GENERALISED COLLAPSE ANALYSIS OF CONCRETE BRIDGES

Campbell MIDDLETON

Department of Engineering, University of Cambridge

Keywords: yield-line, collapse, failure, concrete, slab, bridges.

1 INTRODUCTION

A new technique for evaluating the *ultimate strength* of reinforced concrete bridges has been developed which enables a wide variety of bridge types and plastic failure mechanisms to be analysed.

Many of the existing difficulties with plastic collapse and yield-line methods, which previously limited their application to simple slab configurations, have been overcome using computer graphics and solid modelling concepts. In contrast to the traditional yield-line approach, the effect of any load combination on complex failure modes can be evaluated rigorously without the need for mathematical expressions relating the failure mode geometry to the bridge structure and the applied loading. In addition, provision for compressive membrane strength enhancement, "realistic" steel arrangements, and the effects of steel corrosion and concrete deterioration can be included.

This generalised collapse analysis method has also been incorporated into a *reliability analysis* format to evaluate the *safety* or notional probability of failure of concrete bridges. As a result the effects of variability of the parameters that govern the behaviour of these structures can be studied although the details of this probabilistic investigation are not included in this paper.

A computer program has been written to implement these techniques and evaluate the factor of safety against ultimate collapse for concrete slab bridges, and also some simple configurations of beam-and-slab bridges, under the influence of any loading and/or deterioration of the material components.

2 THE ASSESSMENT OF SHORT-SPAN CONCRETE BRIDGE DECKS

There are numerous research projects currently underway aimed at improving our knowledge of the structural behaviour of bridges. Many of these have been initiated by the Highways Agency in England. In particular, there has been an extensive programme of research on masonry arch bridges [7], which includes a number of full-scale load tests carried out by the Transport Research Laboratory [8].

However, one type of bridge that has received relatively little attention in the recent literature is the short-span concrete slab. Although these may seem somewhat unexciting structures, and would also appear to be very simple to assess, they have in fact created significant problems in the current bridge assessment programme. As highlighted by audit of bridge assessments undertaken for the Highways Agency [9], large numbers of these slab bridges have been ‘failed’ by assessing engineers due to insufficient *flexural* capacity and are scheduled for re-assessment or strengthening. It is the assessment of this type of structure that is considered in more detail here.

Clearly it is important that engineers carefully evaluate the methods of analysis employed and ensure that the most realistic and relevant ones are used for determining the strength of bridges. To be overly conservative could result in expensive and often unwarranted remedial action such as replacement or strengthening being undertaken. Even placing some form of traffic or weight restriction on a strategic bridge can impose a substantial economic burden on a community.

2.1 Methods of analysis

The fundamental philosophy adopted for assessment, as distinct from design, has been to evaluate only the *ultimate* strength as the fundamental criterion for passing or failing a structure. Serviceability criteria are not usually considered. The argument given is that an existing structure is likely to have already exhibited evidence of any serviceability problems and these should have been dealt with in maintenance programmes. Thus the methods of analysis employed need to be able to predict realistically this ultimate capacity.

2.2 Elastic analysis - the conventional approach to assessing bridge load capacity

Current codes of practice are written with the implicit assumption that the design and assessment of bridges will usually be undertaken using linear elastic analysis techniques. Elastic theory is well established and understood, is supported by many computer software packages, and has been found most satisfactory for the design of bridges. As a *lower-bound* method the engineer can be confident that the analysis method should be conservative and hence safe.

This approach is quite understandable for design where a certain degree of conservativeness costs relatively little. However is this appropriate for assessing the load carrying capacity of *existing* short-span concrete slab bridges? What does *failure* actually mean in an elastic analysis and what are the *consequences* of such a failure in terms of both risk to life and economic terms?

Firstly, the conventional approach to the assessment of short-span concrete bridges must be reviewed. Typically, the engineer might initially perform a simple elastic beam analysis using a representative strip of the bridge deck. If this 'quick' check shows the structure to be inadequate, a more detailed linear elastic analysis allowing for transverse distribution of load would probably be performed using either a grillage or finite-element analysis. These results are then examined to identify individual locations at which the maximum calculated moments or shears exceed the estimated ultimate capacity of the section.

The decision to strengthen or replace a structure is commonly made on the basis of these results. However, many older reinforced concrete bridges in the U.K. were built with little or no top steel and very little transverse steel. Such structures almost inevitably are rated at very low flexural capacities using such an elastic failure criterion when, for example, the live load is positioned to one side of the deck resulting in some hogging or transverse sagging moments.

In reality, concrete structures will crack under heavy loads resulting in a change in the stiffness of the slab. Even when the ultimate moment capacity of a section of the deck is exceeded loads will be redistributed elsewhere in the slab provided sufficient ductility is available and it does not fail prematurely in shear. As a result, a linear elastic analysis will not accurately model the distribution of stresses or the *actual* behaviour in the post-elastic range where non-linear effects dominate. Elastic methods can be very conservative since failure of one element in the structure is typically used to define failure of the structure as a whole. In the cases of *flexural* failure, the consequences are likely to be small and may only affect the serviceability of the structure. If one accepts that serviceability criteria do not govern and collapse is the criterion on which to base the assessment, such conservativeness is not warranted for concrete slab bridges for which ductile flexural failure is the critical mechanism of failure. Once an individual section has reached ultimate or yielded, the failure must develop into a full collapse mechanism before the structure will actually fall down.

Despite the enormous cost implications of adopting such an approach, *elastic* methods are still relied upon as the primary analysis tool for assessing concrete deck slabs. This is despite the fact that there is a wealth of evidence from model experiments and full-scale load tests to show that concrete bridges are often able to carry loads well in excess of the 'theoretical' capacity calculated using elastic techniques [2].

It is thus important to investigate the options available to an engineer if, after performing an elastic analysis, the structure still fails to comply with the required standards.

