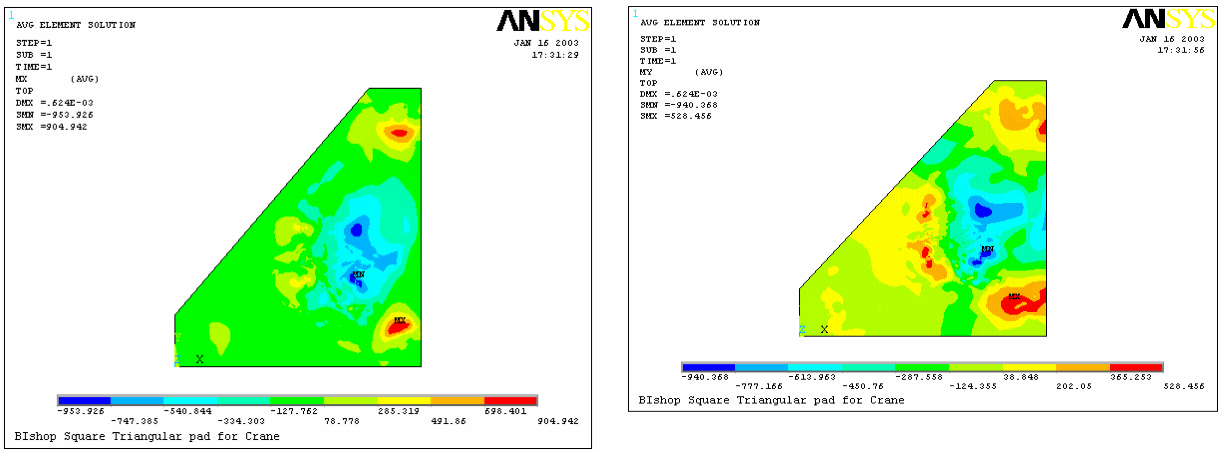


Fig. 5 Principal Bending Moments from FE Analysis

The results from the layered model are compared to that calculated using traditional approach of sizing the reinforcement using individual moments in each direction. Orthotropy ratio of the reinforcement is defined as ratio of steel in each direction for top and bottom layers of the sandwich.

Table 1 Normalised Summary of reinforcement

Foundation Type	Layer 1 - 0^0	Layer 2 - 90^0	Layer 3 -0^0	Layer 4 – 90^0
Rectangular (orthotropy ratio=1.0) Sandwich Model Hand Calcs	1.0 1.05	1.0 1.2	1.0 1.45	1.0 1.35
Triangular (orthotropy ratio = 1.5) Sandwich Model Hand Calcs	1.0 1.10	1.0 1.1	1.0 1.65	1.0 1.75

The differences have been observed due to the geometry and degree of orthotropy of the reinforcement in both types of foundations. If serviceability calculations are performed with the rebar obtained from the sandwich model, then it would be quite possible that the crackwidth exceed the code limit however there is no code requirement to be met for this case.

3.2 Wind Turbine Foundations

Concrete gravity base foundations are generally used to support land based wind turbines. The predominant loading for this is, generally governed by out of service condition although for larger machines (greater than 3. MW) sometimes in-service condition may govern. Generally bottom reinforcement is provided in orthogonal directions for practical reasons although radial and tangential reinforcement is optimum. Top reinforcement in the foundations consists of radial and tangential reinforcement as shown in Photo 2. The design condition for the base is the pressure from the foundation acting on a narrow area of the foundation.

Photo 2 Typical gravity base foundation of wind turbines (courtesy RES)

The foundation shown in Figure 6 is 11.7m diameter and has varying thickness from 0.6m at the free end to 1.3m towards the base. The pressure on the base has a magnitude of 1000 kN/m^2 and is applied on a contact width of 1.2m and 6.96m long. The slab is assumed to have an average thickness of 0.95m except the central pier has a thickness of 2.3m. The pier is 4.6m in diameter and 2.3 m deep. Eight noded Shell 93 elements in ANSYS have been used in the model for the analysis.

The finite element model of the base is shown in Figure 6. The stress resultants from the FE model are used in layered model to size the reinforcement. Reinforcement computed using hand calculations by one way action of the slab ignoring the 3D effects are also presented for comparison.

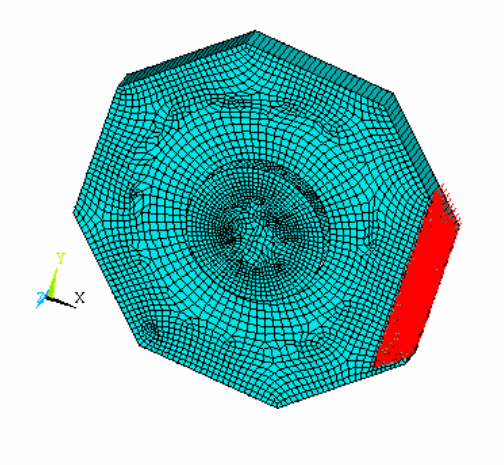


Fig. 6 FE Model of the foundation of wind turbine structure

The stress resultants including displacements from elastic analysis are shown in the following figure 7.