The only practical alternatives to elastic analysis would involve undertaking a more sophisticated analysis of the ultimate strength of the bridge or else carrying out load tests on the bridge itself as a means of verifying the load capacity. In the research environment, where the best possible predictive methods are sought to model the actual behaviour of bridges, researchers have, almost without exception, used *yield-line theory*, and in more recent years *non-linear finite element methods*, to predict the flexural collapse behaviour of concrete slabs and concrete bridge decks.

The question is do these provide a practical alternative to elastic analysis for assessing ultimate strength and could they be adopted widely in practice?

2.3 Non-Linear Finite Element Analysis (NLFE)

Many researchers have applied non-linear finite element techniques to the analysis of concrete structures. Cope [4] used this approach to examine a skew reinforced concrete bridge. He predicted that there would be high stresses in the obtuse corner of the deck slab. This was subsequently verified during a site investigation that found extensive cracking under the asphalt surfacing in this region. Kotsovos [6] has used NLFE to study reinforced and prestressed concrete beams and, in particular, shear failure mechanisms. Collins [3] has also extensively applied NLFE methods to various reinforced concrete structures including complex offshore oil platforms. Wills [11] has analysed a half-scale prestressed concrete beam-and-slab bridge using TRL's in-house research NLFE program. This predicted a collapse load 25% above that observed in the testing.

Although NLFE methods have developed to a sophisticated state, their applicability is severely limited by their high cost in computing time and the advanced level of expertise required to use them. In addition, the technique is load-history dependent and very sensitive to the choice of material parameters. Another disadvantage often cited is that finite element programs (both linear and non-linear) usually generated a large amount of output data and it is often difficult to verify the results using some form of simple hand calculation.

As a result, it is more suited to in-depth, specialised assessments of major structures or for laboratory research, and is not presently considered to be a practical option for use in assessing large numbers of existing bridges. This situation could well change in the future as computing developments continuously result in decreasing costs and greater speed with NLFE programs, although the sensitivity of results, need for calibration, and specialised expertise required are still likely to limit their application.

2.4 Plastic collapse analysis - the yield line method

The other analytical alternative is to use plastic collapse analysis or yield-line methods for assessing the strength of concrete bridges. Yield-line analysis considers the global collapse of a concrete slab rather than the 'failure' of a single element thus utilising the full, distributed strength capacity of a structure. As a result it is usually significantly less conservative than elastic methods. Up until the development of non-linear finite element methods in the 1960's and 70's, researchers investigating the ultimate strength of concrete slabs almost exclusively adopted this approach as the best available theoretical method for predicting flexural strength. Over the last 80 years many large and small-scale models of slabs and bridges have been tested. Several full-sized highway bridges have also been tested to destruction. These experiments have consistently shown that plastic collapse or yield-line theory is an extremely powerful and accurate tool for predicting the ultimate flexural strength of concrete slab, and many beam-and-slab, structures [2].

Although most engineers are taught yield-line theory during their training it has been, until recently, rarely used for the assessment of concrete bridges. Even though nearly all design and assessment codes permit the use of plastic methods, there exists a somewhat paradoxical situation in which yield-line methods have been used extensively in research but have not, as yet, been widely adopted in engineering practice although this position is quickly changing within the UK. A possible explanation for this is that traditional 'hand' yield-line analysis methods can be extremely tedious and, without some form of computer program to facilitate the analysis, they are impractical to apply to anything but the simplest slab geometry, reinforcement, loading and failure mode configurations. In addition, as an upper-bound technique, there is always a degree of uncertainty that the critical failure mode has been found. The usual concern has been that one has to laboriously check a number of possible failure mechanisms for many different loadcases and even then other more intricate geometries might be possible. A further concern is whether or not the bridge has sufficient ductility to justify the assumptions inherent in yield-line theory [1]. Incorporating a ductility check is somewhat difficult using conventional yield-line methods.

Although good predictions of ultimate shear strength can be obtained using plasticity theory for reinforced concrete members that contain sufficient shear reinforcement to ensure ductile behaviour, further research is required to validate its use for the assessment of shear strength in general practice [5]. As a result, shear capacity must still be checked using the conventional elastic approach. Fortunately shear tends not to be the primary cause of failure in most concrete slab bridges and thus yield-line analysis is well-suited to structures that have been found to be adequate in shear but have failed in flexure.

Although yield-line analysis is an upper-bound method, there is a wealth of experience available from the literature on the types of failure mechanisms likely to form under typical highway loading configurations [2]. In addition, significant reserves of strength are found in many bridges resulting from compressive membrane or "arching" action and, to a lesser extent, work hardening of the reinforcing steel. This evidence supports the view that, for appropriate types of concrete bridge decks, the method can be applied with confidence, provided the limitations of the technique are well understood by the assessing engineer.

The Highways Agency in the U.K. recognised the potential for applying yield-line analysis to concrete bridge assessment. Whereas elastic analysis programs are widely available, it was found that there were no generally available yield line analysis programs in widespread use anywhere in the world. The few programs referred to in the literature tend to be very specific and restricted in their applicability to quite simple structural configurations. As a result, the Highways Agency commissioned the Department of Engineering at Cambridge University to develop such a program. This project resulted in a novel collapse analysis program called COBRAS (for COncrete BRidge ASsessment), which was validated against model tests, theoretical solutions, and the Transport Research Laboratory's NLFE research program. The development of this yield-line program has provided a very powerful tool with which plastic collapse analyses of concrete bridges can be undertaken for a wide selection of possible failure modes and assessment loadcases.

3 GENERALISED ANALYSIS METHOD FOR YIELD LINE ANALYSIS

The breakthrough that allows a generalised yield line solution scheme to be computerised is the realisation that the yield-line problem can be reduced to what is fundamentally a problem of geometry. Although, with the benefit of hindsight, this might appear trivial it must be recognised that no general solution scheme has previously been developed. Using developments in *computer graphics* and *solid modelling* theory, an analysis technique has been developed which creates a three-dimensional 'picture' of the bridge. This is used to derive all the required geometrical relationships for the failure mechanisms, whilst incorporating features describing the component material properties and the applied loads.

Perhaps the most significant feature of this modelling technique is its ability to analyse rigorously realistic configurations of loading, bridge geometry, support fixity and failure mechanisms without the need to derive mathematical expressions describing the inter-relationship between these parameters. Multi-layered, banded and curtailed reinforcement layers can be included. It is also possible to make some provision for the effects of steel corrosion and concrete deterioration.

The analyst selects from a pre-defined library of failure mechanisms, as shown schematically in Figure 2, and the program then iterates through a large number of possible geometries for each mechanism in search of the lowest, and hence critical, failure mode. As a result, structures that were hitherto impractical to assess by hand can now be analysed automatically. With a modern portable computer a typical concrete bridge assessment can be performed in a couple of minutes.

The new method revolves around performing the following five tasks:

1. Modelling the bridge and its structural components
2. Modelling the applied live loads
3. Modelling the failure mechanisms
4. Optimising the failure mode geometry
5. Calculating the ultimate strength and factor of safety

3.1 Modelling the bridge and its structural components

The fundamental parameters governing the collapse behaviour of a concrete bridge are the geometry in plan, the support fixity, the cross-sectional dimensions, the concrete strength and density, the details of the various layers of steel reinforcement and the applied loading. The modelling method developed permits each of these features to be separately specified and then merged together using computer graphics solid modelling techniques to form a single "bridge structure model". For example, a layer of reinforcing steel can be defined by the outline plan of the bars and also properties such as area of steel (per metre width), yield strength, effective depth, and orientation in plan etc.

Many other parameters can also be represented in this same way. For example parameters such as *strength reduction factors*, to allow for deterioration in the concrete and/or steel, and *membrane strength enhancement factors*, can be defined and merged with the bridge model.

This process of combining all the components together uses principles from set-theory, and the actual merging of component parts is performed using a generalised 3D solid modelling package, specifically written for this purpose. The final model represents the entire bridge and incorporates all the required analysis parameters of material components and geometry.

3.2 Building the applied load models

Complex loading combinations allowing, for example, for lane loads, vehicle or individual wheel loads or line (knife-edge) loads can also be represented in terms of regions on which a given intensity of load acts. Since the magnitude and position of applied live loading is independent of the structural components of the bridge, the various loadcases to be assessed are "assembled" in the computer in the specified location on top of the structure model of the bridge deck. However they are not combined with the bridge model. In this way a separate, independent graphical representation of each load case is stored in the computer and allows the engineer to represent any desired combination or complexity of loading on the structure.

3.3 Modelling the failure mechanisms

The generalised analysis method generates 3D *polyhedral failure models*, which are 3D “pictures” of each of the chosen yield-line failure mechanisms, and stores these in the computer. These solid failure models provide all the required geometric information needed for the work calculation in the yield line analysis.

One of the major strengths of this approach is that the failure modes are described totally independently of the loading and the structural properties of the bridge, depending only on the shape of the bridge perimeter taken from the boundary representing the plan of the bridge deck.

By incorporating an extensive library of pre-defined yield-line patterns within the program the user can easily choose a selection of collapse modes for assessment. Figure 2 shows the library of standard failure modes currently incorporated into the COBRAS package. This includes a selection of some of the most commonly reported failure modes for bridge slabs and also some complex fan mechanisms. The library can easily be extended to any practical failure mechanism geometry if required.

By merging the three models representing the *structure*, the *loading* and the *failure mechanism*, a single 3D solid model or “picture” of the entire bridge in its collapsed state is produced and stored in a data structure within the computer. This merging of the three component models to form a *solid bridge model* is accomplished using a solid modelling package developed specifically for this purpose.

Contained within this new solid bridge model are full details of all the information necessary to describe the structural parameters of the material components and dimensions of the bridge, the external loading acting on the bridge, and the required geometric information needed for the collapse mechanism analysis using the work method. This includes the location and length of all the yield lines, the details of abutment fixity at each of the boundaries and the relative rotations between adjacent rigid plate elements of the failure surface.

3.4 Optimisation of failure mode geometry

By changing the position of some of the vertices of the solid bridge model, a rapid “step-like” iteration of the failure mode geometry can be performed (Figure 1). By using this procedure for all the different failure mechanisms chosen from the library of failure modes, a search can be made for the critical global collapse mechanism with the lowest factor of safety.

The significant advantage of this approach lies in its speed and simplicity as it avoids the necessity to derive expressions for the work equation or undertake an often-difficult partial differentiation calculation to obtain an estimate of the critical failure mode geometry. With a computer, a large number of iterations can be examined quickly, thus ensuring that the critical geometry for the particular mode is found to within the accuracy dictated by the selected iteration step size.

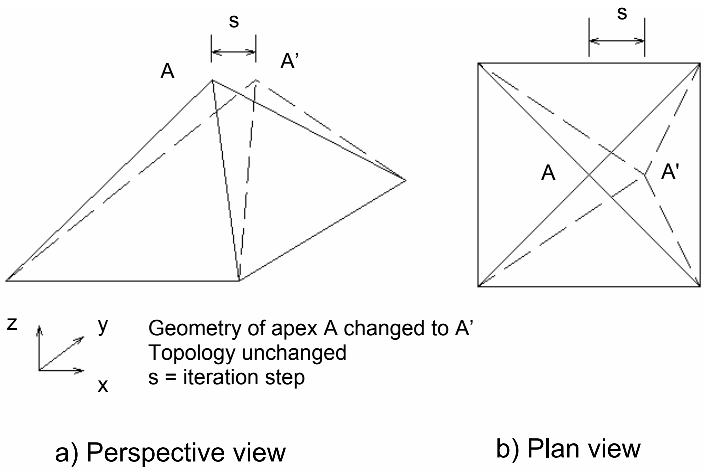


Fig. 1 Iteration of the failure mechanism geometry.