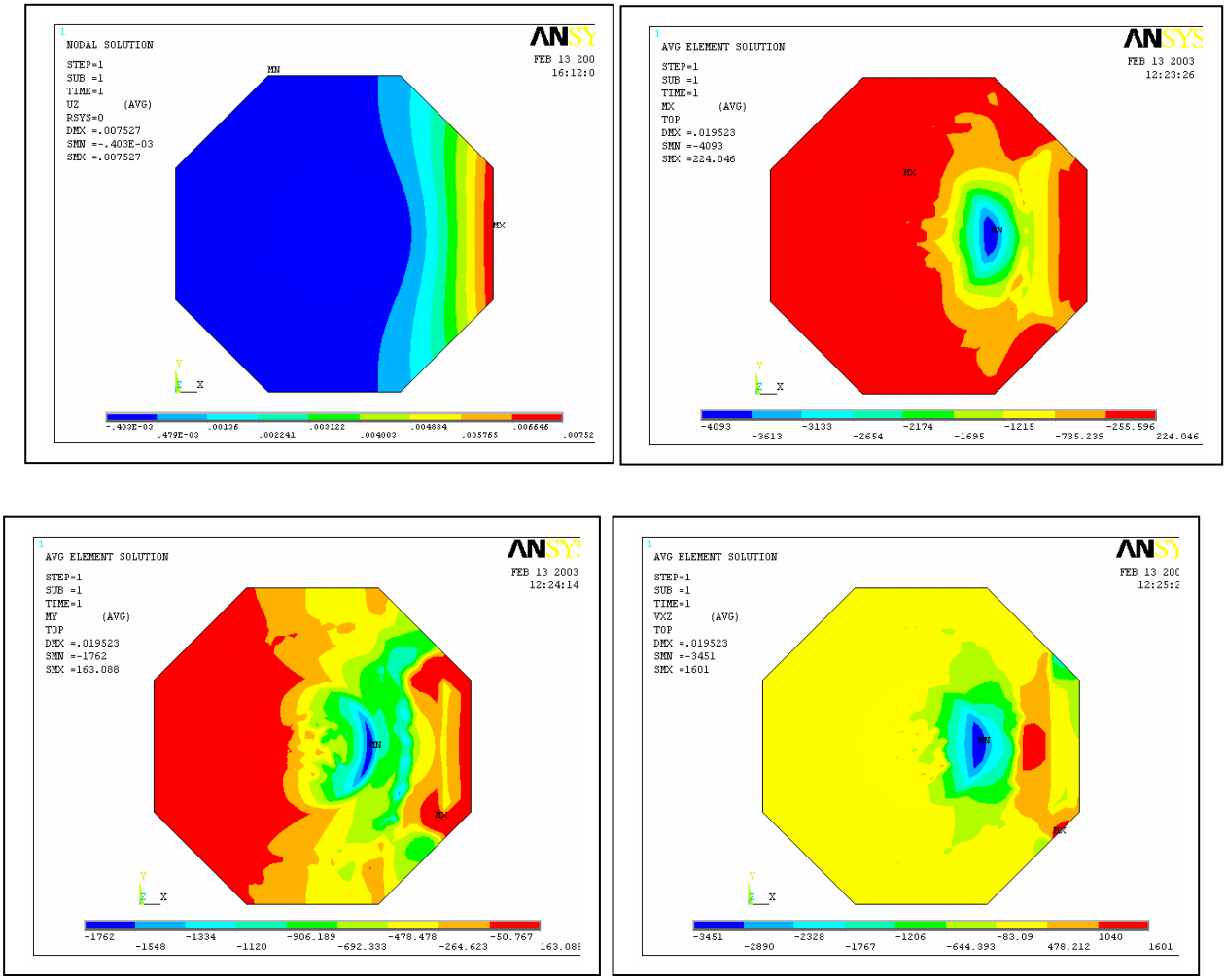


Fig. 7 Displacements (m) and stress resultants (kN, m)

The optimum reinforcement computed from the layered method is shown in the table below.

Table 2 Normalised Summary of reinforcement

Foundation Type	Bottom layer - 0^0	Bottom Layer– 90^0
Rebar from plastic theory Hand calculations	1.0 1.1 to 1.23	1.0 -

It can be seen from the above table that positive rate of strain in both directions gives minimum steel in the foundations as demonstrated earlier in the literature by Morley [4].