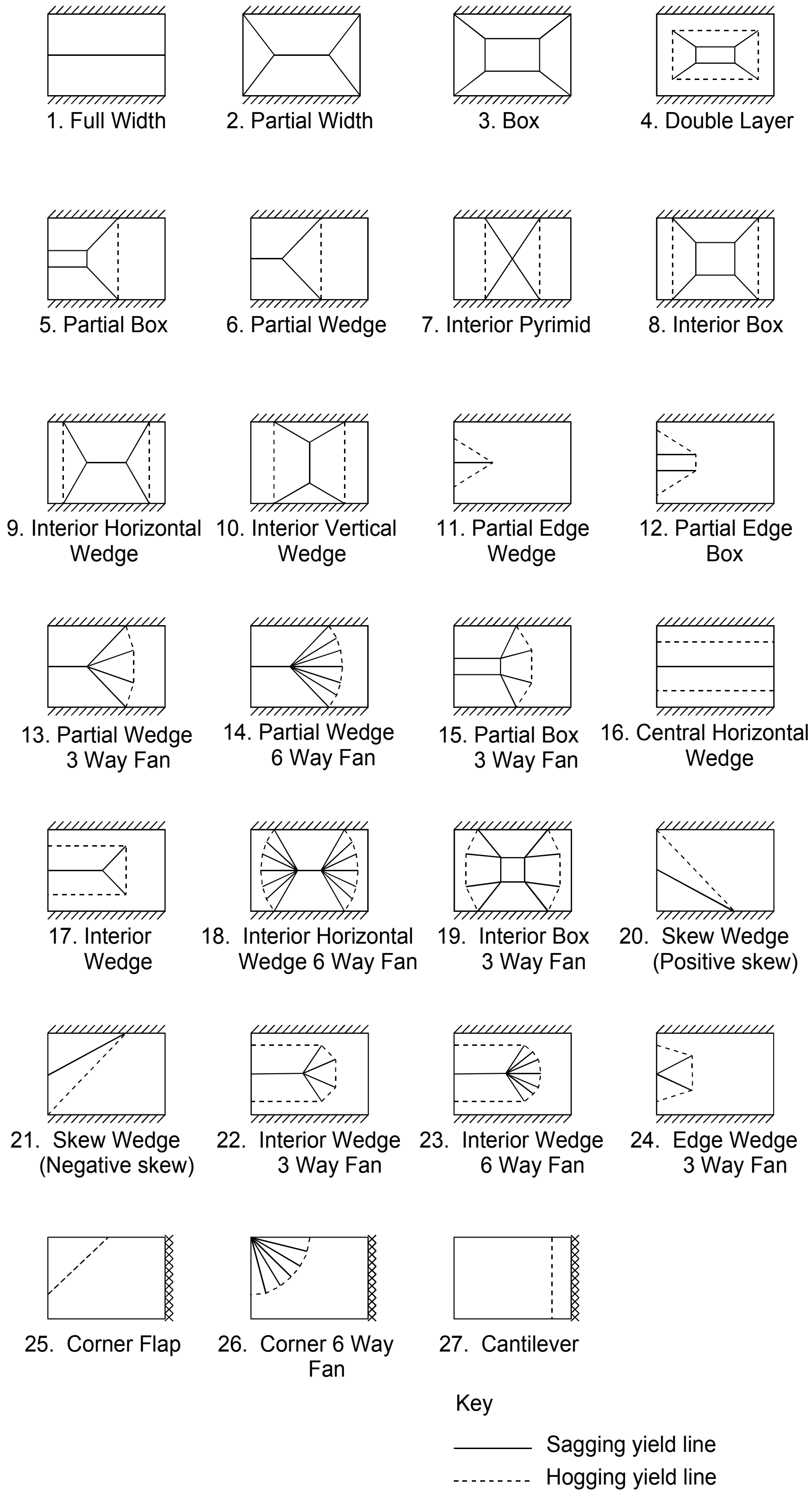


Fig. 2 Schematic diagram of failure mechanism library.

3.5 Calculating the ultimate strength and factor of safety

The load capacity of a bridge is determined using the yield-line method in which a global *factor of safety (F.O.S.)* is derived. Having applied a given assessment load to a postulated failure mechanism and derived a factor of safety, the parameters defining the failure mode geometry are varied to minimise the factor of safety and hence determine the load–capacity of the bridge. The factor of safety at collapse is given by the ratio of the calculated strength to the effects of the applied loading.

$$\textit{Factor of safety (F.O.S.)} = \frac{\textit{Strength}}{\textit{Load Effects}}$$

In the work method this equation can be expressed in terms of the energy dissipated in the yield lines or plastic zones (*ED*) and the work done by the applied loads and the self-weight of the structure (*WD*).

$$F.O.S. = \frac{WD}{ED}$$

where *ED* = energy dissipated in the yield lines
WD = work done by the loads (i.e. self-weight, superimposed dead load and live load)

An alternative measure of safety, commonly used in non-linear finite element analyses and bridge assessments, is the *live load factor* or λ-factor, which is defined as the ratio of the applied live load required to cause failure to the initially specified assessment live load, with the self-weight and superimposed dead load remaining constant.

$$\textit{Live Load Factor (LLF or } \lambda) = \frac{\textit{Failure Live Load}}{\textit{Assessment Live Load}}$$

In terms of the work method used here, the factor of safety at collapse is 1.0.

$$F.O.S = \frac{ED}{WD} = \frac{ED}{\lambda.WD_{LL} + WD_{SW} + WD_{SDL}} = 1.0$$

$$\lambda = \frac{ED - WD_{SW} - WD_{SDL}}{WD_{LL}}$$

where

WD_{SW} = work done by the self-weight of the bridge
WD_{SDL} = work done by the superimposed dead loads
WD_{LL} = work done by the specified live loading

These two measures of safety, *F.O.S.* and *λ*, will usually have different magnitudes. As the work done by the self-weight and superimposed dead load increases as a proportion of the total work, the difference between the two measures widens. This is likely to become more pronounced with larger span bridges where self-weight dominates the loading.

The default setting on the COBRAS package determines the ultimate capacity of the structure by minimising the factor of safety (*F.O.S.*), but there is an option for the user to undertake the optimisation by minimising the live load factor (λ-factor). In the program, the values of both the factor of safety and the load factor are determined. The critical failure mode found by minimising one of these factors will not necessarily be equivalent to that obtained by minimising the other.

4 OTHER FEATURES OF THE GENERALISED YIELD LINE ANALYSIS METHOD

4.1 Generalised moment capacity program

Because all the components of the bridge deck are incorporated in the bridge structure model, the rigorous “theoretical” moment capacity of the actual concrete section about the axis of the yield line, allowing for all the orientations, depths and types of reinforcement that cross the selected yield line, can be calculated. In contrast, when attempting such calculations by hand it is usual to simplify the analysis by adopting Johansen's *stepped yield criterion* or the *affinity theorems* to account for orthotropic reinforcement layouts. Such simplifications are not necessary in the computerised approach.

4.2 Ductility

Since most code measures of ductility, defined in terms of rotation capacity of a section, are related to the geometry and material components along the yield lines of the structure, each of which is fully defined in the solid bridge model, the rotation capacity at all yield-line sections can be checked directly. The method does not ensure ductility is available, however it does enable all the yield-line sections to be checked for compliance and a warning is given if any section does not satisfy user defined limits on neutral axis depths, and percentage of steel reinforcement.

The limits currently checked are:

i) The percentage of steel crossing all yield lines to warn of heavily reinforced sections:- A warning is given if the steel reinforcement percentage, $\rho \geq 3.5\%$.
ii) The neutral axis depth, x:- A warning is given if this exceeds 40% of the overall depth of the slab, D (i.e. If $x \geq 0.4 \times D$).

4.3 Geometric compatibility of failure mechanism

The 3D solid bridge model representation of the collapsed shape of the structure provides a simple method for checking *geometric compatibility*. For the solid shape to form a valid failure mechanism, all the faces of the solid must be planar. A routine incorporated in the program checks the planarity of all the faces of each of the chosen failure modes and prevents analysis of an incompatible mechanism. Thus one of the primary requirements of yield-line theory can be automatically checked.

4.4 Reinforcement corrosion and concrete deterioration

Allowance for deterioration in the steel reinforcement due to corrosion or reduction in the concrete strength of a bridge is allowed for by specifying regions of the deck to which *Steel Corrosion* and/or *Concrete Deterioration* are assigned to represent deteriorated elements. An affected region is defined by the geometric location of the deterioration and region and the magnitude of the deterioration factor appropriate to the materials in the region specified. For reinforcement, a factor on the area of steel is used to model the effects of corrosion. This reduces the cross-sectional area of the bars. Thus any reinforcement within the "corrosion affected zone" has the area of steel reduced by the *corrosion reduction factor* that must be selected by the assessor. In an identical manner, areas of concrete with reduced strength are identified and a *concrete deterioration factor* applied. This is recognized to be a simplistic approach to the problem of deterioration and makes no allowance for potential problems from loss of bond, de-lamination or spalling; however, it does provide a means by which some measure of the effects of deterioration on flexural strength can be made.

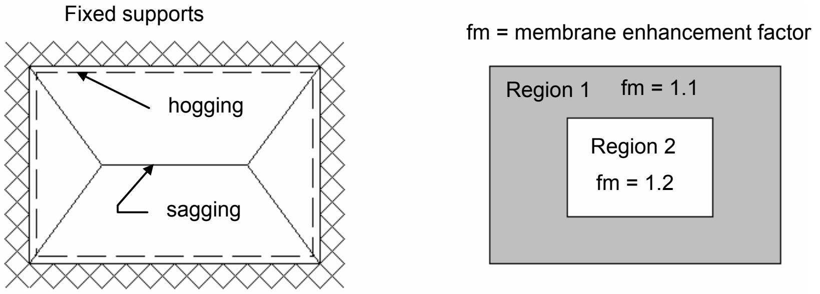
4.5 Membrane action in concrete bridges

It is well known that in many practical situations there is often some restraint to the lateral expansion of a slab deflecting under external loading. This restraint can be provided by edge beams, diaphragms or the supports to the slab and results in internal arching action within the depth of the slab which can significantly enhance both the flexural moment capacity of the structure as well as its resistance to punching shear under concentrated loading.

There are several publications which detail the many experimental and theoretical studies of compressive membrane action in concrete slabs and the various analytical models that have been proposed. The major difficulty faced by all researchers in this field has been finding a method for quantifying the amount of enhancement for all the possible variations of reinforcement percentages, span/depth ratios, edge fixity conditions and load configurations. Johansen's classical yield-line theory makes no allowance for membrane action. Consequently the method has often been found to significantly underestimate the load carrying capacity of restrained slabs. The COBRAS package does not attempt to derive membrane enhancement factors for any given bridge type; however, the program does provide a means by which the results of various theoretical studies can simply be included.

The method is based on the assumption that the degree of membrane strength enhancement can be represented in geometric terms by a region of the bridge deck, defined by a *membrane region*, in which there is a specified increase in moment capacity of the slab. This allows for variation in the amount of membrane enhancement in different areas of the deck or in zones with differing degrees of restraint (Figure 3). This method is based on the approach presented by Rankin et al. [10] for the case of restrained slabs subjected to uniformly distributed loads. The moment capacity of the slab within each membrane feature polygon is multiplied by the user specified membrane enhancement factor, f_m.

It is recognised that this is a rather simplistic approach to membrane action but until further research provides solutions to this problem, the above approach is one technique that allows some consideration of membrane enhancement to be included in an assessment.