3.3 Post Tensioned Building Floors

Post tensioned building slabs have been quite popular in the last 10 years due to the advantages offered by such floors. The design of a post tensioned exhibition floor is selected here to demonstrate that the serviceability design gives much higher reinforcement despite the partial prestressing used in the structure. The structure is designed to take a live load of 15 to 20 kN/m^2. Photo 3 shows the arrangement of tendons in a flat duct. The structures consists of slabs spanning onto banded beams as shown.

Photo 3 Exhibition Floor

The analysis results are presented for worst case scenario of loading on the floor. The results showed that generally partial prestressed structures where live load is much higher than dead load do require higher amounts of steel to satisfy serviceability crack widths.

Table 3 Normalised Summary of Reinforcement

Partial prestressed floor slab	Span	Support
Rebar from plastic theory- ULS Rebar from serviceability criterion	1.0 1.08 to 1.40	1.0 1.06 to 1.2

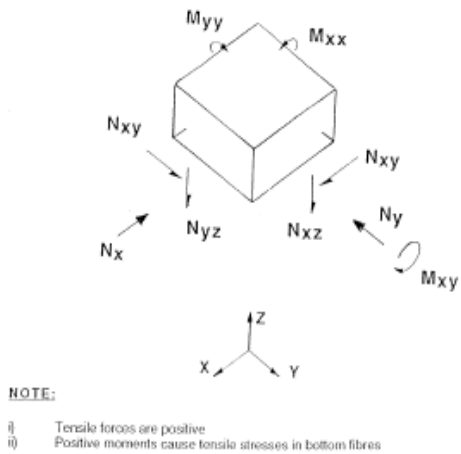
3.4 Offshore Gravity Based Structures

Figure 8 below shows the finite element model of a concrete gravity based structure analysed in ANSYS. This prestressed concrete structure at a conceptual stage and front end design engineering phase has been analysed for a range of loads consisting of installation loads, wave, and live loads from the deck. Vaious ULS and SLS combinations have been analysed that resulted a in a total of 132 load combinations - not uncommon in this type of structures.

Figure 8 Finite element model of a gravity based concrete structure

The finite elements from the model (developed with shells) was grouped into so called "Design Groups" which represented zones of the structure with similar thicknesses and prestress/reinforcement details. Results for the design groups DG1 to DG 42 were then filtered by the in-house program MEP (Minimax Enveloping Program) in order to obtain for each Design Group the load combinations that contained 8 key parameters (inplane forces – 3, bending moment - 3 and out of plane shear -2) resulting in a total of 26 criterion as:

Maximum and minimum of; (2*13 =26)
Nx in-plane direct force
Ny in-plane direct force
Nxy in-plane shear force
Mxx bending moment
Myy bending moment
Mxy bending moment
Vxz transverse shear
Vyz transverse shear
Ni principal force
Mi principal moment
Vi principal shear
S1 principal skin stress at top face
S2 principal skin stress at bottom face



Thus for each design group there are a maximum of 26 critical parameters to check both for SLS and ULS conditions. The operation is repeated for all design groups in SLS and ULS conditions. These forces are then read into a CONCRETE software that has layered approach for estimating the reinforcement.

Table 4 below shows the ratio of ULS to SLS forces generated for each component of stress resultant in the structure. It can be seen that the loading regimes are completely different for both cases and a high degree of nonlinearity exists. Therefore a single loading regime is not applicable for sizing the reinforcement from plastic theory. Not surpisingly the reinforcement computed is vary complex patterns to accommodate the forces generated in the structure from various loading conditions.