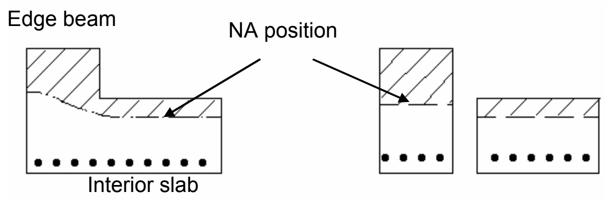
a) Restrained slab with uniformly distributed loading

b) Feature regions for membrane enhancement

Fig. 3 Example regions for membrane enhancement factor

4.6 T-Beam effect in beam-and-slab structures

The current analysis model adopts a simplified, approximation technique for calculating the flexural moment capacity of a bridge deck with different cross-sectional thicknesses such as with a beam-and-slab deck or slab deck with edge beams. In this approach, the moment capacity of each section of deck of different thickness is assessed separately. Thus a different neutral axis position will be derived for a section of slab adjacent to an edge beam element as shown in Figure 4.



a) Actual N.A. position

b) Discrete element N.A. position used by COBRAS

Fig.4 The T-beam effect

At present, only pure rotation about the yield-line axis is considered. This has been shown experimentally to produce good agreement with tests for a variety of slab and beam–and–slab models. However difficulties begin to arise when a yield line is found to cross a structure such as an M-beam (UK) or I-beam (US & Australia) deck at an acute angle to the longitudinal axis of the beam in which case it is unlikely that a long, flexural yield line will form along the beam itself. In such a situation it is far more probable that some local deviations in the yield-line pattern will occur and some form of stepped yield line incorporating a predominantly flexural failure of the beam (or else a flexural-shear or pure shear failure) or else some form of torsional hinge might form. Modelling such behaviour introduces a number of complexities that are not allowed for in the methodology developed here.

5 LIMITATIONS OF THE ANALYSIS METHOD

- The primary limitations on the new generalised collapse analysis method are those inherent in the fundamental assumptions of classical plasticity theory and yield-line theory. These include the assumption that the yield-line sections will display rigid plastic behaviour and that all the energy dissipated in the structure at collapse will be concentrated in localised plastic zones between undeforming, rigid plate elements.

- As an upper-bound method, there is always the possibility that other, more critical, failure mechanisms exist that are not considered by the program.
- The analysis method at this stage has only been implemented for flexural failure modes. Shear is not checked with this method and must be assessed separately.
- Beam-and-slab structures are currently assumed to act as independent elements in flexure and, although conservative, this could be improved by including effective flange widths into the analysis.
- Deep or heavily reinforced beams in a beam-and-slab bridge may not satisfy the ductility requirements of yield-line theory and must be considered with caution. The yield-line method was original developed for concrete slabs although there is much evidence to also support its use with beam-and-slab structures, provided the limitations of the method are understood. The program incorporates a facility for identifying sections with potential ductility problems. Any section with high reinforcement percentages or a neutral axis location greater than user-defined limits will be identified with a warning message during execution of the program.
- Provision has been made within the program for including a membrane enhancement component, although the difficulty in selecting appropriate values still remains.

6 VALIDATION OF THE COBRAS YIELD LINE ANALYSIS PROGRAM

To confirm the validity of the program, a number of forms of calibration were undertaken. Firstly, the program was checked against a number of published analytical solutions which confirmed the correct theoretical result was obtained in each case.

Secondly, a calibration study was undertaken in collaboration with the Transport Research Laboratory's (TRL) Structural Analysis Unit to compare predictions for collapse load and failure mode geometry obtained using the COBRAS program with those obtained by the TRL's non-linear finite element program, NFES, for a number of different bridge structures under a variety of load configurations.

In this study with TRL excellent agreement between the collapse load and failure mode geometry predicted by COBRAS and NFES was obtained for all the bridges analysed. The difference in predicted ultimate capacity was less than 4% in all the examples assessed except for four specific cases where a conservative assumption about the strength of edge-beams in the COBRAS program resulted in a maximum of 13% underestimation of the collapse load.

Thirdly, the program was used to predict the failure mode and collapse load for a number of experimental tests on concrete slabs. Although there have been numerous tests over the years to verify yield-line theory, a series of tests was conducted at Cambridge University specifically aimed at validating the theoretical predictions of collapse load obtained using the COBRAS program. To date, a total of 13 different tests have been carried out as part of an ongoing validation programme. The model slabs were scaled at approximately 1/10th the size of a full-scale bridge in Scotland that had been tested to destruction by the TRL in 1992. This resulted in the model slabs being nominally 600 mm in length by 1000 mm wide and 40 mm thick. In one set of tests the slabs were skewed at 30 degrees, and in another the slabs were widened to 1500 mm to examine failure mechanisms contained wholly within the central region of the slab. Various reinforcement configurations were considered, with and without transverse and top steel, and with varying percentages of each.

Truck loading was simulated using a two-wheel axle load which was applied at mid-span in all but one of the 13 tests. In the exception a solitary point load was used. The goal was to force the model structures to fail in some form of complex fan mechanism rather than just a full-width transverse yield-line at mid-span (which is often found to be critical under the uniformly distributed HA lane load pattern that is specified in the UK assessment code). Such a fan mechanism puts a greater demand on the ductility of the slab as well as on the predictive capabilities of the computer program.

6.1 Results from the Cambridge model tests

The results from these model tests are shown in Table 1, which compares the failure loads predicted using the COBRAS plastic collapse program (P_{COBRAS}) and the actual failure loads measured in the laboratory (P_{test}).

In Table 1 it can be seen that in all but the final two tests (A1 & A2), the yield-line method was conservative in predicting the capacity of the model slabs. The mean value of the ratio of the measured failure load to the predicted load was 1.13, with standard deviation 0.13. Values ranged from 0.87 in test A2 to 1.33 in test C2, with the range being between 1.04 and 1.33 for the first 11 tests. By way of example, Figures 6(a) and 6(d) show the failure mechanism patterns obtained in two of these tests (K4 & C3) and Figures 6(b) and 6(c) show the corresponding critical yield-line pattern predicted using COBRAS overlaid on the observed soffit crack pattern. (Slab K4 was tested twice – once with a single axle load at each side of the slab).

Examination of the specimens in tests A1 and A2 after failure suggested that a breakdown in the bond between the concrete and the smooth, shiny 4 mm diameter bars used to reinforce the slab may have caused these two lower than expected results. Further tests subsequently confirmed this hypothesis which emphasises the importance of bond in the collapse behaviour of reinforced concrete structures.

Table 1 Predicted versus actual ultimate loads for model bridge slabs.

	Hudson tests				Kite tests				Collins tests			Antill tests	
Test No.	**H1**	**H2**	**H3**	**H4**	**K1**	**K2**	**K3**	**K4**	**C1**	**C2**	**C3**	**A1**	**A2**
P_{test} **(kN)**	29.2	28.3	27.9	19.0	23.4	23.5	24.1	22.7	36.8	37.2	31.3	18.0	25.6
P_{COBRAS} **(kN)**	24.5	25.3	23.0	17.2	22.5	22.3	20.5	17.7	N/A	28.5	25.7	19.0	29.3
$\frac{P_{test}}{P_{COBRAS}}$	1.19	1.12	1.21	1.10	1.04	1.05	1.18	1.28	-	1.33	1.22	0.95	0.87

An important observation in all these experimental tests in the laboratory was that substantial deformation and cracking developed well before the maximum load capacity was reached. Thus if a structure has been in service for many years and there is no visual evidence of distress the assessing engineer can be reasonably confident that the structure is capable of sustaining significantly higher loads than those *already* experienced by the structure. Clearly this does not mean the structure is necessarily capable of sustaining the full 40 tonne load, as it may never have been subjected to loads near the maximum legal limit. However it does give some reassurance to the assessor that collapse is not imminent at the loads to which the structure has already been subjected.

Fundamental to this statement is the assumption that the critical failure mode will be *flexural* and the structure is sufficiently *ductile* to allow such a mechanism to form. Shear failures may occur in a brittle manner and may not give warning of impending failure (although an experimental programme at Cambridge on shear in beam and slab bridges has indicated that significant cracking usually precedes shear failure at loads well below the ultimate collapse load in most (but not all) cases [5].)

As a result of the development of this yield line analysis method and program to implement this approach, yield-line analysis is fast becoming one of the most commonly adopted alternative methods of analysis in the UK for evaluating the ultimate strength of concrete bridge decks found to be inadequate in flexure. The program has been employed by over 40 bridge authorities or consultants to re-assess dozens of bridges that had been “failed” using conventional analysis.

The effectiveness of this approach was demonstrated in a survey of 35 bridges which had failed their original assessment and were then re-assessed using this yield-line program. 28 (80%) were found to upgraded to the full assessment load capacity of 40 tonnes, 3 (9%) were upgraded to 38 tonnes and 4 (11%) remained at, or were slightly upgraded to 16 tonnes. These results are shown graphically in Figure 5. It is evident that this approach can result in very substantial savings to bridge owning authorities if applied in the appropriate circumstances to short-and-medium span concrete bridge decks.

7 CONCLUSIONS

There is clearly an economic imperative to refine and extend the current methods of analysis used for assessing the load carrying capacity of existing short-to-medium span bridges with hundreds of millions of pounds at stake. There are a number of ways in which bridge engineers could use more advanced analysis methods to more realistically predict bridge behaviour. The first would be to more widely accept the definition of failure at the ultimate limit state as “collapse of part or all of a structure” rather than first yield of an individual component. Secondly, there is significant scope for much greater utilisation of the lower bound theorem to manipulate element stiffness properties and hence optimise the distribution of stress resultants in a structure such that an improved estimate of load capacity is obtained. Thirdly, far wider use of inelastic and plastic methods of analysis, and in particular yield-line analysis, would result in far more realistic predictions of the load capacity of our bridges.

It would seem that the potential benefits of adopting more advanced computational tools for bridge assessment are widely recognised and acknowledged by practitioners but there has nevertheless been a reluctance to implement them.

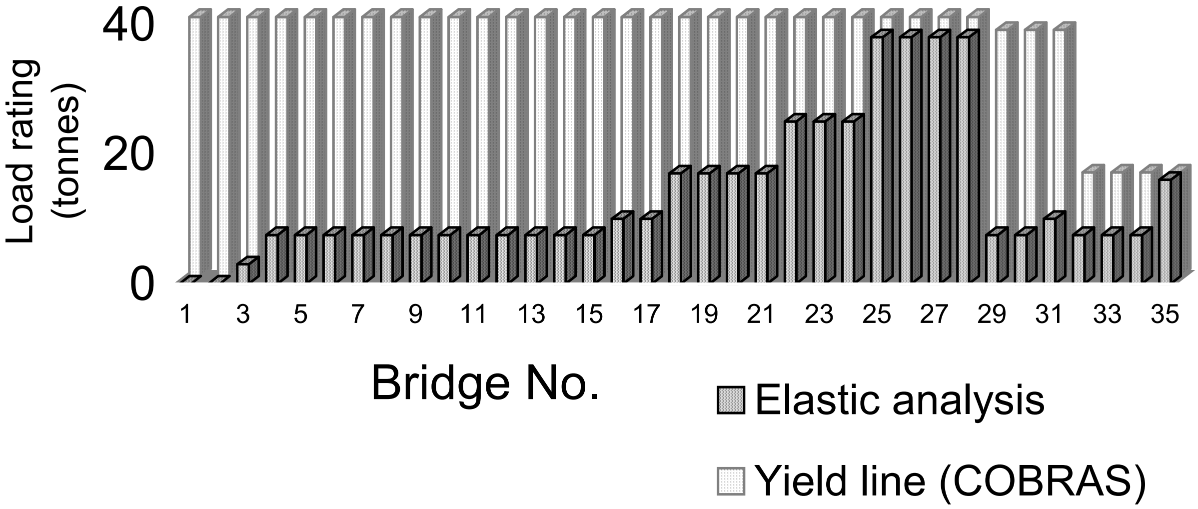


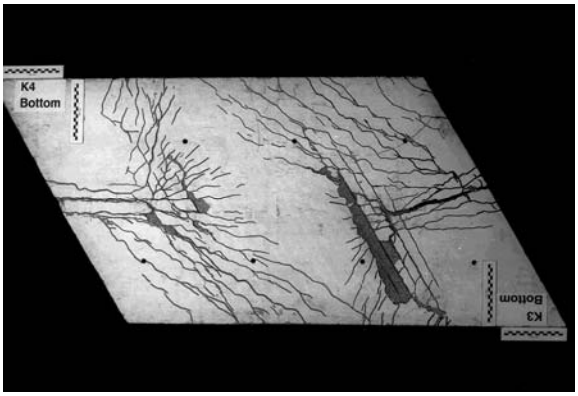
Fig. 5 Comparison of bridge load ratings obtained using elastic and yield-line analysis.