Table 4 Ratio of ULS to SLS forces in various design groups of structure

ULS/SLS	Nx	Ny	Nxy	Mx	My	Mxy	Vxz	Vyz
DG1	3.81	1.81	3.06	71.20	0.24	1.09	15.10	2.21
DG2	0.90	0.25	2.10	2.16	9.67	0.68	0.03	0.00
DG3	1.47	5.45	1.91	1.50	1.06	3.47	1.00	1.02
DG4	1.98	0.84	1.37	0.56	0.12	0.89	1.07	1.38
DG5	1.85	0.80	1.38	0.43	0.63	0.87	1.12	1.51
DG6	1.96	0.79	0.52	1.56	1.75	0.65	1.81	1.42
DG7	0.04	0.75	2.36	0.40	1.08	47.33	1.26	1.12
DG8	2.27	0.96	3.55	1.23	1.22	1.53	1.20	2.15
DG9	1.87	0.76	0.60	1.26	1.27	0.87	1.15	2.12
DG10	0.22	0.75	0.45	0.29	1.86	0.88	0.94	0.90
DG11	1.24	0.06	1.87	2.29	0.84	1.0	3.53	0.29
DG12	1.24	0.90	1.60	1.53	0.06	1.32	1.36	1.56
DG13	1.27	0.50	2.02	2.63	1.43	2.69	0.04	1.25
DG14	1.26	0.96	1.50	1.26	1.26	7.00	0.72	0.66
DG15	1.21	1.05	0.96	4.19	49.50	1.83	1.91	1.48
DG16	1.27	1.05	1.12	1.71	1.50	0.50	1.94	1.44
DG17	0.44	1.17	7.96	3.47	0.50	0.23	0.08	0.06
DG18	0.34	0.89	8.73	1.51	2.30	1.24	0.40	1.46
DG19	0.79	0.85	0.39	1.27	12.65	1.01	1.14	0.46
DG20	1.48	0.98	1.55	1.81	0.95	1.09	2.19	1.10
DG21	5.63	24.68	41.17	7.96	18.06	94.55	2.03	271.09
DG22	3.35	12.00	1.38	1.64	1.57	5.00	1.81	3.00
DG23	1.38	1.34	2.75	1.44	1.24	0.64	0.96	1.00
DG24	1.50	1.13	1.18	0.81	1.38	2.14	0.85	1.74
DG25	0.16	0.41	1.87	0.75	0.10	1.00	2.61	4.14
DG26	0.43	0.60	2.16	2.64	0.22	1.19	4.89	39.11
DG27	0.26	0.55	0.00	28.60	0.14	2.61	0.34	0.10
DG28	0.22	0.73	0.59	4.10	3.98	1.11	1.48	49.70
DG29	0.11	0.03	1.02	0.85	1.16	1.20	0.87	1.16
DG30	0.11	0.36	3.85	0.30	0.15	5.05	2.42	1.65
DG31	3.63	0.88	1.91	1.30	1.44	1.63	11.54	1.51
DG32	3.72	0.86	1.72	2.13	1.39	2.38	1.53	1.49
DG33	1.53	1.23	1.72	1.00	1.00	0.91	1.03	1.06
DG34	1.45	1.25	0.52	1.31	0.98	1.46	1.36	0.79
DG35	1.77	0.21	1.13	0.08	0.10	0.06	0.04	0.12
DG36	1.48	1.51	0.98	1.91	1.82	1.66	2.27	1.52
DG37	1.17	1.09	1.02	2.84	1.00	1.85	0.83	1.43
DG38	1.29	0.45	1.08	0.93	0.70	1.65	0.88	1.34
DG39	1.32	1.07	0.91	1.20	1.39	5.75	1.0	0.89
DG40	1.56	0.14	1.07	1.21	1.03	0.58	1.42	0.97
DG41	0.65	0.29	1.91	1.05	1.03	0.01	8.17	1.51
DG42	0.01	0.50	0.72	1.00	0.99	2.00	1.00	0.70

Table 5 below shows the reinforcement computed from layered model of each section of the design group. The ratios are normalized with respect to the optimum steel computed for ULS condition, The results show due to the highly non linear loading regime that exists between SLS and ULS loads, the optimum steel computed from ULS condition need to be increased in order to satisfy the serviceability criterion with respect to crack width, minimum compression zone, limit on concrete stress in working conditions etc.

Table 5 Normaised Reinforcement Ratios in each category of the design group

Design Group category	Steel required from SLS conditions	Optimum Steel from ULS
Outer Skirts	1.0 to 1.2	1.0
Inner Skirts	1.1 to 2.5	1.0
Skirt Stiffeners	1.4 to 1.98	1.0
Base Slab	1.0 to 1.5	1.0
Base Slab Upstands	1.0	1.0
Caisson Walls	1.03 to 1.2	1.0
Caisson stiffeners	1.0 to 2.0	1.0
Tanks	1.05 to 2.0	1.0
Utility Shaft	1.07 to 1.2	1.0
Drill Shaft	1.0 to 1.35	1.0

4 CONCLUSIONS

The following conclusions are deduced from various case studies presented here:

1) Clark/Nielsen/Morley approach give a very good first estimate of the steel required for most practical applications. The reinforcement thus computed can be further optimized if a sophisticated layering technique is used instead of a simple three layer sandwich model.
2) The steel computed by the plastic theory is optimum for most practical cases wherein the serviceability criterion is not required.
3) Where serviceability criterion is required in terms of crack width, max service stress in concrete, and minimum compression depths etc., the reinforcement required will be much higher that that computed using plastic theory. The ratios of steel required from SLS to ULS condition varies with the loading regime and could be in the order of 100 to 200% for some cases. Case studies are presented here to demonstrate such effects.

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