Overall, the use of elastic analysis methods for assessing the ultimate load capacity of concrete bridges may in many situations result in a significant under-estimate of strength. The development of the COBRAS yield-line program provides a very powerful alternative tool with which plastic collapse analyses of these bridges can be undertaken for a wide selection of possible failure modes and assessment loadcases. As an upper-bound approach, care must be used in applying this technique however there is substantial theoretical and experimental evidence to support its validity for concrete bridge decks in which sufficient ductility exists to justify the assumptions inherent in yield-line theory.

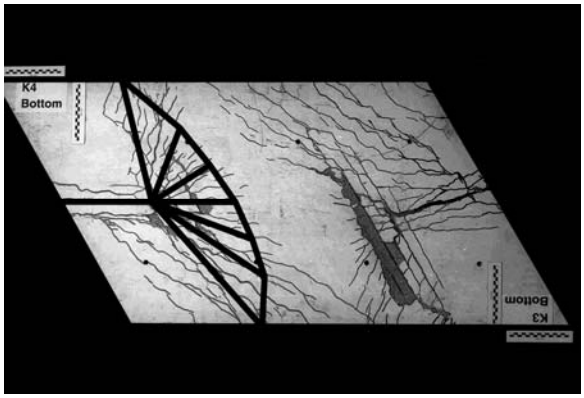
There are enormous opportunities for those in the profession willing to employ more advanced methods of analysis to improve the modelling of the complex interaction between the applied live loads and the bridges which are required to support them.

ACKNOWLEDGEMENTS

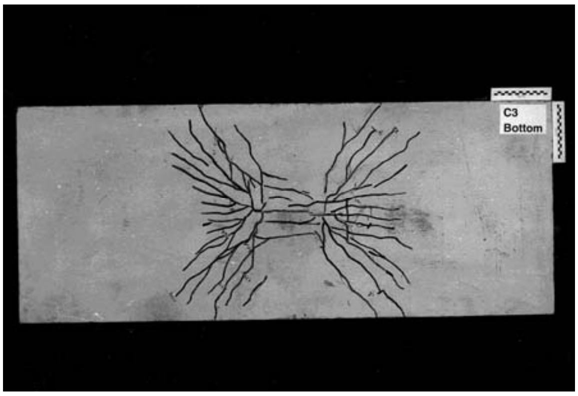
The support of the Highways Agency and the Transport Research Laboratory in the development of the COBRAS analysis program described here is gratefully acknowledged. The assistance of Dr C.T.Morley, Mr P.Fidler and Mr T.Love of Cambridge University, Dr G.Tilly and Mr J.Wills of TRL (both now retired) and Mr B.Sadka and Mr S.Chakrabarti of the Highways Agency is also gratefully acknowledged. Any views expressed are not necessarily those of the supporting organisations.



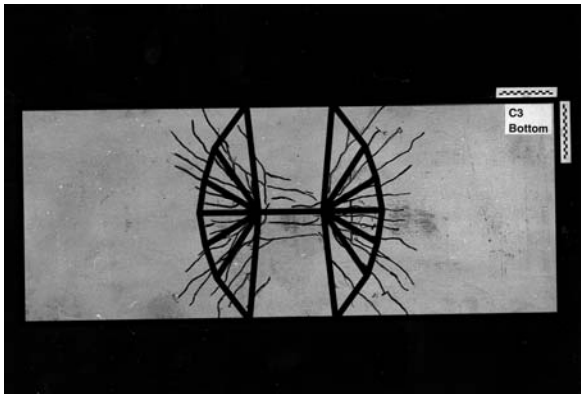
(a) Test slab K4 – actual crack pattern



(b) Test slab K4 – predicted yield-line pattern



(c) Test slab C3 – actual crack pattern



(d) Test slab C3 – predicted yield-line pattern

Fig. 6 Examples of actual & predicted failure modes for concrete bridge models

REFERENCES

[1] Beeby,A.W., "Ductility in reinforced concrete: why is it needed and how is it achieved?", *The Structural Engineer,* **75**, No.18, September 1997, pp311-318.

[2] Clark,L.A., "Collapse Analysis of Short-to-Medium Span Concrete Bridges", *Contractor's Report* CRR 528/577/124*,* Transport and Road Research Laboratory, Crowthorne, 1984.

[3] Collins,M.P., Mitchell,D., *Prestressed concrete structures,* Prentice Hall, New Jersey, 1991.

[4] Cope,R.J.(ed.), *Concrete bridge engineering: performances and advances*, Elsevier Applied Science, London, 1987.

[5] Ibell,T.J., Morley,C.T., and Middleton,C.R., "A plasticity approach to the assessment of shear in concrete beam-and-slab bridges", *The Structural Engineer,* **75**, No.19, October 1997, pp331-338.

[6] Kotsovos,M.D., Pavlovic,M.N., *Structural concrete. Finite element analysis for limit state design,* Thomas Telford, London, 1995.

[7] Melbourne,C. (ed.), *Arch bridges*, Thomas Telford, London, 1995, pp693.

[8] Page,J.: "Masonry arch bridges". *TRL state of the art review,* HMSO, London, 1993.

[9] Parsons Brinckerhoff, "A review of bridge assessment failures on the motorway and trunk road network", *HA Contract 2/419 - Technical Audit of the Application of BA49*, December 2003.

[10] Rankin,G., Niblock,R., Skates,A., and Long,A., "Compressive membrane action enhancement in uniformly loaded, laterally restrained slabs". *The Structural Engineer*, Vol.69, No.16, August 1991,pp.287-295.

[11] Wills,J., Crisfield,M.A., "No-tension finite-element and mechanism analyses of a half-scale beam-and-slab bridge deck", *Research Report* 217, Transport and Road Research Laboratory, Crowthorne, 1989.